Nonlinear Analysis of Direct Sampling Mixer using F-matrix

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- Background
- Analysis with Fundamental Matrix
  - Nonlinear System
  - Periodic Time-Variant System
- Analysis of Charge Sampling Circuit
- Simulation
- Conclusion and Future Work

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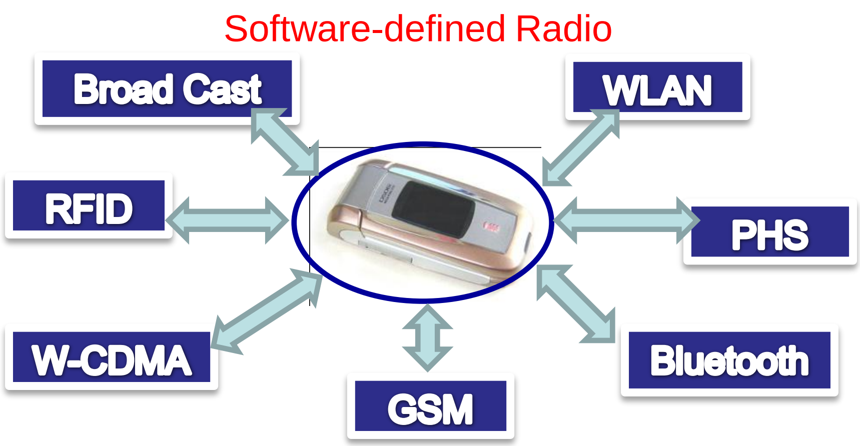
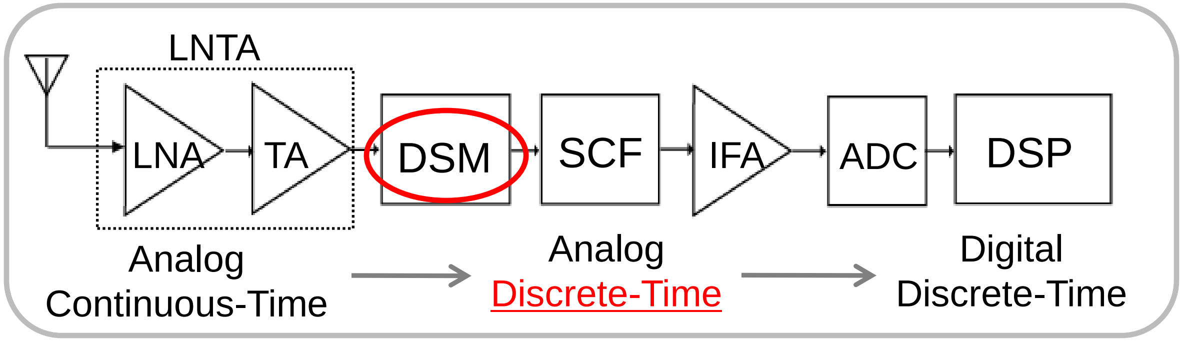
Background

◆Software-defined Radio



- System with various modulation method and frequency on a RF circuit.
  ⇒ Software-defined Radio

- Discrete Time Receiver



TA : Voltage ⇒ Current
DSM : charge sampling decimation
SCF : RF Filtering

✓Down conversion and decimation
⇒ Demand Moderation of ADC
✓Discrete time processing
⇒ Reconfigurable Technology

•Power consumption
•Small size

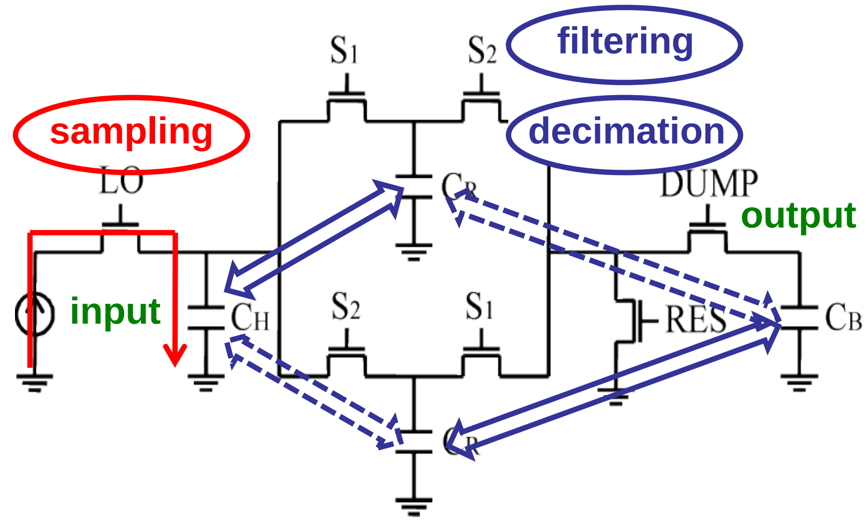
Background

◆Direct Sampling Mixers (DSM)

