Scalar-field approach of IE-MEI Method for the Three-dimensional Scattering Problem

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IEEE AP-S 2002 (covering: Objectives of the Research; Overview of the Scalar-field approach of IE-MEI (SIE-MEI) Method; Formulation of the SIE-MEI Method; Numerical Implementation)

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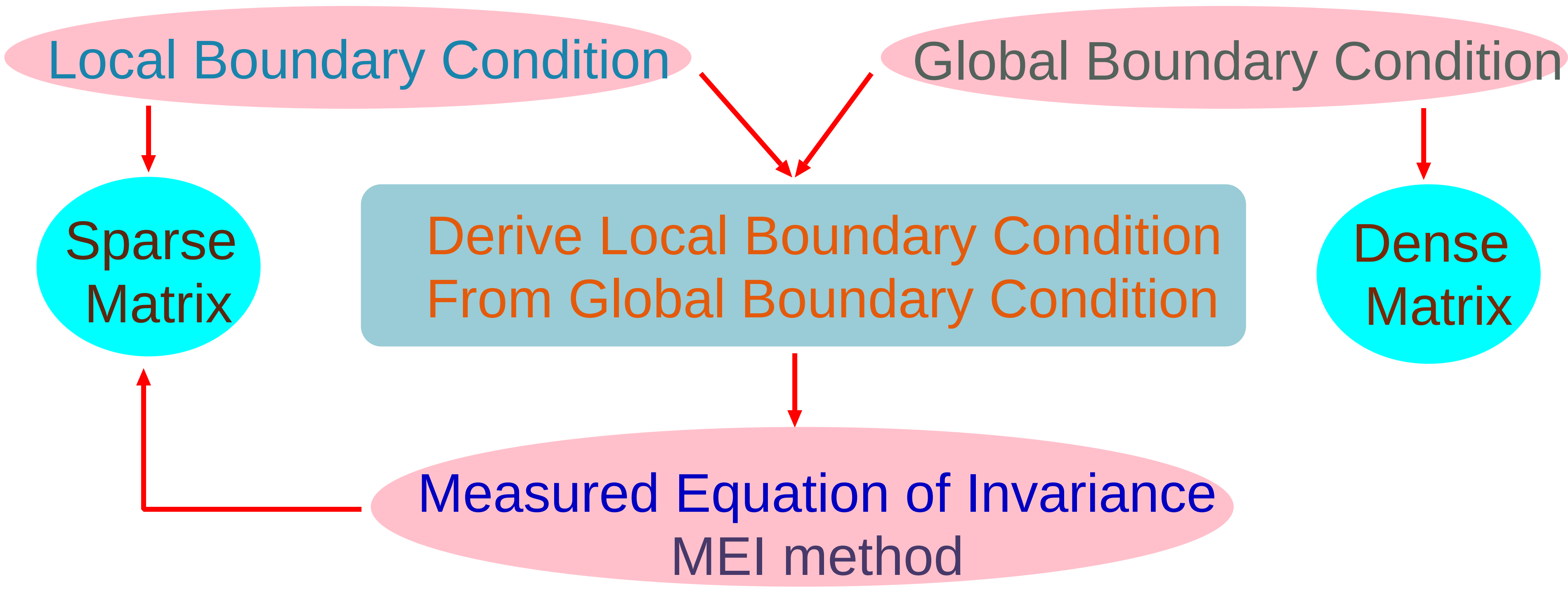
MEI Method

(Mei *et al.*, 1994, *Ref.* [1])



By means of Green’s function in the boundary integral equations, the measured equation include both radiation and evanescent fields.

MEI Method + FD Method

According to the MEI method, boundary nodes are satisfies the local linear equation which are :

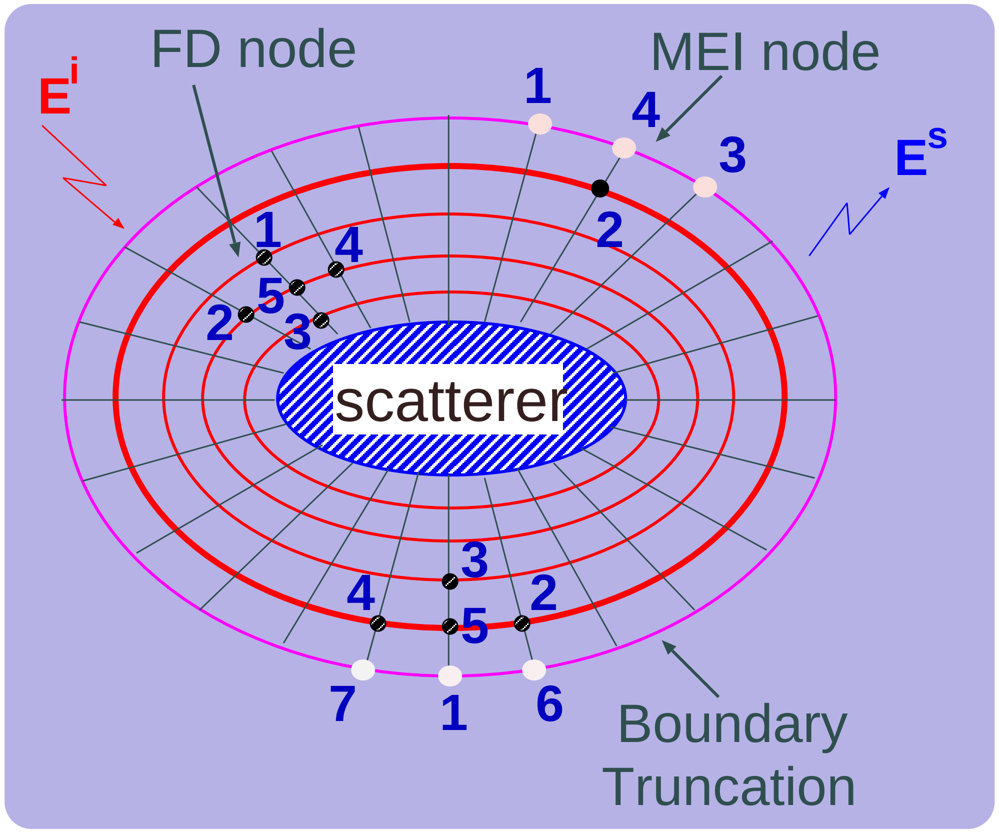
- location dependent,
- geometry specific, and
- invariant to field excitation.

MEI postulates

$$\sum_{i=1}^{4} C_i \phi_i = 0$$

scattered field

MEI coefficients



FD = Finite Difference

Solution Technique :

One of the coefficients is always chosen arbitrarily.
Local equation is valid for possible set of current distributions.
metrons

For $C_4=1$ with Q metrons,

$$C_1\phi_{11} + C_2\phi_{12} + C_3\phi_{13} = -\phi_{14}$$

$$C_1\phi_{21} + C_2\phi_{22} + C_3\phi_{23} = -\phi_{24}$$

$$\cdots \quad \cdots \quad \cdots \quad \cdots$$

$$C_1\phi_{Q1} + C_2\phi_{Q2} + C_3\phi_{Q3} = -\phi_{Q4}$$

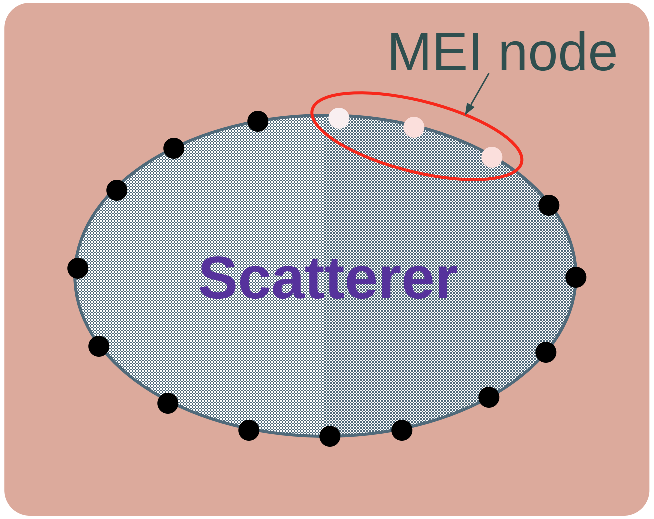
IE-MEI Method

(Rius *et al.*,1996, *Ref.* [2] & Hirose *et al.*,1999)

Surface IE derived from reciprocity relation.
On surface MEI postulates.
Sparse matrix with same number of unknowns as BEM.

Savings in computational time, and memory needs.

Suitable for arbitary 2D boundaries, but not efficient for 3D boundaries (*Ref.* [3]).



BEM = Boundary Element method

Motivation of the Research

- IE-MEI method has the same number of unknowns as BEM with matrix sparsity, which reduces computational time and memory needs.
- IE-MEI method is suitable for 2D boundaries but not efficient for 3D problem.
- In 3D, choice of suitable metron set, mesh generation, set of adjacent nodes, etc., are not established yet.

To approach these problems, we introduce Scalar-field IE-MEI for the 3D boundaries (*Ref.* [4]).

Objective of the Research (1)

- Derive the formulation for Scalar-field approach of IE-MEI (SIE-MEI) method.
- Implement this method to the 3D uniform shape and arbitrary shape scalar-field scattering problem.
- Compare the results with the available solutions.

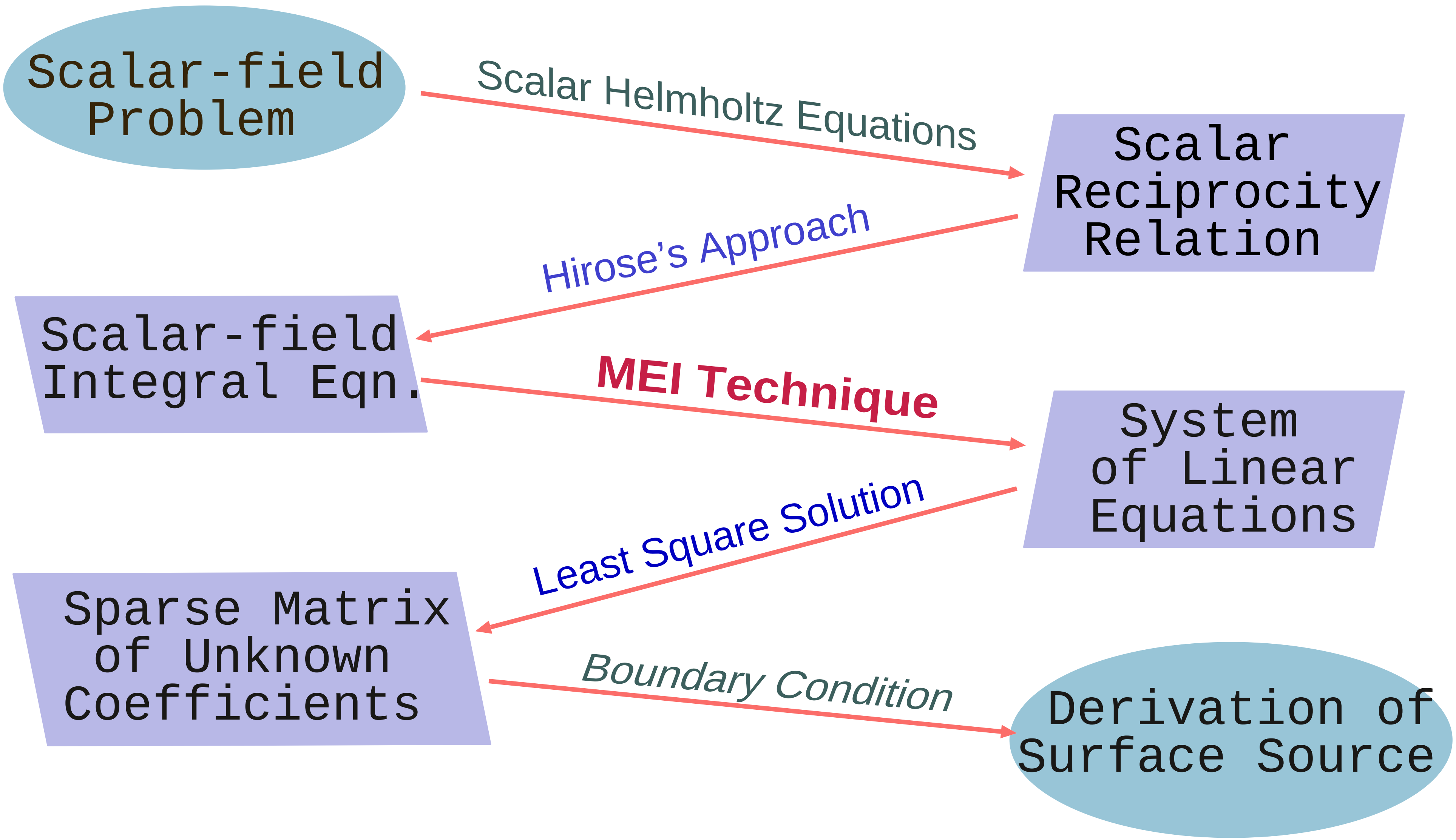
Obtain the suitable metron set and computational technique to implement SIE-MEI method in 3D problem efficiently.

Objective of the Research (2)

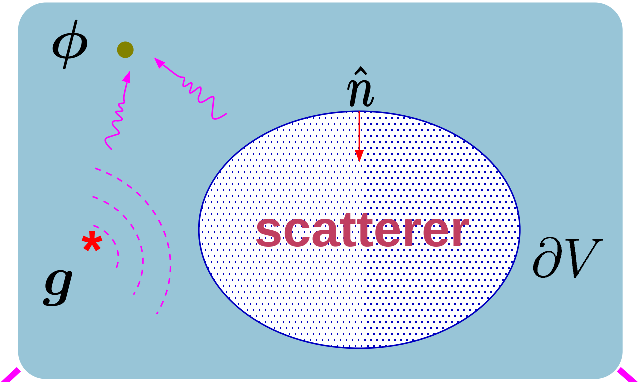
- Derive the Matrix Localization (ML) technique to reduce the computational time.
- Implement the insensitive properties of MEI coefficients for the scattering computation of modified scatterer with minimum CPU time.
- Compare the CPU time & memory requirements with the available solutions.

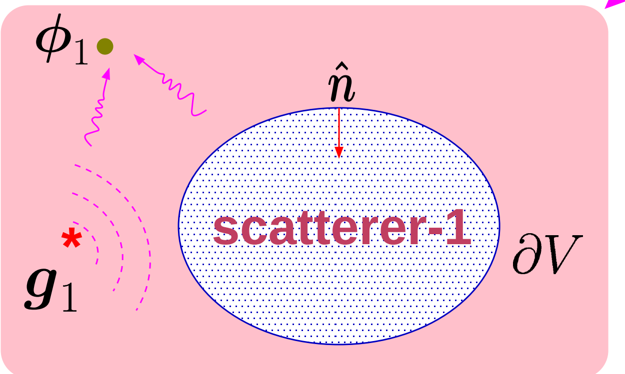
Additional techniques are implemented to enhance the applicability of SIE-MEI method (*Ref.* [5]).

SIE-MEI Method



Scalar-field problem :

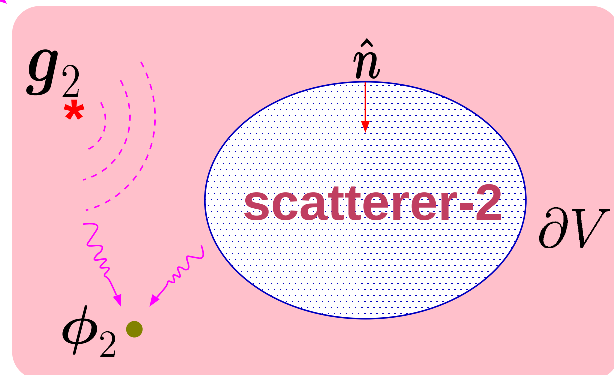




Scalar Helmholtz Equation

$$\nabla^2 \phi_1(\boldsymbol{r}) + k^2 \phi_1(\boldsymbol{r}) = -g_1(\boldsymbol{r})$$

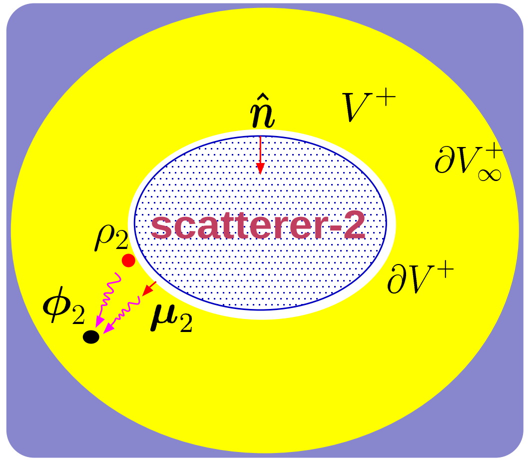
$$\nabla^2 \phi_2(\boldsymbol{r}) + k^2 \phi_2(\boldsymbol{r}) = -g_2(\boldsymbol{r})$$



$$\int_V (\phi_2(\boldsymbol{r}) g_1(\boldsymbol{r}) - \phi_1(\boldsymbol{r}) g_2(\boldsymbol{r})) dV = \oint_{\partial V} \left(\phi_1(\boldsymbol{r}) \frac{\partial \phi_2(\boldsymbol{r})}{\partial n} - \phi_2(\boldsymbol{r}) \frac{\partial \phi_1(\boldsymbol{r})}{\partial n} \right) dS$$

Scalar reciprocity relation

Integral Equation (IE) Formulation:



$$\oint_{\partial V^+} \left(\phi_1(\boldsymbol{r}) \frac{\partial \phi_2(\boldsymbol{r})}{\partial n} - \phi_2(\boldsymbol{r}) \frac{\partial \phi_1(\boldsymbol{r})}{\partial n} \right) dS = -\int_{V^+} \phi_1(\boldsymbol{r}) g_2(\boldsymbol{r}) dV$$

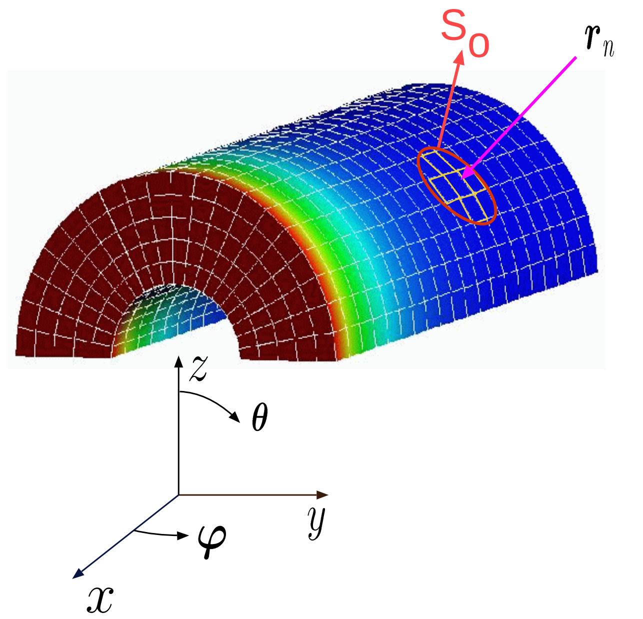
$\partial V^+ \rightarrow \partial V$

$$\oint_{\partial V} \left(\phi_1(\boldsymbol{r}) \tilde{\rho}_2(\boldsymbol{r}) - \frac{\partial \phi_1(\boldsymbol{r})}{\partial n} \tilde{\boldsymbol{\mu}}_2(\boldsymbol{r}) \cdot \hat{\boldsymbol{n}} \right) dS = 0$$

scattered field

equivalent local sources near the scatterer

Where, $\tilde{\rho}_2(\boldsymbol{r}) = g_2 \Delta w + \frac{\partial \phi_2(\boldsymbol{r})}{\partial n}$ and $\tilde{\boldsymbol{\mu}}_2(\boldsymbol{r}) \cdot \hat{\boldsymbol{n}} = \phi_2(\boldsymbol{r})$



According to the MEI technique, localize the IE which satisfies the MEI postulates

$$\int_{S_o} \left(\phi_1(\boldsymbol{r})\tilde{\rho}_2(\boldsymbol{r}) - \frac{\partial \phi_1(\boldsymbol{r})}{\partial n} \tilde{\boldsymbol{\mu}}_2(\boldsymbol{r}) \cdot \hat{\boldsymbol{n}} \right) dS = 0$$

depends on scatterer geometry,
depends on position, and
is invariant to excitation field.

Discretize the surface and expand the local sources around $\boldsymbol{r}_n$

unknown invariant local sources

$$\sum_{m \in R_n} \left[\phi_1(\boldsymbol{r}_m)\tilde{\rho}_{2,n}(\boldsymbol{r}_m) - \frac{\partial \phi_1(\boldsymbol{r}_m)}{\partial n} \tilde{\boldsymbol{\mu}}_{2,n}(\boldsymbol{r}_m) \cdot \hat{\boldsymbol{n}} \right] = 0$$

$$R_n = \{m_{1,n}, m_{2,n} \cdots, m_{M,n}\}$$
$$n = 1, 2, \cdots, N$$

$$\phi_{1,q}(\boldsymbol{r}_m) = \int_s \rho_q(\boldsymbol{r}') G(\boldsymbol{r_m}, \boldsymbol{r}') \, dS'$$

called Metrons
$q = 1, 2, \cdots, Q$

In matrix form $[\mathbf{C}\ \mathbf{D}]\begin{bmatrix}\mathbf{a}\\ \mathbf{b}\end{bmatrix} = 0$

column vector of $2M$ unknown invariant local sources

$[Q \times 2M]$ matrix of metron fields and their normal derivatives

Local matrix around $\boldsymbol{r}_n$ may be underdetermined (if $Q < 2M$) or overdetermined (if $Q > 2M$) system of linear equations which can be solved with least square solution using SVD.

SVD of local matrix : $[\mathrm{C}\ \mathrm{D}] = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$

V_{min} is the solution of the least square problem, to get the coefficients for particular node, which are $a_1,\ a_2,\ \cdots\ ,a_M$ and $b_1,\ b_2,\ \cdots\ ,b_M$

Repeat the procedure for each nodal point n (= 1, ..., N) and get the sparse matrices **A** and **B** with M nonzero elements in each row.