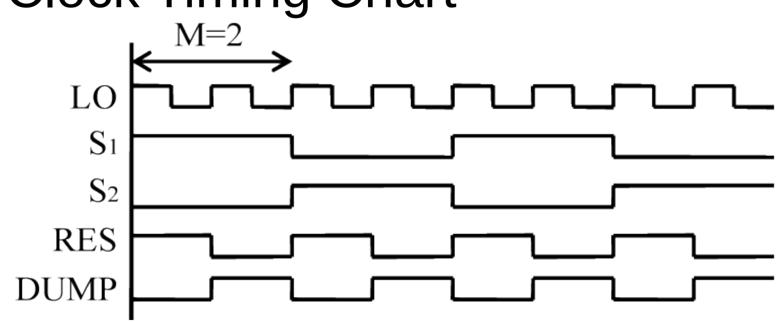
Features

■**<u>Charge Sampling</u>**
⇒Frequency Conversion
■ **<u>Charge Shared Filtering</u>**
⇒RF Filtering
■**<u>Decimation</u>**
⇒ Less power loss due to low speed ADC
■**<u>MOS Switches and capacitors</u>**
⇒ One chipping operation
■ **<u>Reconfigurability</u>**
⇒ Filter response change easily.

•Basic of DSM



•Clock Timing Chart



Problem

■**Noise-reduction**
■**Improvement of filter property**
■**Nonlinear distortion**

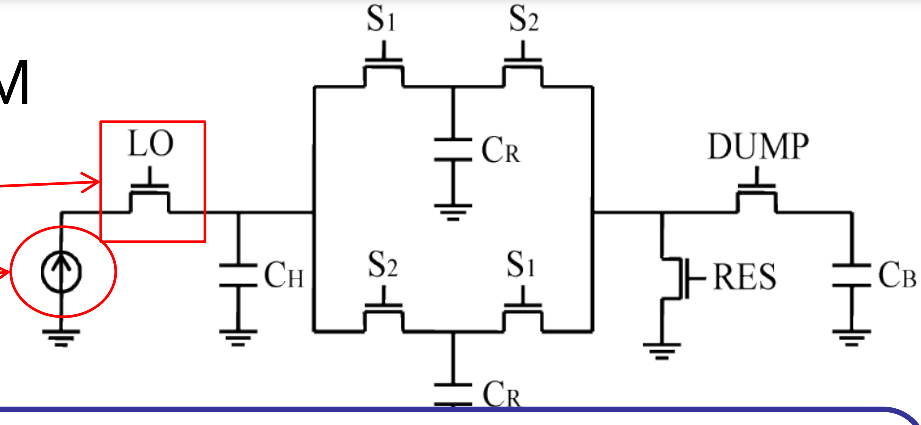
Research Propose : Design of distortion compensation method

Background

◆Complexity of Nonlinear DSM

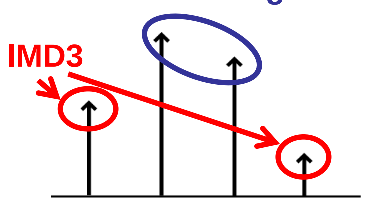
Nonlinear MOS Switch

Nonlinear TA



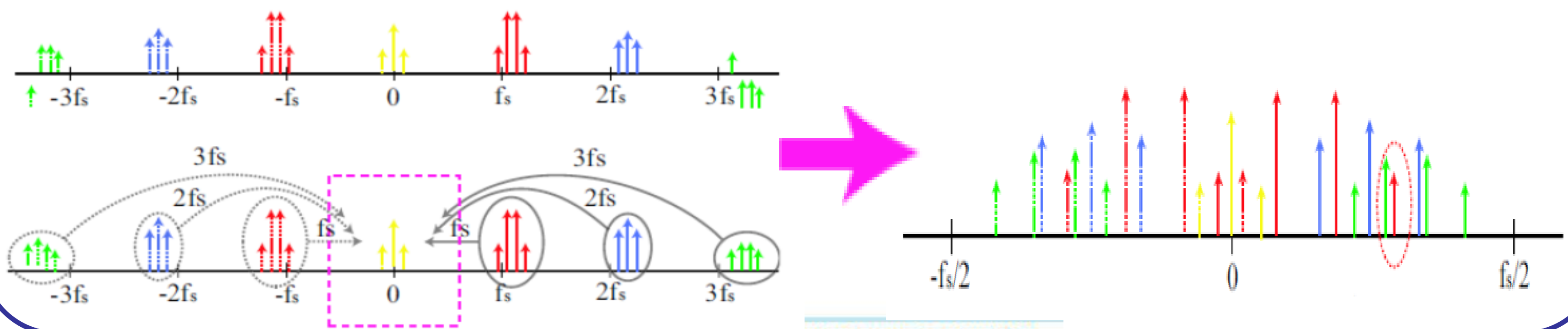
Memory Effect

DSM has frequency characteristic (memory effect) , so nonlinearity is expressed as a function of <u>not only amplitude, also operation frequency.</u>



Periodic Time-Variant System

Periodic time-variant system has frequency conversion.