Scattered Field for Soft Surface:

$$\phi^{\text{inc}}(\boldsymbol{r}_s) + \phi^{\text{sc}}(\boldsymbol{r}_s) = 0 \qquad \boldsymbol{r}_s \in \partial V$$

Boundary Condition

$$[\mathbf{A}][\boldsymbol{\phi}^{\text{inc}}] + [\mathbf{B}]\left[\frac{\partial \boldsymbol{\phi}^{\text{sc}}}{\partial n}\right] = 0$$

Boundary Condition in Matrix Equation

$$\frac{\partial \phi(\boldsymbol{r}_n)}{\partial n} = \frac{\partial \phi^{\text{inc}}(\boldsymbol{r}_n)}{\partial n} + \frac{\partial \phi^{\text{sc}}(\boldsymbol{r}_n)}{\partial n}$$

Equivalent Surface Source

$$= \left[\frac{\partial \boldsymbol{\phi}^{\text{inc}}(\boldsymbol{r}_n)}{\partial n}\right] - [\mathbf{B}]^{-1}[\mathbf{A}][\boldsymbol{\phi}^{\text{inc}}]$$

$$\phi^{\text{sc}}(\boldsymbol{r}) = \sum_{n=1}^{N} \frac{\partial \phi(\boldsymbol{r}_n)}{\partial n} G(\boldsymbol{r}, \boldsymbol{r}_n)\, \Delta s$$

Derivation for Scattered Field

Numerical Implementation

Incident field, $\phi^{\mathrm{inc}}(\boldsymbol{r}) = e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}$

Scattered field, $\phi^{\mathrm{sc}}(\boldsymbol{r}) = \int_s \rho_q(\boldsymbol{r}')\, G(\boldsymbol{r}, \boldsymbol{r}')\, ds'$

Spherical wave function as metron set,

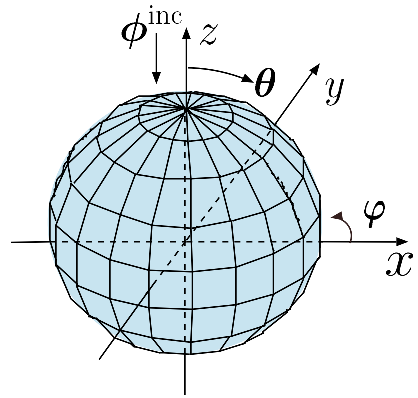
$$\rho(r, \theta, \varphi) = \sum_{n=0}^{\infty} h_n^{(2)}(kr) \sum_{m=-n}^{n} P_n^{|m|}(\cos\theta) e^{jm\varphi}$$

Segment length $= \dfrac{1}{10}\lambda$ (wavelength)

Use rectangular patch discretization.

Example : Sphere (1)

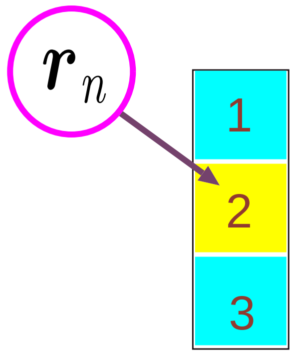
(2D Body of Revolution)



Incident field, $\phi^{\text{inc}}(\boldsymbol{r}) = e^{jka\cos\theta}$

Zonal harmonics as metrons,

$$\rho_q = P_q(\cos\theta), \qquad q = 1, 2, \cdots, Q$$



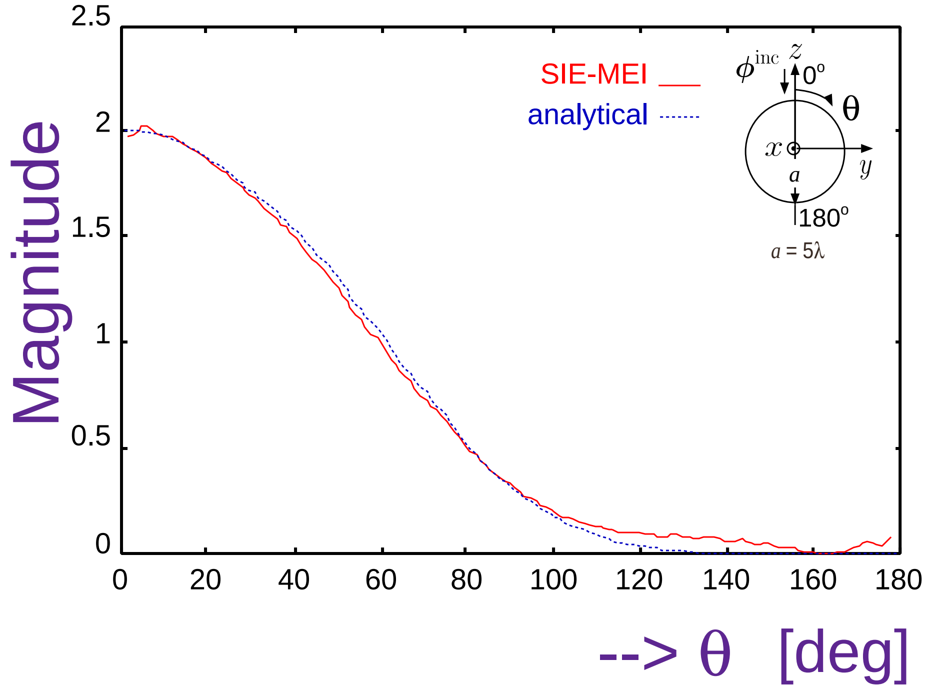
segments in
the local region

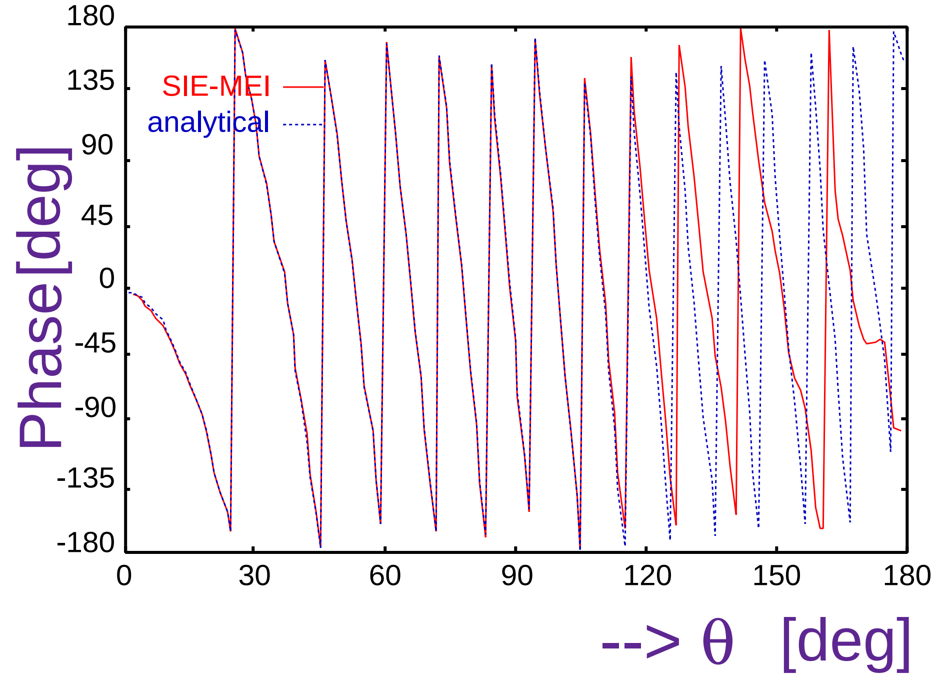
Local sources expanded into M = 3

Measuring function varies only along the polar direction, θ

Results (1)

Plot of Equivalent Surface Source on the Sphere along the Polar Direction by Using SIE-MEI

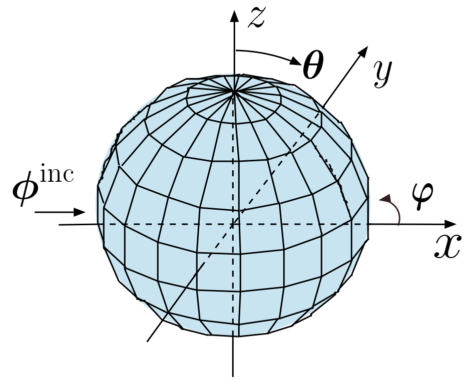




Radius = 5 wavelengths (λ)

Example : Sphere (2)

(Full 3D Body)



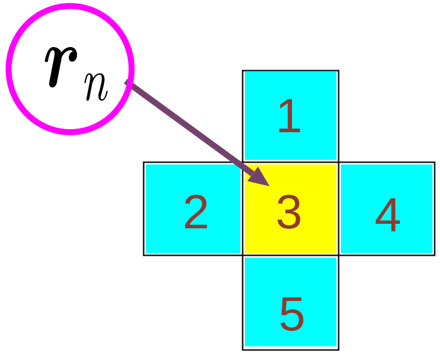
Incident field, $\phi^{\text{inc}}(\boldsymbol{r}) = e^{-jkx}$

Constant radial distance (r = a) with polar, and equatorial variation, the metron set,

$$\rho_q = h^{(2)}_{n_{max}}(ka)P^{|m|}_{n_{max}}(\cos\theta)e^{jm\varphi}$$

$$n_{max} = 0,1,2,\cdots,N_{max} \quad m = -n_{max},\cdots,0,\cdots,n_{max}$$

$$q = 1,2,\cdots,Q \quad Q = 1+(n_{max}-1)^2+2(n_{max}-1)$$



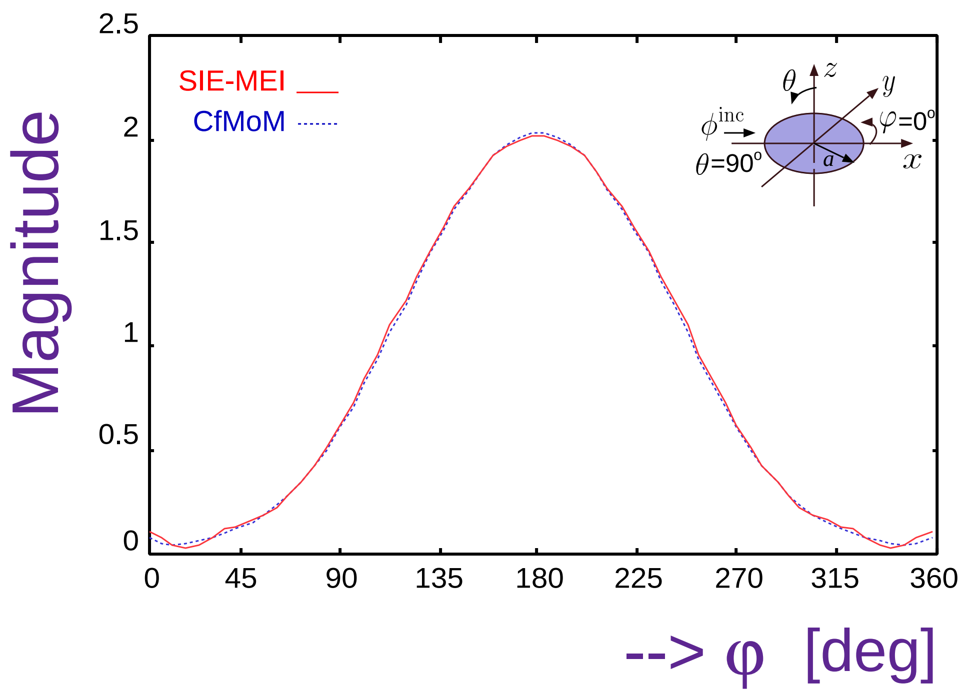
segments in the local region

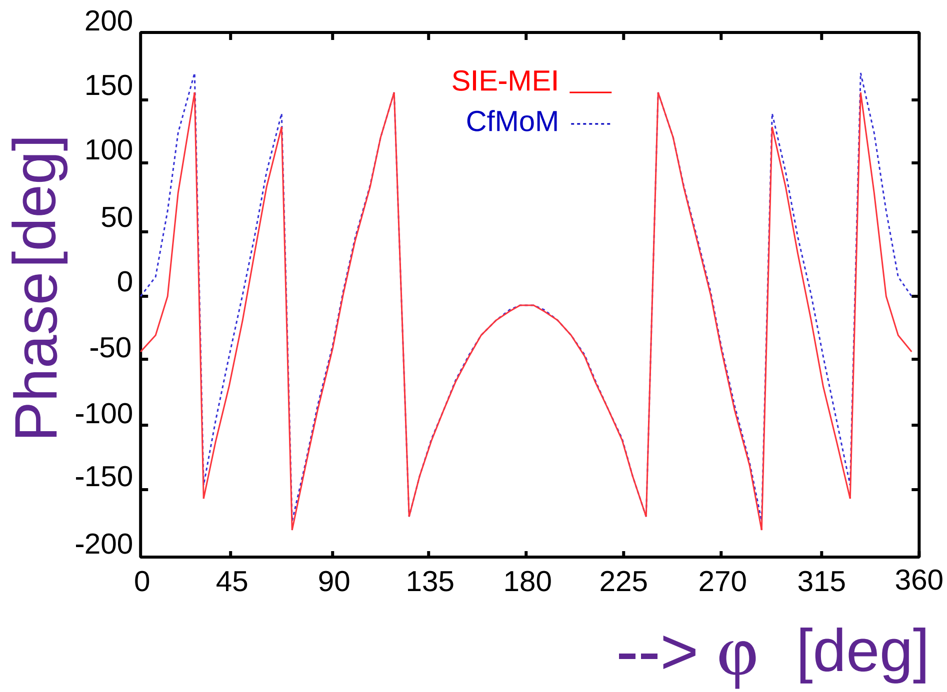
Local sources expanded into M = 5

Measuring function are varies in both polar θ and equatorial φ directions.

Results (2)

Plot of Equivalent Surface Sources on the Sphere along $x - y$ Plane at $z = 0$, by Using SIE-MEI

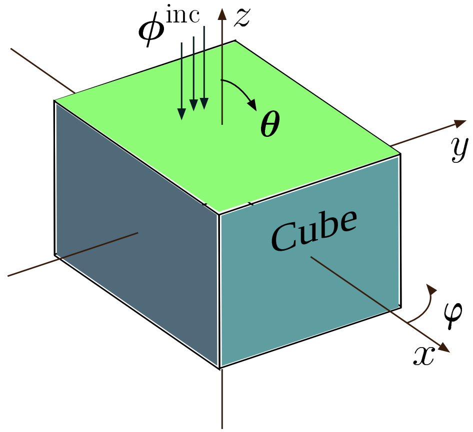




Radius = 1 wavelength (λ)

CfMoM = Combined-field Method of Moments

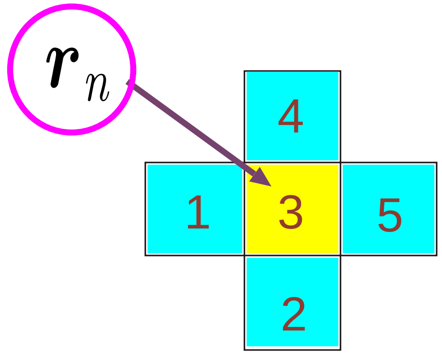
Example : Cube



Incident field, $\phi^{\rm inc}(\boldsymbol{r}) = e^{jkz}$

Cube has radial, polar, and equatorial variations, thus the metron set,

$$\rho_q = h^{(2)}_{n_{max}}(kr)P^{|m|}_{n_{max}}(\cos\theta)e^{jm\varphi}$$



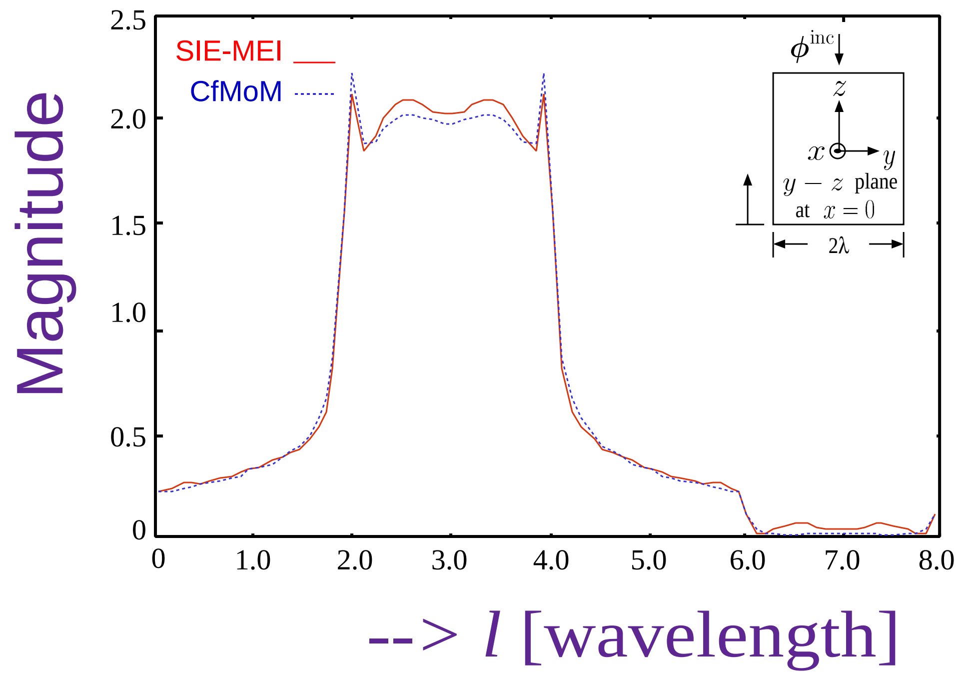
segments in the local region

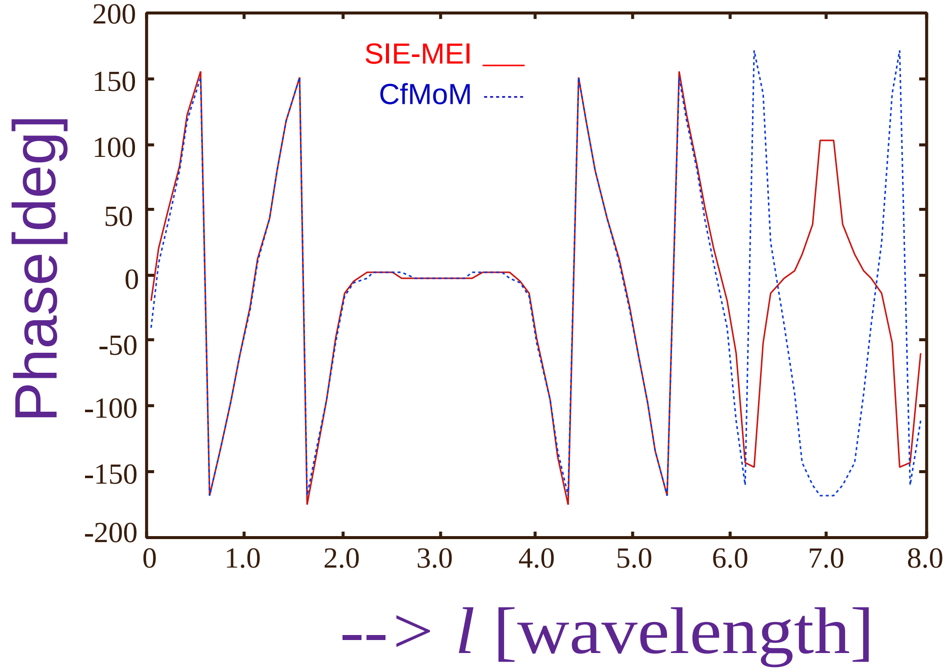
Measuring function varies in θ and φ directions

Local sources expanded into M = 5

Results (3)

Plot of Equivalent Surface Sources on the Cube along $y-z$ Plane at $x=0$, by Using SIE-MEI



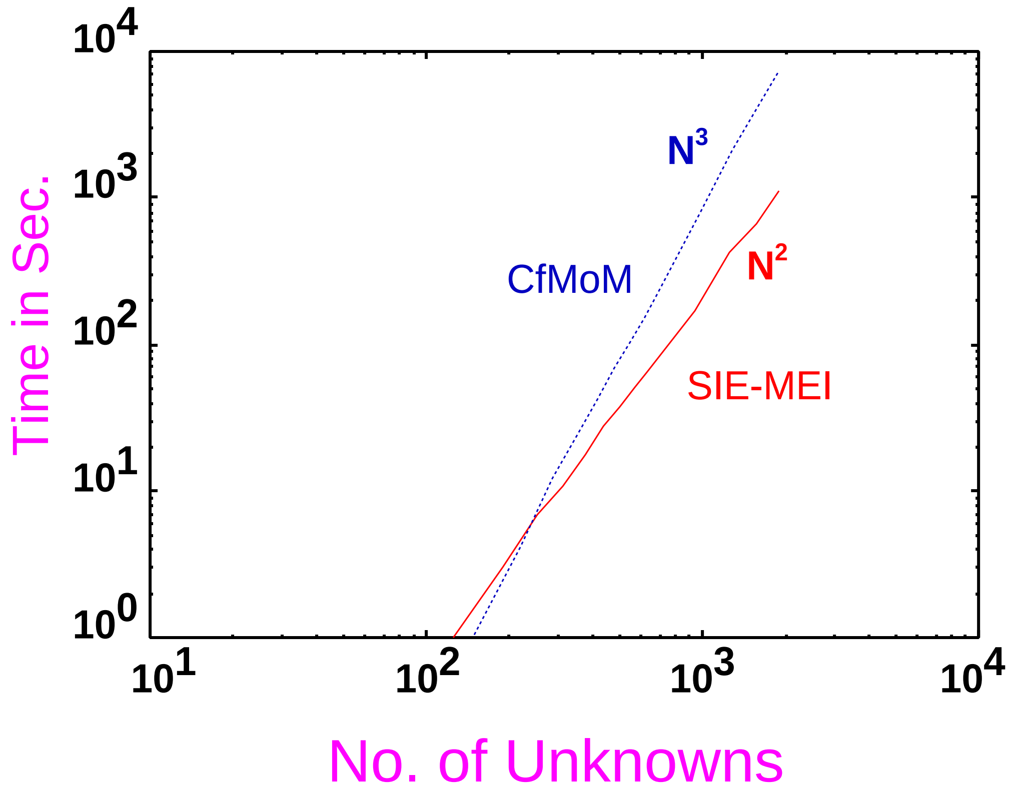


Cube side length = 2 wavelengths (λ)

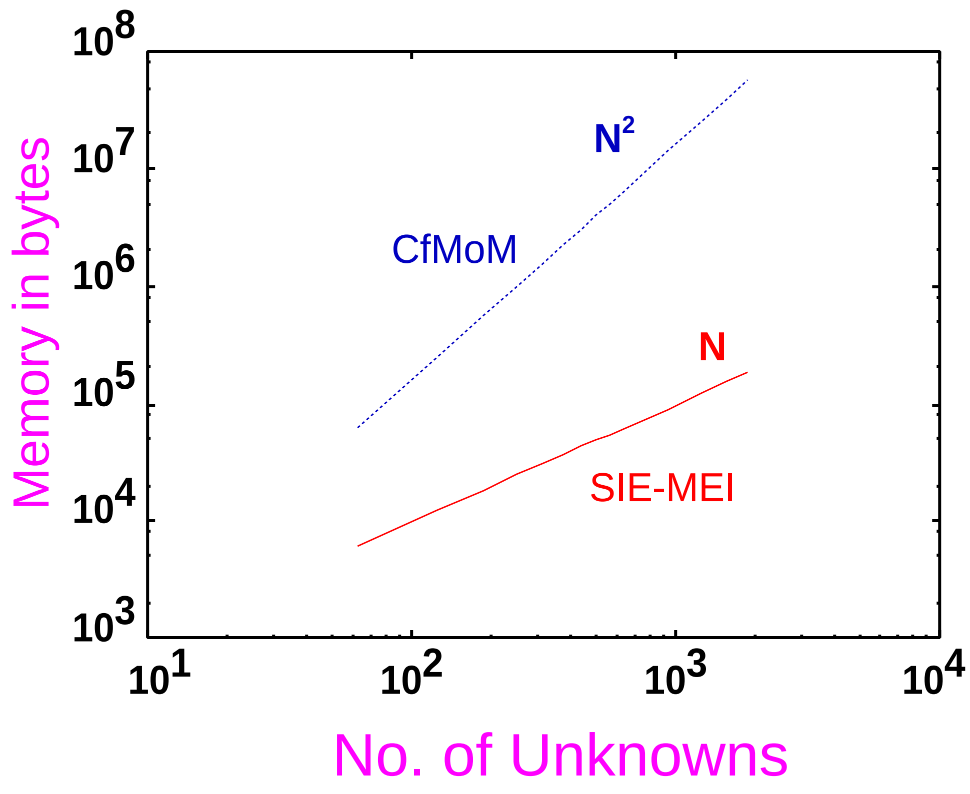
Comparison with CfMoM

Savings in CPU time & Memory needs

Total Computational time



Memory needs for Matrix storage



Matrix Localization (ML) Technique

Conventional Solution Process :

1. Discretize the scatterer surface,
2. Localize the surface for each node,
3. Derive the metron fields of the node and its neighbors,
4. Create the local matrix and solve it for the MEI coefficients,
5. Repeat steps 2 to 4 for each node to get the sparse matrices.