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Analysis with Fundamental Matrix

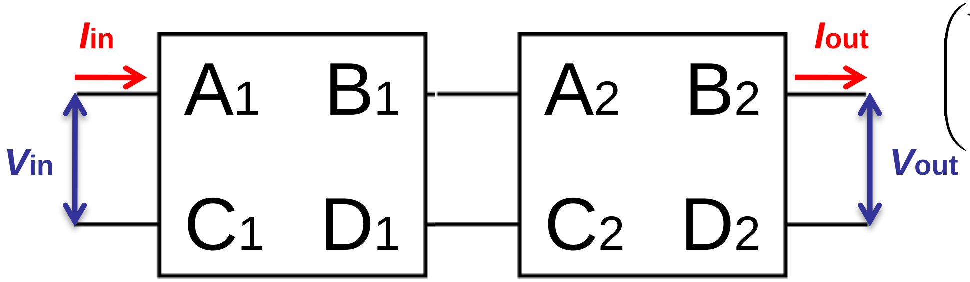
◆New Analysis Method

The <u>frequency conversion and Volterra series function</u> are introduced into **Fundamental matrix**.

Advantage

- ➢Complex circuit can be divided into some F-matrix. ⇒ <u>flexibility</u>
- ➢Calculation of cascade connection is matrix calculation. ⇒ <u>easy</u>

•Cascade Connection with F-matrix



$$\begin{pmatrix} V_{in} \\ I_{in} \end{pmatrix} = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} \begin{pmatrix} V_{out} \\ I_{out} \end{pmatrix}$$

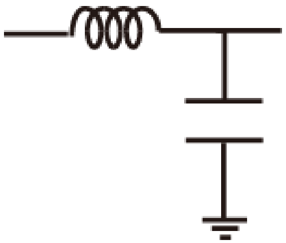
$$= \begin{pmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ C_1A_2 + D_1C_2 & C_1B_2 + D_1D_2 \end{pmatrix} \begin{pmatrix} V_{out} \\ I_{out} \end{pmatrix}$$

Inverse of (1, 1) component ⇒ Frequency Characteristic : V_{out}/V_{in}

Analysis with Fundamental Matrix

◆Approach

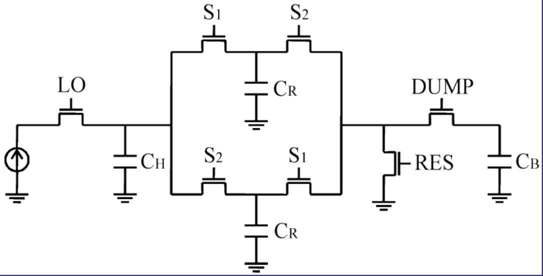
Linear Time-Invariant :
LTI System (ex. LC Filter)
⇒ Conventional F-matrix target



Nonlinear Time-Invariant System (ex. PA)

Linear Periodic Time-Variant :
LPTV System (ex. Linear Mixer)

Nonlinear Periodic Time-Variant :
NLPTV system (ex. Nonlinear Mixer, DSM)



Nonlinear System

◆Volterra Series Function

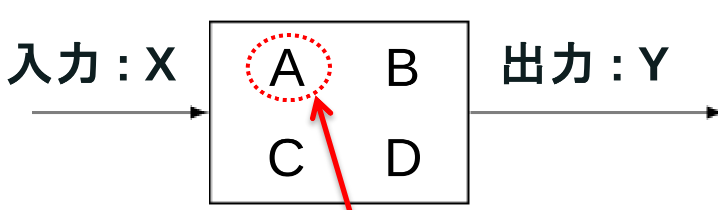
➢Overall property of DSM can't be expressed as power series function because DSM have frequency characteristic . ⇒ **Volterra Series**

Volterra Series Function

$$Y(\omega)=H_1(\omega)X(\omega)$$
$$+\int H_2(\omega_1,\omega-\omega_1)X(\omega_1)X(\omega-\omega_1)d\omega_1$$
$$+\iint H_3(\omega_1,\omega_2,\omega-\omega_1-\omega_2)X(\omega_1)X(\omega_2)X(\omega-\omega_1-\omega_2)d\omega_1 d\omega_2$$

→ Multidimensional Transfer Function

➢Expression of F-matrix



$$\begin{pmatrix} V_X(\omega) \\ I_X(\omega) \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \bullet \begin{pmatrix} V_Y(\omega) \\ I_Y(\omega) \end{pmatrix}$$

$A_1(\omega_1)$, $A_2(\omega_1, \omega_2)$, $A_3(\omega_1,\omega_2,\omega_3)$ ⇐ This **A** has each dimensional functions

Nonlinear System

◆Cascade Connection of Volterra Series

G : X ⇒ Y
H : Y ⇒ Z
F : X ⇒ Z

F

$$\begin{pmatrix} V_X(\omega) \\ I_X(\omega) \end{pmatrix} = \begin{pmatrix} G & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} H & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} V_Z(\omega) \\ I_Z(\omega) \end{pmatrix} = \begin{pmatrix} G \bullet H & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} V_Z(\omega) \\ I_Z(\omega) \end{pmatrix}$$