Proposed Solution Process :

1. Discretize the scatterer surface,
2. Derive the metron fields of each node and create the global matrix,
3. Localize the global matrix for each node with its neighbors,
4. Create the local matrix and solve it for the MEI coefficients,
5. Repeat steps 3, 4 for each node to get the sparse matrices.

Mathematical Formulation: Conventional

SIE-MEI Integral Equation, $\oint_{\partial V} \left(\phi_1(\boldsymbol{r})\tilde{\rho}_2(\boldsymbol{r}) - \frac{\partial \phi_1(\boldsymbol{r})}{\partial n} \tilde{\boldsymbol{\mu}}_2(\boldsymbol{r}) \cdot \hat{\boldsymbol{n}} \right) dS = 0$

Localized Integral Equation, $\int_{S_o} \left(\phi_1(\boldsymbol{r})\tilde{\rho}_2(\boldsymbol{r}) - \frac{\partial \phi_1(\boldsymbol{r})}{\partial n} \tilde{\boldsymbol{\mu}}_2(\boldsymbol{r}) \cdot \hat{\boldsymbol{n}} \right) dS = 0$

Discretized Integral Eqn, $\sum_{m \in R_n} \left[\phi_1(\boldsymbol{r}_m)\tilde{\rho}_{2,n}(\boldsymbol{r}_m) - \frac{\partial \phi_1(\boldsymbol{r}_m)}{\partial n} \tilde{\boldsymbol{\mu}}_{2,n}(\boldsymbol{r}_m) \cdot \hat{\boldsymbol{n}} \right] = 0$

Metron field, $\phi_{1,q}(\boldsymbol{r}_m) = \int_s \rho_q(\boldsymbol{r}') G(\boldsymbol{r_m}, \boldsymbol{r}') \, dS'$

Metron Set
$q = 1, 2, \cdots, Q$

Each metron field requires N times of operation,
Q metrons require N x Q times of operation.

Local matrix for n-th node, $[\mathbf{C}\ \mathbf{D}]\begin{bmatrix}\mathbf{a}\\ \mathbf{b}\end{bmatrix} = 0$

column vector of $2M$ MEI coefficients

$[Q \times 2M]$ matrix of metron fields and their normal derivatives,

which requires $Q \times 2M \times N$ times of operation.

For $n = 1$, $M = 3$,

$$\begin{bmatrix} C_{1,N} & C_{1,1} & C_{1,2} & D_{1,N} & D_{1,1} & D_{1,2} \\ C_{2,N} & C_{2,1} & C_{2,2} & D_{2,N} & D_{2,1} & D_{2,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_{Q,N} & C_{Q,1} & C_{Q,2} & D_{Q,N} & D_{Q,1} & D_{Q,2} \end{bmatrix} \begin{bmatrix} a_N \\ a_1 \\ a_2 \\ b_N \\ b_1 \\ b_2 \end{bmatrix} = 0$$

Use least square solution to get,

$a_N,\ a_1,\ a_2$ and $b_N,\ b_1,\ b_2$

Repeat the procedure for each nodal point n (= 1, ... , N) and get the sparse matrices **A** and **B**.

$$\mathbf{A} = \begin{bmatrix} a_{1,1} & a_{1,2} & 0 & \cdots & a_{1,N} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{N,1} & 0 & \cdots & a_{N,N-1} & a_{N,N} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} b_{1,1} & b_{1,2} & 0 & \cdots & b_{1,N} \\ b_{2,1} & b_{2,2} & b_{2,3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{N,1} & 0 & \cdots & b_{N,N-1} & b_{N,N} \end{bmatrix}$$

Conventional technique requires totally

$Q \times 2M \times N \times N = 2QMN^2$ times of operation.

Mathematical Formulation:Proposed

SIE-MEI Integral Equation, $\oint_{\partial V} \left(\phi_1(\boldsymbol{r})\tilde{\rho}_2(\boldsymbol{r}) - \frac{\partial \phi_1(\boldsymbol{r})}{\partial n} \tilde{\boldsymbol{\mu}}_2(\boldsymbol{r}) \cdot \hat{\boldsymbol{n}} \right) dS = 0$

Discretized Integral Eqn, $\sum_n \left[\phi_1(\boldsymbol{r}_n)\tilde{\rho}_{2,n}(\boldsymbol{r}_n) - \frac{\partial \phi_1(\boldsymbol{r}_n)}{\partial n} \tilde{\boldsymbol{\mu}}_{2,n}(\boldsymbol{r}_n) \cdot \hat{\boldsymbol{n}} \right] = 0$

Global Matrix for *N* nodes, $[\mathcal{C} \ \mathcal{D}] \begin{bmatrix} \mathcal{A} \\ \mathcal{B} \end{bmatrix} = 0$

column vector of $2N$ MEI coefficient

$[Q \times 2N]$ matrix of metron fields and its normal derivatives,

which requires $Q \times 2N \times N$ times of operation.

Global matrix for ***N*** nodes with ***Q*** metrons,

$$
\begin{bmatrix}
C_{1,1} & C_{1,2} & C_{1,3} & C_{1,4} & \cdots & C_{1,N-1} & C_{1,N} & D_{1,1} & \cdots & D_{l,N} \\
C_{2,1} & C_{2,2} & C_{2,3} & C_{2,4} & \cdots & C_{2,N-1} & C_{2,N} & D_{2,1} & \cdots & D_{2,N} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
C_{Q,1} & C_{Q,2} & C_{Q,3} & C_{Q,4} & \cdots & C_{Q,N-1} & C_{Q,N} & D_{Q,1} & \cdots & D_{q,N}
\end{bmatrix}
\begin{bmatrix}
a_1 \\ a_2 \\ a_3 \\ a_4 \\ \cdots \\ a_{N-1} \\ a_N \\ b_1 \\ \cdots \\ b_N
\end{bmatrix}
= 0
$$

Localize the matrix with *M* (= 3) nodes.

For $n = 1$, $M = 3$,

$$\begin{bmatrix} C_{1,N} & C_{1,1} & C_{1,2} & D_{1,N} & D_{1,1} & D_{1,2} \\ C_{2,N} & C_{2,1} & C_{2,2} & D_{2,N} & D_{2,1} & D_{2,2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_{Q,N} & C_{Q,1} & C_{Q,2} & D_{Q,N} & D_{Q,1} & D_{Q,2} \end{bmatrix} \begin{bmatrix} a_N \\ a_1 \\ a_2 \\ b_N \\ b_1 \\ b_2 \end{bmatrix} = 0$$

Solve the matrix according to least square solution.

Repeat the procedure for each nodal point, $n = 1,\ldots,N$
and get the sparse matrices **A** and **B**.

Proposed ML technique requires totally,

$$Q \times 2N \times N = 2QN^2$$ times of operation.

Conventional Technique :

Time required in the integration process is $O(2QMN^2)$

Proposed ML Technique :

Time required in the integration process is $O(2QN^2)$

ML technique can save M times of operation time.

In 2D case, $M = 3$ is sufficient.

For 3D arbitrary shape body M of more than 3 (= 5, 7, 9, etc.) is required.

In 3D ML technique can save more than 3 times of operation time.

Time Comparison

Time Comparison for Cylinder

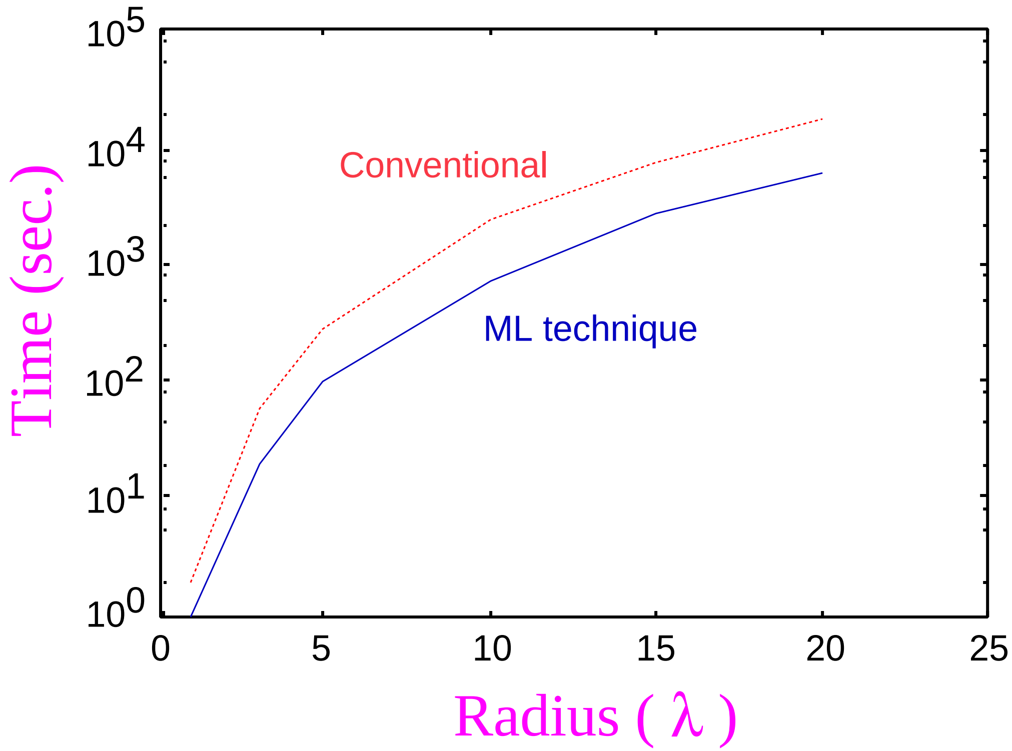
Radius (λ)	Matrix Generation	
	Conventional Method	Matrix Localization
1.	: : 2	: : 1
3.	: :59	: :21
5.	: 4:36	: 1:35
10.	:36:46	:12:29
15.	2:04:28	:42:28
20.	4:54:21	1:43:06

Time Comparison for Sphere

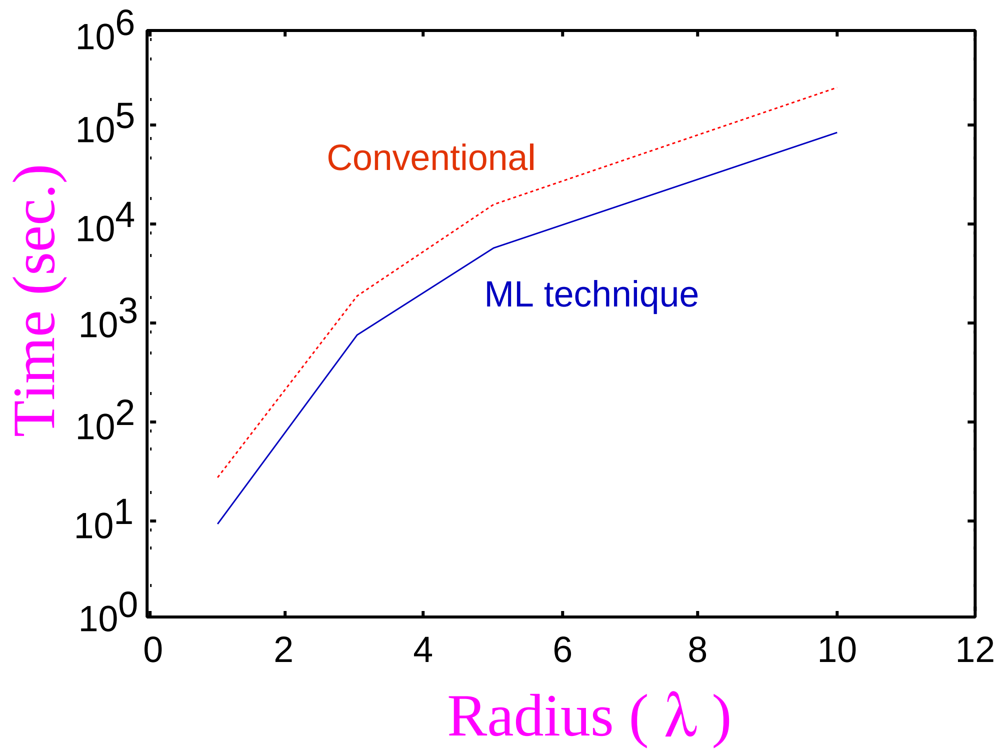
Radius (λ)	Matrix Generation	
	Conventional Method	Matrix Localization
1.	: :25	: : 9
3.	:35:49	:12:30
5.	4:40:52	1:37:12
10.	75:51:03	26:03:47

Time Comparison

Time Comparison for Cylinder



Time Comparison for Sphere



Insensitive Properties of MEI Coefficients

Sparse Matrices

$$\mathbf{A} = \begin{bmatrix} a_{1,1} & a_{1,2} & 0 & \cdots & a_{1,N} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{N,1} & 0 & \cdots & a_{N,N-1} & a_{N,N} \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} b_{1,1} & b_{1,2} & 0 & \cdots & b_{1,N} \\ b_{2,1} & b_{2,2} & b_{2,3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{N,1} & 0 & \cdots & b_{N,N-1} & b_{N,N} \end{bmatrix}$$

N nodes

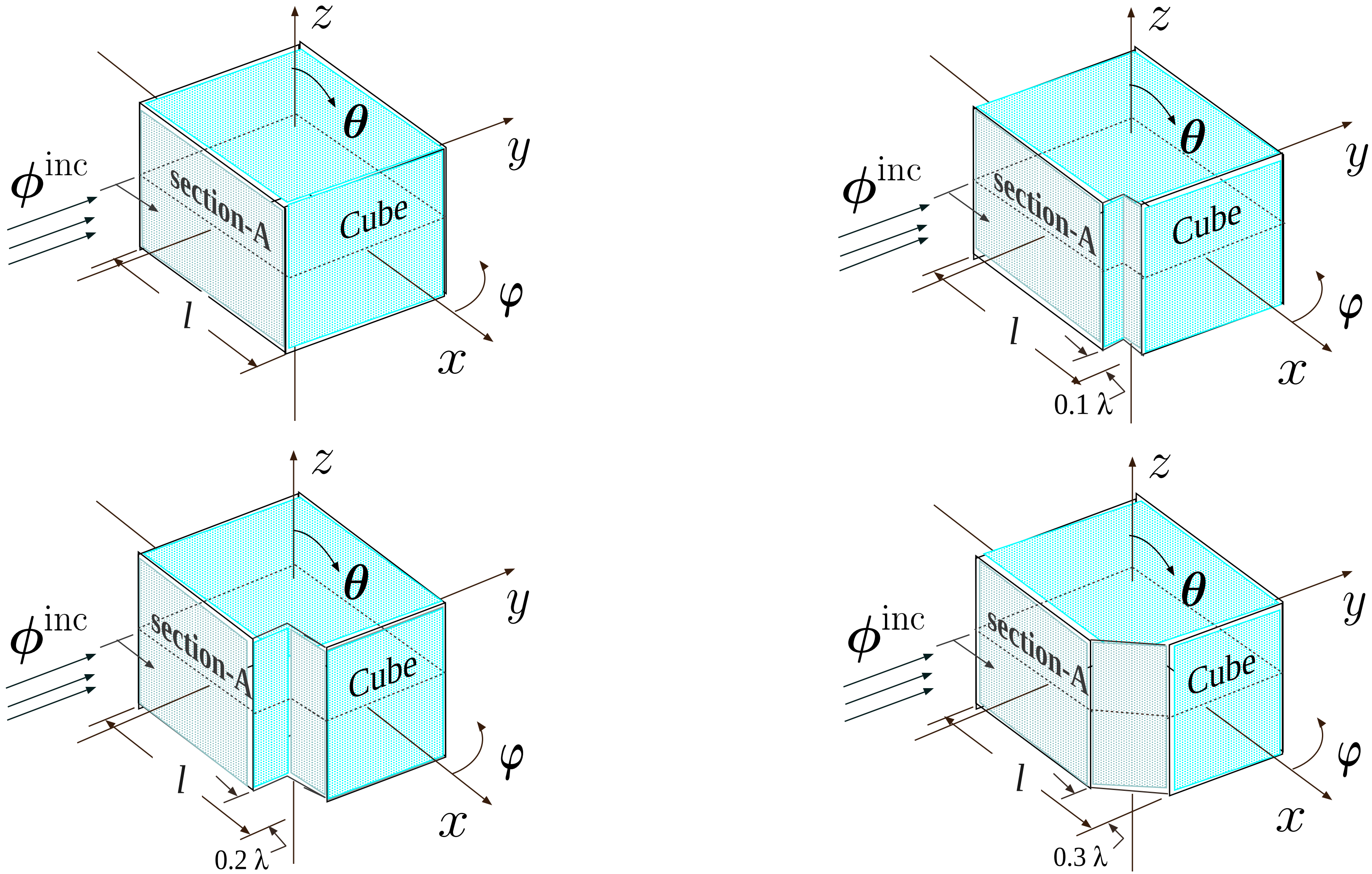
Local Matrix : $[\,\mathbf{C}\ \mathbf{D}\,] \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} = 0$

n^{th} node

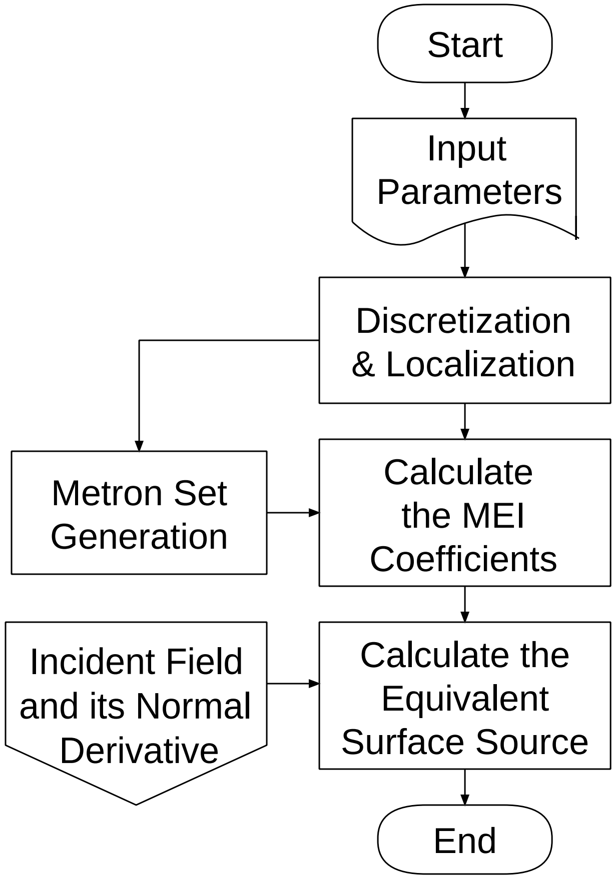
$$\phi_{1,q}(\boldsymbol{r}_m) = \int_s \rho_q(\boldsymbol{r}')\, G(\boldsymbol{r_m}, \boldsymbol{r}')\, dS'$$

Localized Integral Eqn.: $\displaystyle\sum_m \left[a_{nm}\phi_1(\boldsymbol{r}_m) - b_{nm}\frac{\partial \phi_1(\boldsymbol{r}_m)}{\partial n} \right] = 0$