Eq. 1

$$F_1(\omega_1) = G_1(\omega_1) H_1(\omega_1)$$

$$F_2(\omega_1,\omega_2) = G_1(\omega_1+\omega_2) H_2(\omega_1,\omega_2) + G_2(\omega_1,\omega_2) H_2(\omega_1) H_1(\omega_2)$$

$$F_3(\omega_1,\omega_2,\omega_3) = G_1(\omega_1+\omega_2+\omega_3) H_3(\omega_1,\omega_2,\omega_3)$$

$$+ 2\mathrm{P}\langle G_2(\omega_1,\omega_2+\omega_3) H_2(\omega_2,\omega_3) H_1(\omega_1) \rangle / 3$$

$$+ G_3(\omega_1,\omega_2,\omega_3) H_1(\omega_1) H_1(\omega_2 H_1(\omega_3)$$

$$\left[\text{where P}\langle\ \rangle : \text{permitation operation}\,(\omega_1,\omega_2,\omega_3)\right]$$

Periodic Time-Variant System

◆Linear Periodic Time-Variant (LPTV) System

PTV transfer function : $H(\omega, t)$

$$y(t) = H(\omega, t)x(t)$$

$$H(\omega, t) = H(\omega, t+T)$$

$$\begin{cases} x(t): \text{input} \\ y(t): \text{output} \\ T: \text{time period} \end{cases}$$

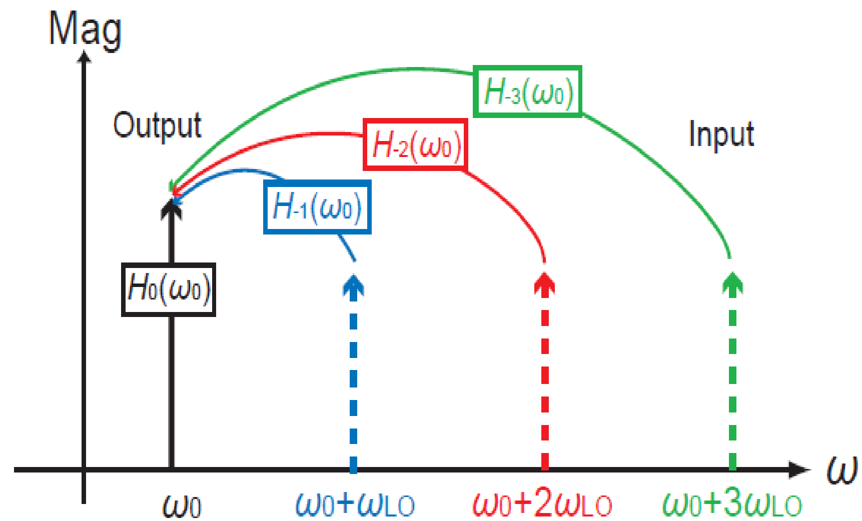
Fourier series expansion (Eq. 2)

$$H(\omega, t) = \sum_n H_n(\omega) e^{jn\omega_{\text{LO}} t}$$

$$H_n(\omega) = \frac{1}{T} \int_0^T H(\omega, t) e^{-jn\omega_{\text{LO}} t} dt$$

$$y(t) = \sum_n H_n(\omega) e^{jn\omega_{\text{LO}} t} \cdot x(t)$$

➢Frequency Conversion



$$(\omega_{\text{LO}} = 2\pi / T : \text{clock frequency})$$

Fourier Transform

$$Y(\omega) = \sum_n H_n(\omega) X(\omega - n\omega_{\text{LO}})$$

Frequency Conversion

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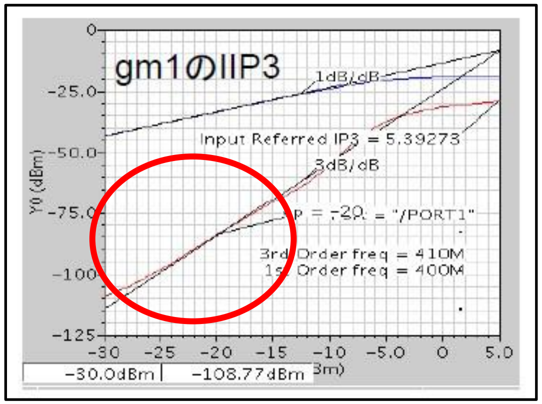
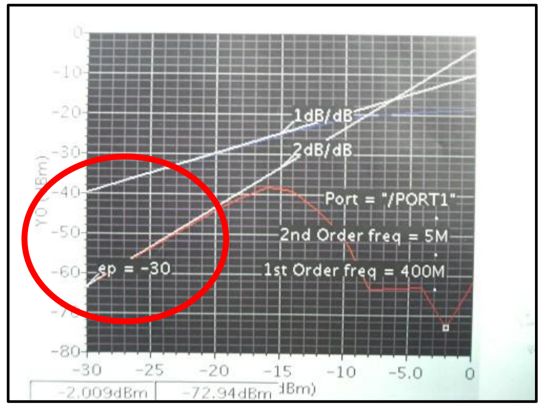
Analysis of Charge Sampling Circuit

◆Nonlinearity of TA

➢Power series expansion is approximated from measurement of the prototype TA.