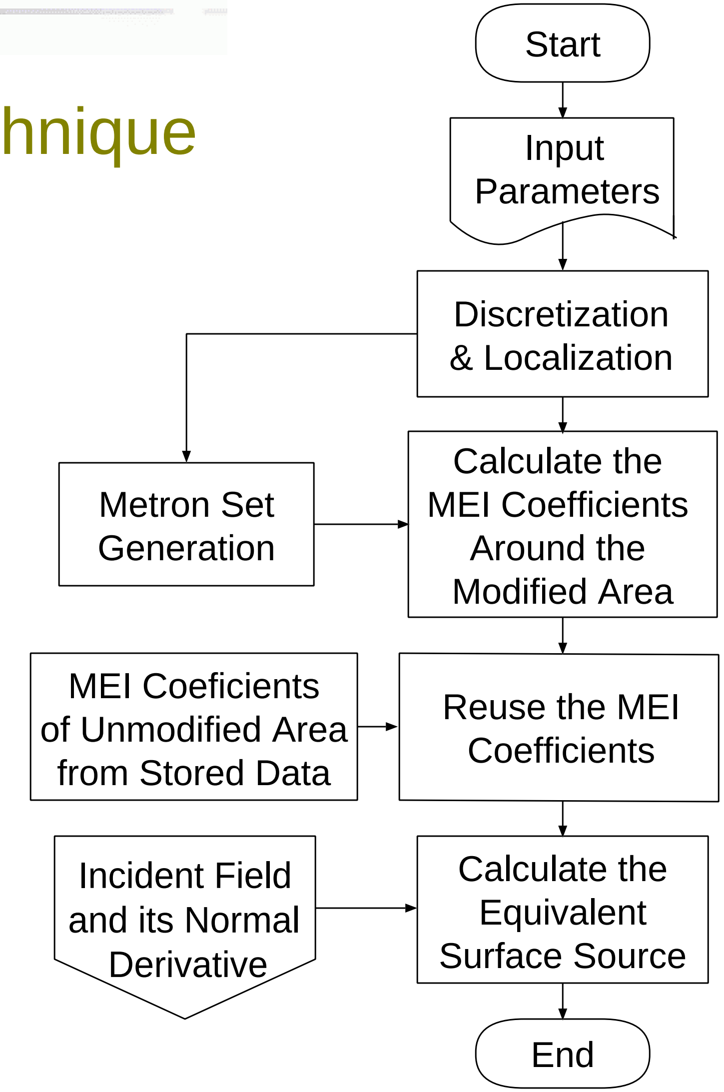
Applicability of Insensitive Properties



Flow-chart of the Solution Technique

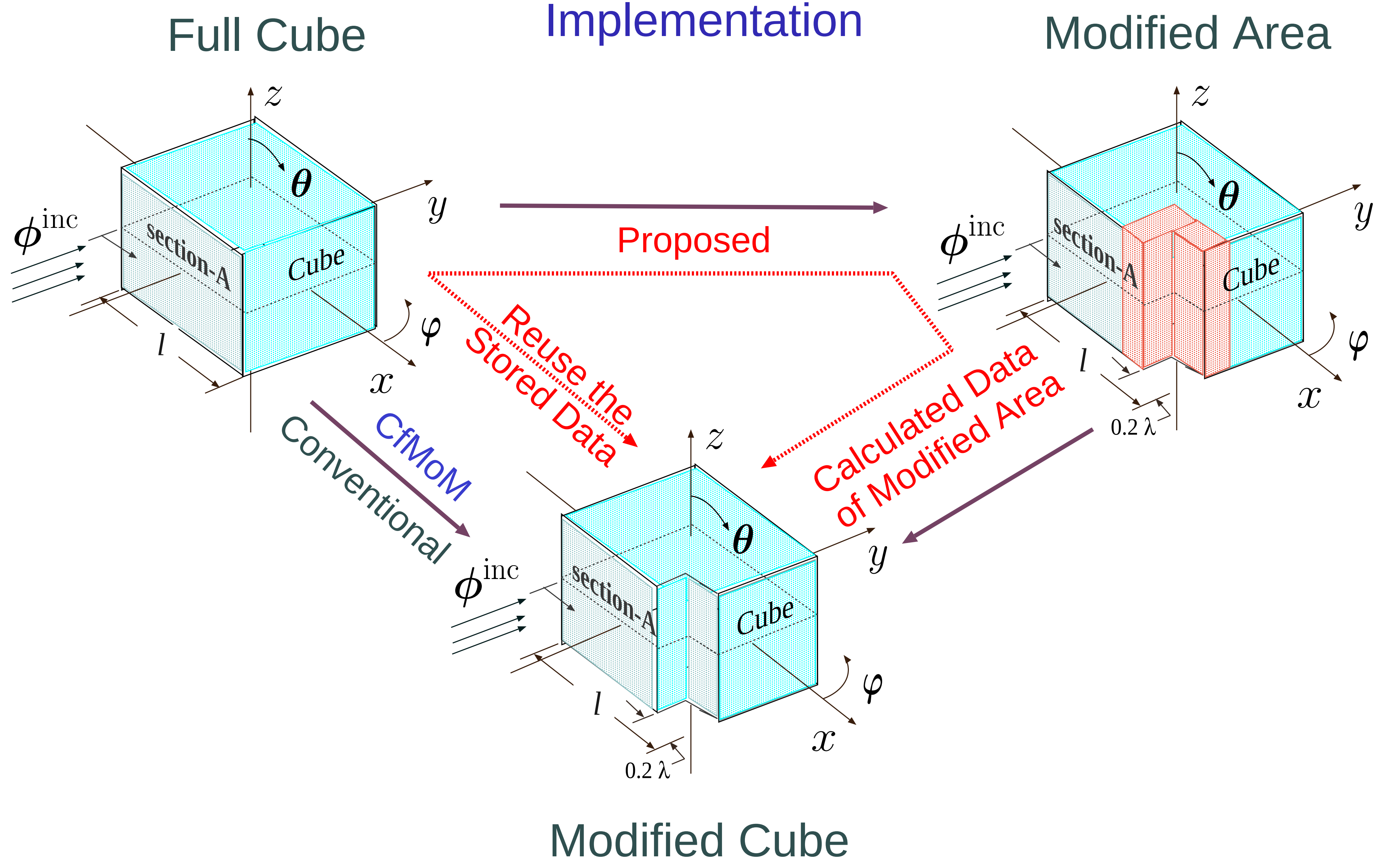


Conventional Solution Technique



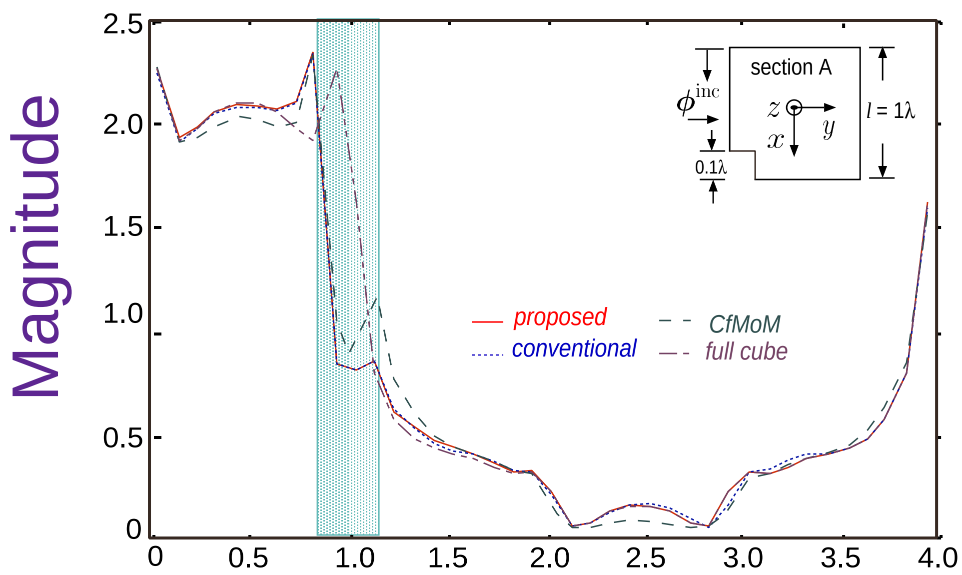
Proposed Solution Technique

Implementation

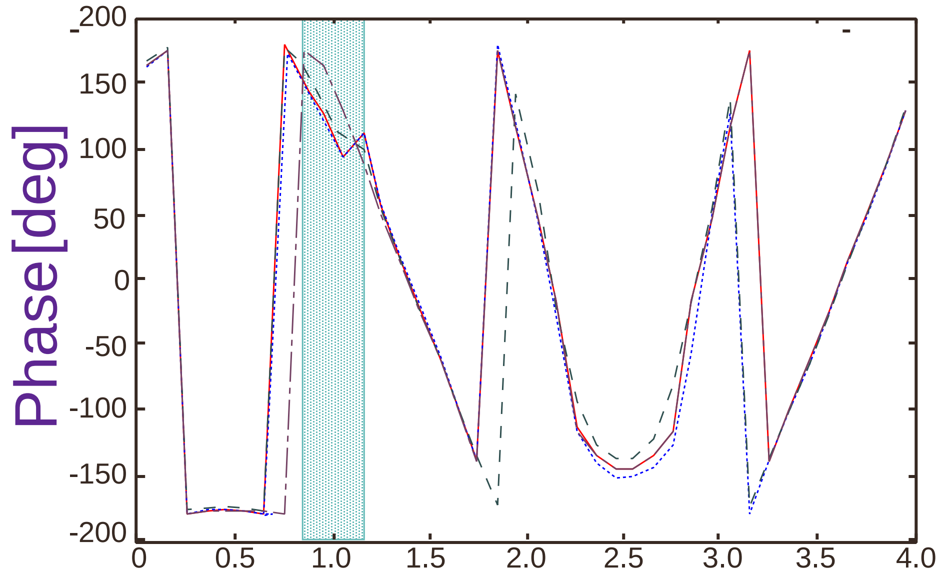


Results (1)

Plot of Equivalent Surface Sources on the 0.1λ Modified Cube along $x - y$ Plane at $z = 0$, by Using SIE-MEI



--> l [wavelength]

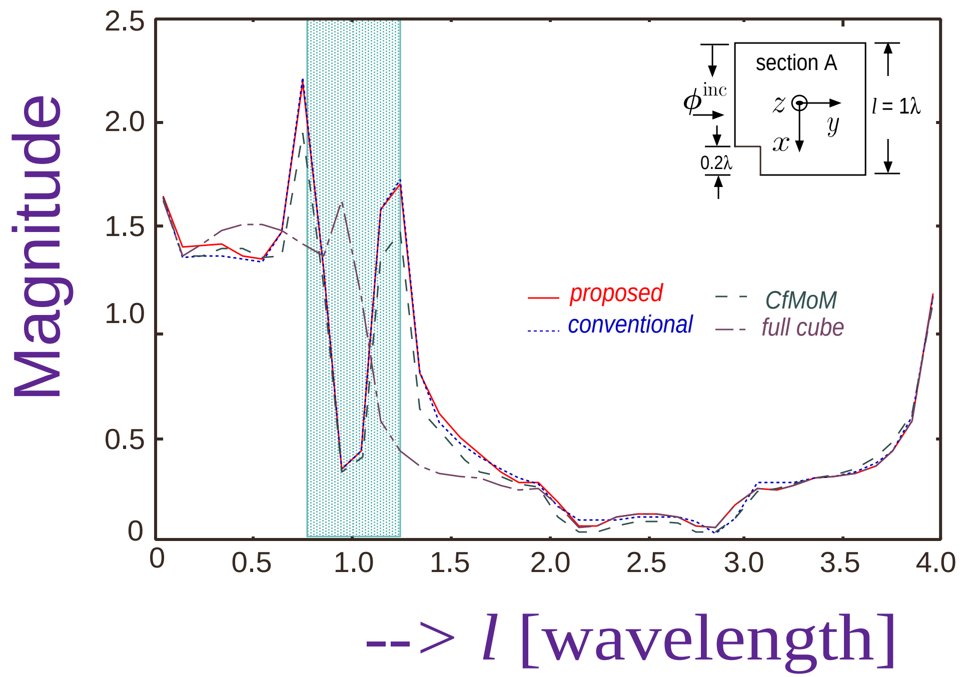


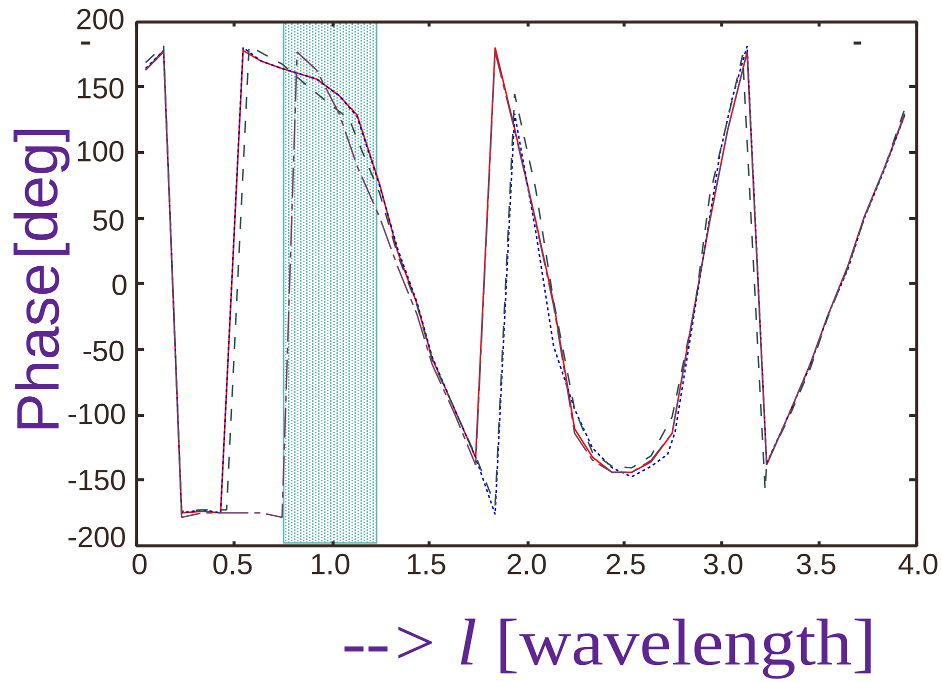
--> l [wavelength]

Cube side length = 1 wavelength (λ)

Results (2)

Plot of Equivalent Surface Sources on the 0.2λ Modified Cube along $x-y$ Plane at $z=0$, by Using SIE-MEI

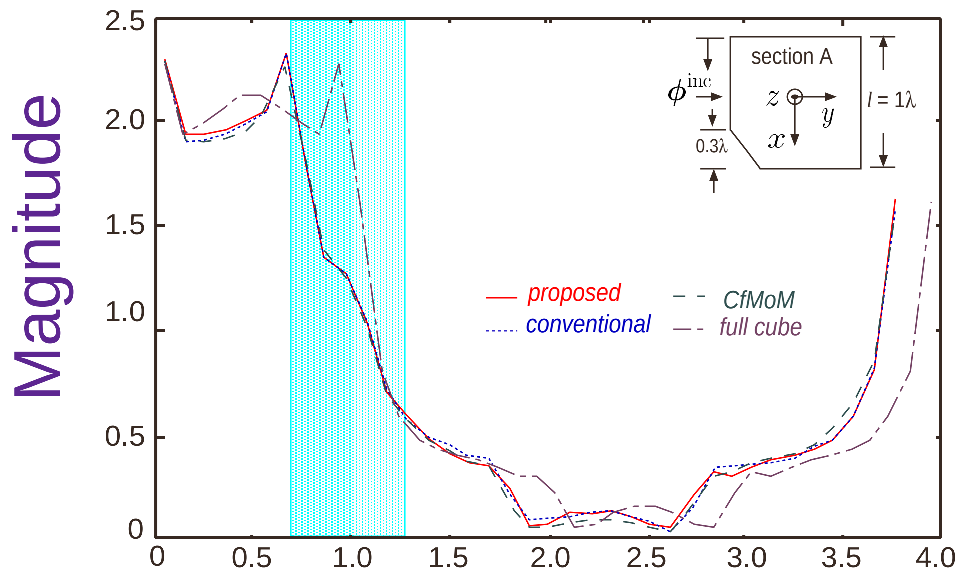


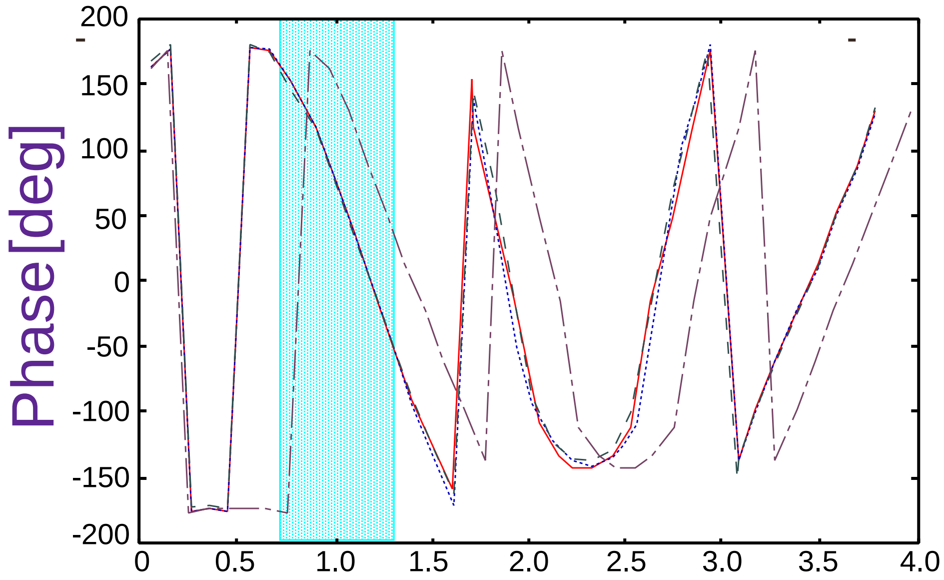


Cube side length = 1 wavelength (λ)

Results (3)

Plot of Equivalent surface sources on the 0.3λ Modified Cube along $x-y$ Plane at $z=0$, by Using SIE-MEI





--> *l* [wavelength] --> *l* [wavelength]

Cube side length = 1 wavelength (λ)

Inner Product Between Two Vectors

Local Matrix : $[\mathbf{C}\ \mathbf{D}] \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix} = 0$

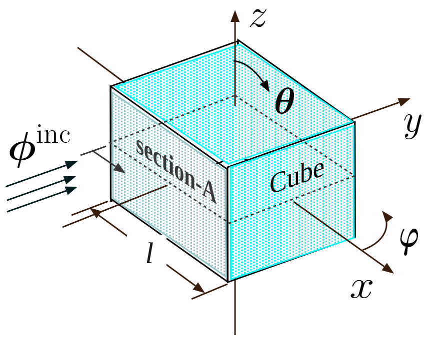
For each node with $M = 5$

$$\boldsymbol{a} = \begin{bmatrix} \boldsymbol{a}_1 \\ \boldsymbol{a}_2 \\ \boldsymbol{a}_3 \\ \boldsymbol{a}_4 \\ \boldsymbol{a}_5 \end{bmatrix} \Rightarrow \widetilde{\boldsymbol{a}} = \begin{bmatrix} \widetilde{\boldsymbol{a}}_1 \\ \widetilde{\boldsymbol{a}}_2 \\ \widetilde{\boldsymbol{a}}_3 \\ \widetilde{\boldsymbol{a}}_4 \\ \widetilde{\boldsymbol{a}}_5 \end{bmatrix}$$

$$\widetilde{\boldsymbol{a}}_1 = \frac{\boldsymbol{a}_1}{||\boldsymbol{a}||_2}$$

$$\widetilde{\boldsymbol{a}}'_1 = \frac{\boldsymbol{a}'_1}{||\boldsymbol{a}'||_2}$$

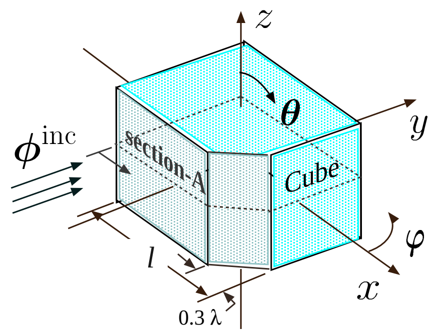
$$\boldsymbol{a}' = \begin{bmatrix} \boldsymbol{a}'_1 \\ \boldsymbol{a}'_2 \\ \boldsymbol{a}'_3 \\ \boldsymbol{a}'_4 \\ \boldsymbol{a}'_5 \end{bmatrix} \Rightarrow \widetilde{\boldsymbol{a}}' = \begin{bmatrix} \widetilde{\boldsymbol{a}}'_1 \\ \widetilde{\boldsymbol{a}}'_2 \\ \widetilde{\boldsymbol{a}}'_3 \\ \widetilde{\boldsymbol{a}}'_4 \\ \widetilde{\boldsymbol{a}}'_5 \end{bmatrix}$$



Full Cube

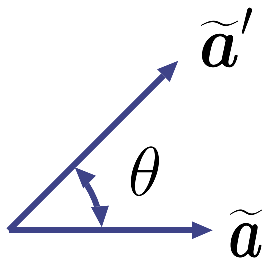
Inner Product

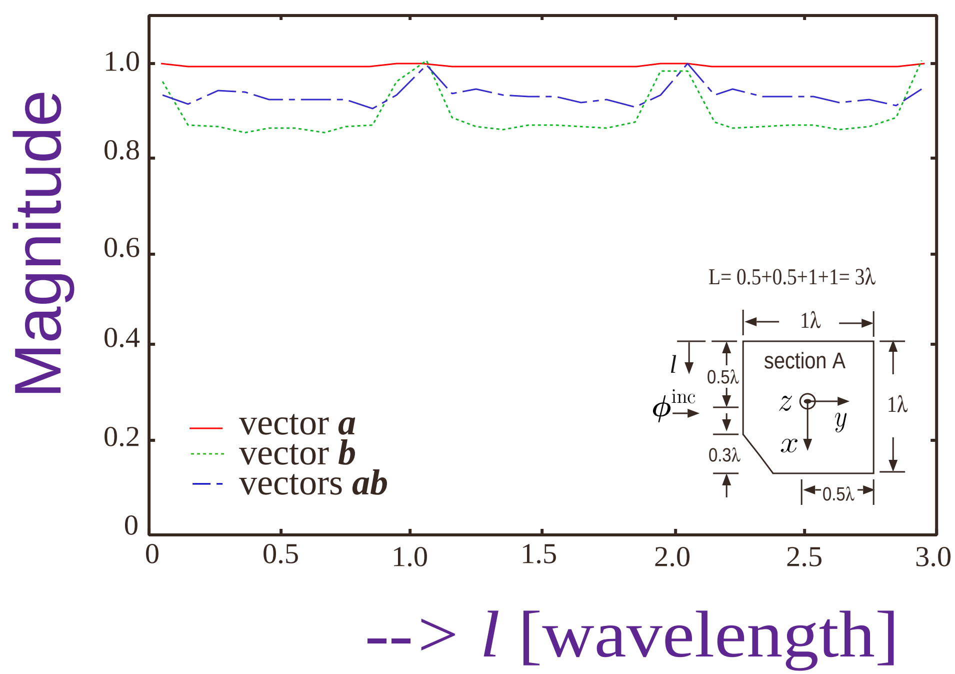
$$\widetilde{\boldsymbol{a}} \cdot \widetilde{\boldsymbol{a}}' = \left| \widetilde{\boldsymbol{a}}^H \widetilde{\boldsymbol{a}}' \right|$$

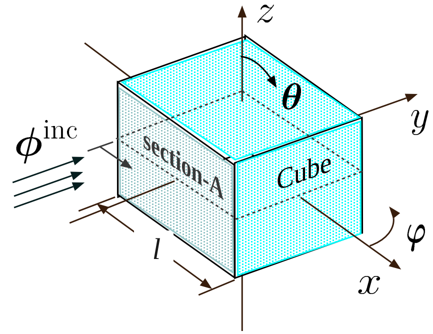


Modified Cube

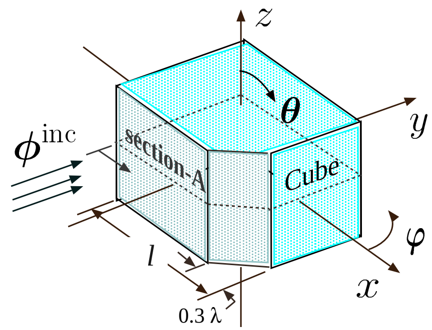
Plot of Normalized Inner Product Between Two Vectors







$$[\mathbf{C}\ \mathbf{D}]\begin{bmatrix}\mathbf{a}\\ \mathbf{b}\end{bmatrix} = 0$$



Time Comparison

Time Comparison for 3D Soft Cube

Scatterer	Cube Side (λ)	Time (sec.)	No. of MEI Coefficients
Full Cube	1	202	6,000
	2	11,280	24,000
Modified Cube	1	179	5,640
	2	10,743	23,340
Modified Area	1	32	940
	2	754	1,640

10 segments/wavelength, $M = 5$

Application of SIE-MEI

SIE-MEI method can be applied to high contrast 3D acoustic scattering problems.

Application of ML Technique

Matrix Localization technique can be used in IE-MEI and SIE-MEI method for the reduction of computational time required in integration process.

By reducing the integration time it can reduce the overall CPU time.

Application of Insensitive Properties of MEI Coefficients

Insensitive properties of MEI coefficients can be used for the computation of scattering from modified structured bodies with minimum CPU time.

This properties is useful for the repeated computation of the modified structure w.r.to the some specified parameters.

This properties are not only applicable for SIE-MEI method, but can also be used for any kind of MEI technique.

Remarks on Present Work

SIE-MEI method :

- uses surface integral equation with MEI postulates, thus the matrix sparsity is preserved, hence the computational time and the memory requirements are reduced.

- is successfully implemented to uniform & arbitrary shape 3D scalar field (acoustic) problems.

- Results have fair agreements with the analytical and numerical solutions.

Future Works

- Implement the SIE-MEI method to other arbitrary shape 3D scalar-field problems and verify the results.
- Use the hybrid method for the bodies with concave structure and compare the result with the other numerical solutions.
- Utilize the insensitive properties of MEI coefficients to the IE-MEI method and verify the results.

Future Works

★ Derive the mixed-potential approach of SIE-MEI method for 3D EM vector field scattering problem.

Conclusions

- Scalar-field approach of IE-MEI method has been successfully implemented to three-dimensional scalar-field scattering problem.
- This method has the same number of unknowns as BEM with matrix sparsity, which reduces computational time and memory needs.
- ML technique and insensitive properties of MEI coefficients can save additional computational time required in the integration process.
- ML technique and insensitive properties of MEI coefficients can be implemented to IE-MEI method without any modification.

References

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- [4] N.M.Alam Chowdhury, J.Takada, and M.Hirose, "Novel Formulation for the Scalar-field approach of IE-MEI method to Solve the Three-dimensional Scattering Problem", to be published IEICE Trans. on Fundamentals.
- [5] N.M.Alam Chowdhury, J.Takada, and M.Hirose, "Applicability of Insensitive Properties of MEI Coefficients for Modified Scattering Objects", submitted to the IEICE Trans. on Fundamentals for publication.