✓IP3 → 3rd coefficient

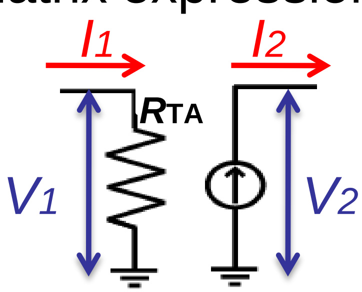


✓IP2 → 2nd coefficient

$$I_{TA}(t) = g_{m1}V_{in}(t) + g_{m2}\{V_{in}(t)\}^2 + g_{m3}\{V_{in}(t)\}^3$$

$$\begin{cases} g_{m1} = 11.3\,\mathrm{mA/V} \\ g_{m2} = -10.3\,\mathrm{mA/V^2} \\ g_{m3} = -86.1\,\mathrm{mA/V^3} \end{cases}$$

➢F-matrix expression of nonlinear TA



$$\begin{pmatrix} V_1(\omega) \\ I_1(\omega) \end{pmatrix} = \begin{pmatrix} 0 & \mathrm{A} \\ 0 & \mathrm{A}/R_{TA} \end{pmatrix} \bullet \begin{pmatrix} V_2(\omega) \\ I_2(\omega) \end{pmatrix}$$

$$a_1 = g_{m1}^{-1}$$
$$a_2 = -g_{m2}g_{m1}^{-3}$$
$$a_3 = -g_{m3}g_{m1}^{-4} + 2g_{m2}^{2}g_{m1}^{-5}$$

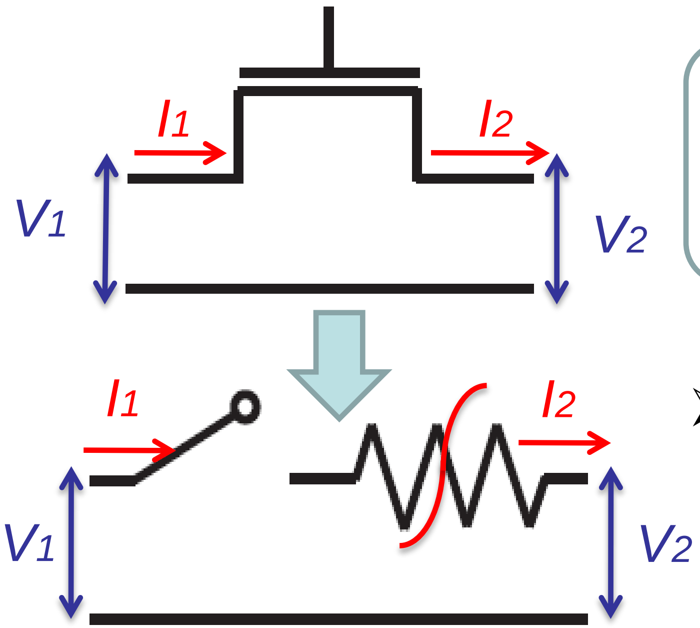
(Eq. 3)

$$\mathrm{A} \bullet I_2(\omega) = a_1 I_2(\omega) + a_2 \int I_2(\omega_1) I_2(\omega-\omega_1) d\omega_1 + a_3 \int I_2(\omega_1) I_2(\omega_2) I_2(\omega-\omega_1-\omega_2) d\omega_1 d\omega_2$$

Analysis of Charge Sampling Circuit

◆Nonlinearity of MOS Switch

➢MOS transistor is dealt as ideal switch and nonlinear resistor.



$$\text{ON state}: V_1(t)=V_2(t)+\underline{\underline{r_{\text{on1}}I_2(t)+r_{\text{on2}}\{I_2(t)\}^2+r_{\text{on3}}\{I_2(t)\}^3}}$$

$$\Updownarrow$$

$$\text{OFF state}: V_1(t)=V_2(t)+\underline{\underline{r_{\text{off1}}I_2(t)+r_{\text{off2}}\{I_2(t)\}^2+r_{\text{off3}}\{I_2(t)\}^3}}$$

➢F-matrix expression of nonlinear MOS switch

$$\begin{pmatrix} V_1(\omega) \\ I_1(\omega) \end{pmatrix} = \begin{pmatrix} 1 & \mathrm{R}(t) \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} V_2(\omega) \\ I_2(\omega) \end{pmatrix}$$

$$\left(\begin{aligned} \mathrm{R}(t)\bullet I_2(\omega) = {} & r_1(t)I_2(\omega) \\ & + r_2(t)\int I_2(\omega_1)I_2(\omega-\omega_1)d\omega_1 \\ & + r_3(t)\int I_2(\omega_1)I_2(\omega_2)I_2(\omega-\omega_1-\omega_2)d\omega_1 d\omega_2 \end{aligned} \right)$$

➢Nonlinear Resistor **R**(t) (Table.1)

r_{on1}	50 Ω	r_{off1}	1 MΩ
r_{on2}	-10 kV/A²	r_{off2}	0
r_{on3}	-200 MV/A³	r_{off3}	0

Analysis of Charge Sampling Circuit

◆Analysis Procedure

1. Expression as F-matrix

$$\begin{pmatrix} V_{\rm in}(\omega) \\ I_{\rm in}(\omega) \end{pmatrix} = \mathrm{F_{TA}} \bullet \mathrm{F_P} \bullet \mathrm{F_{MOS}} \bullet \mathrm{F_C} \bullet \begin{pmatrix} V_{\rm out}(\omega) \\ I_{\rm out}(\omega) \end{pmatrix}$$

⇩

2. Matrix and Volterra series operation for cascade connection. (Eq.1)

$$\mathrm{F}_{11} \Rightarrow G_1(\omega,t), G_2(\omega_1,\omega_2,t), G_3(\omega_1,\omega_2,\omega_3,t)$$

⇩

3. Derivation of Inverse of (1.1) component at F-matrix. (Appendix.1)

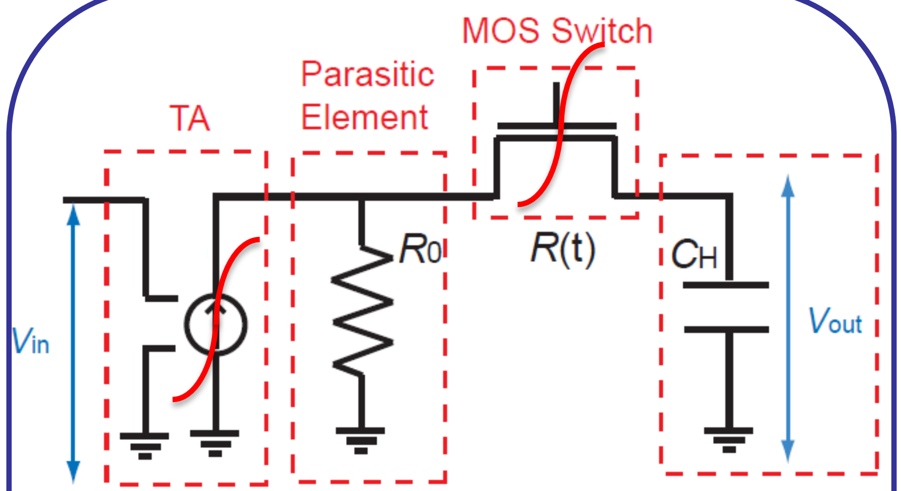
$$H_1(\omega,t), H_2(\omega_1,\omega_2,t), H_3(\omega_1,\omega_2,\omega_3,t)$$

⇩

4. Fourier series expansion. (Eq.2)

$$H_{n,1}(\omega), H_{n,2}(\omega_1,\omega_2), H_{n,3}(\omega_1,\omega_2,\omega_3)$$

➢Charge Sampling Circuit



$$\mathrm{F_{TA}} = \begin{pmatrix} 0 & A \\ 0 & A/R_{\rm TA} \end{pmatrix}, \mathrm{F_P} = \begin{pmatrix} 1 & 0 \\ 1/R_0 & 1 \end{pmatrix}$$

$$\mathrm{F_{MOS}} = \begin{pmatrix} 1 & R(t) \\ 0 & 1 \end{pmatrix}, \mathrm{F_C} = \begin{pmatrix} 1 & 0 \\ j\omega C_H & 1 \end{pmatrix}$$

➜ **Multidimensional & PTV Transfer Function**

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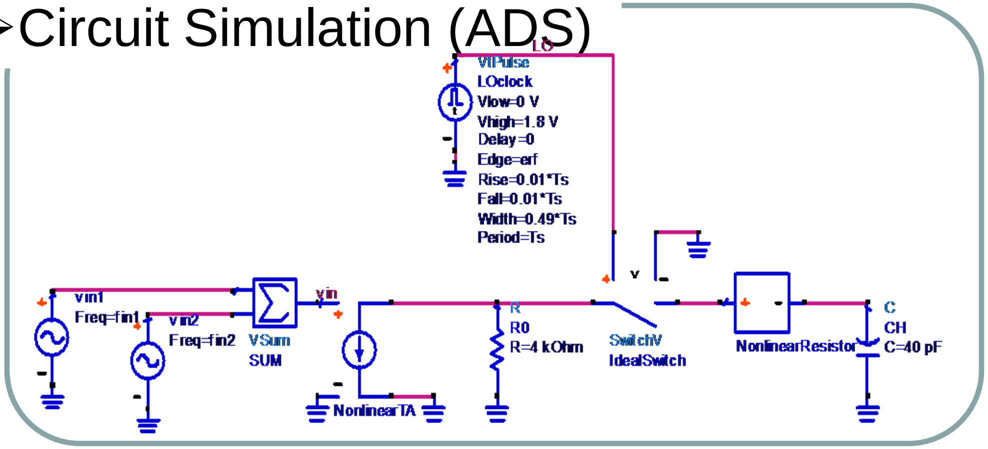
◆Simulation

➢Simulation Parameter

P_{in}	-15 dBm	C_H	40 pF
f_{LO}	500 MHz	C_0	0.3 pF
R_0	4 kΩ	V_G	0 ~ 1.8 V
A	(Eq. 3)	**R(t)**	(Table.1)

➢Circuit Simulation (ADS)



•Input Frequency : $f_{in1} + f_{LO}$ and $f_{in2} + f_{LO}$ (2 tone signal)

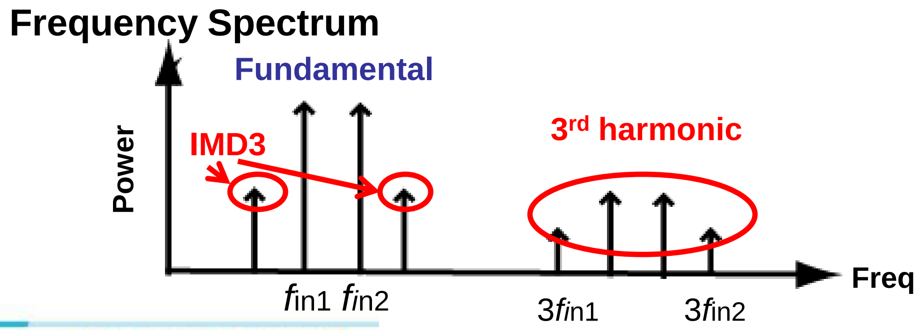
⇒ Output Frequency are f_{in1}, f_{in2}, and 3rd harmonic signal are down converted near $3f_{in1}$ and $3f_{in2}$ due to nonlinearity.

•Differential operation

⇒2nd distortion is canceled

∗ V_G →gate voltage of switch

∗ IMD3 → 3rd Inter Modulation Distortion

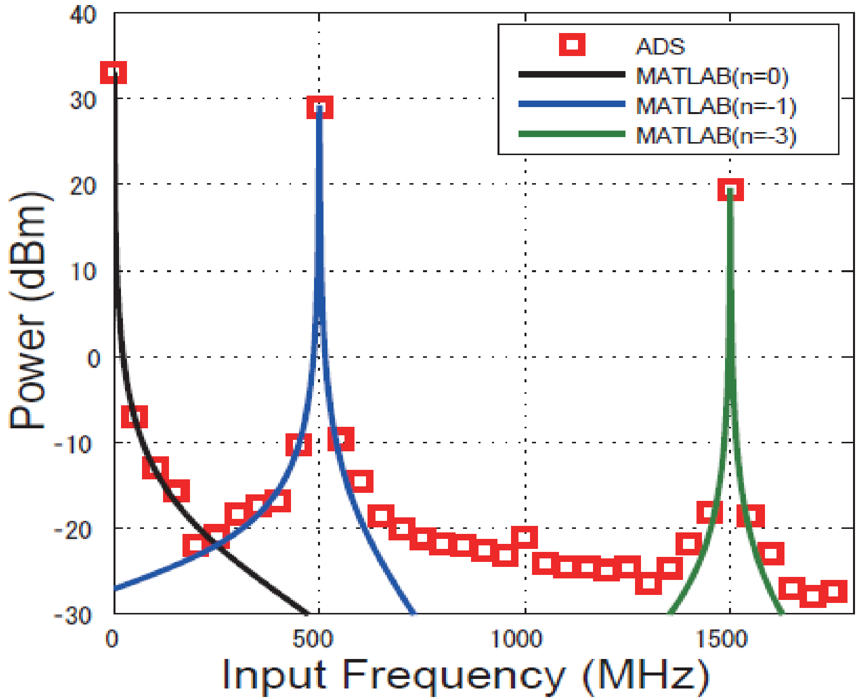


Simulation

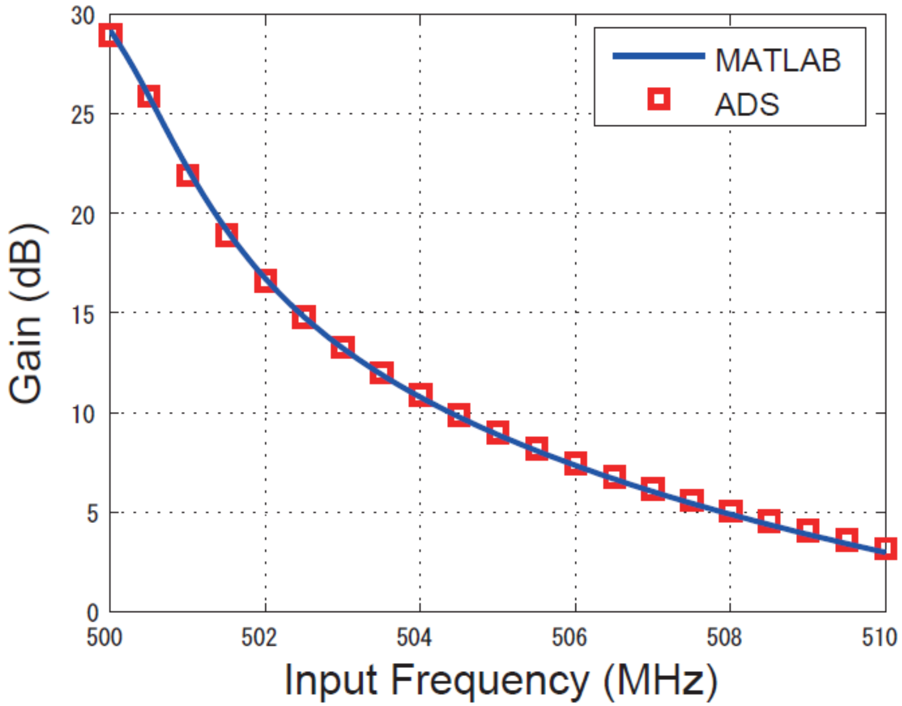
◆LPTV System : Frequency Response

➢First, we assume that charge sampling circuit is linear ($a_2=a_3=0$, $r_{on2}=r_{on3}=0$)

•Frequency Response



•Frequency Response near f_{LO}

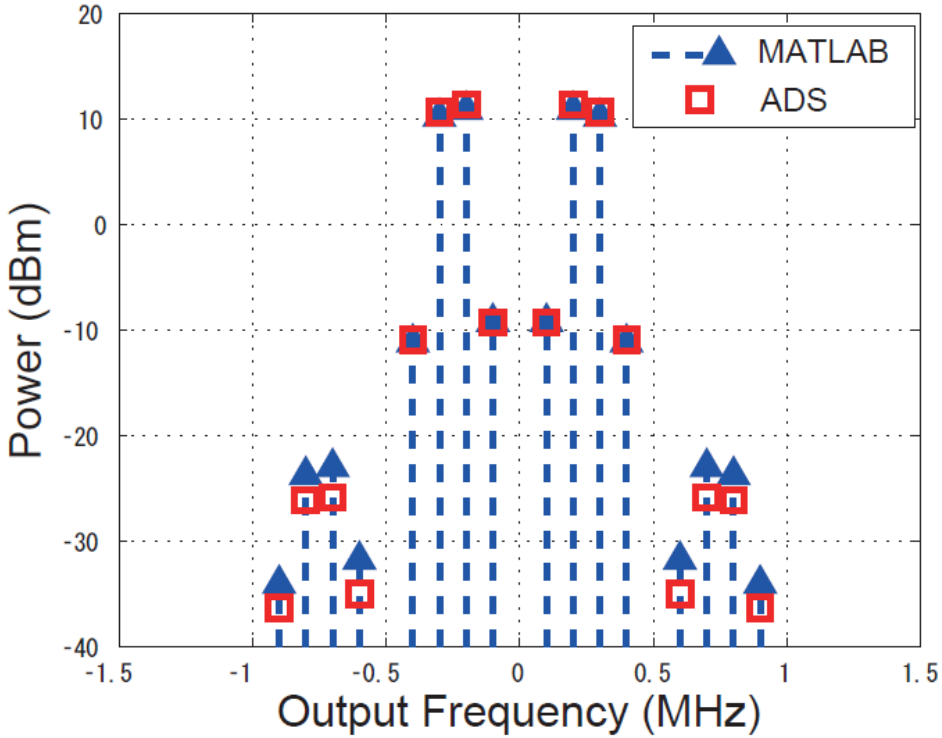


✓If input frequency is near nf_{LO}, MATLAB and ADS results are almost same.

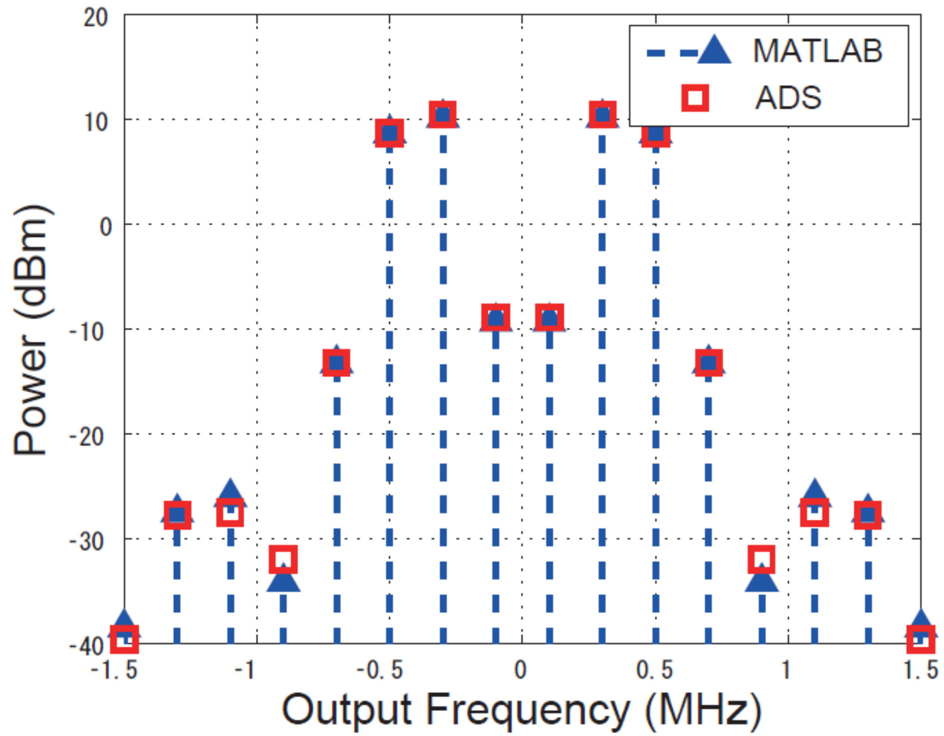
Simulation

◆NLPTV System : Frequency Spectrum

- f_{in1} = 0.2MHz, f_{in2} = 0.3MHz



- f_{in1} = 0.3MHz, f_{in2} = 0.5MHz



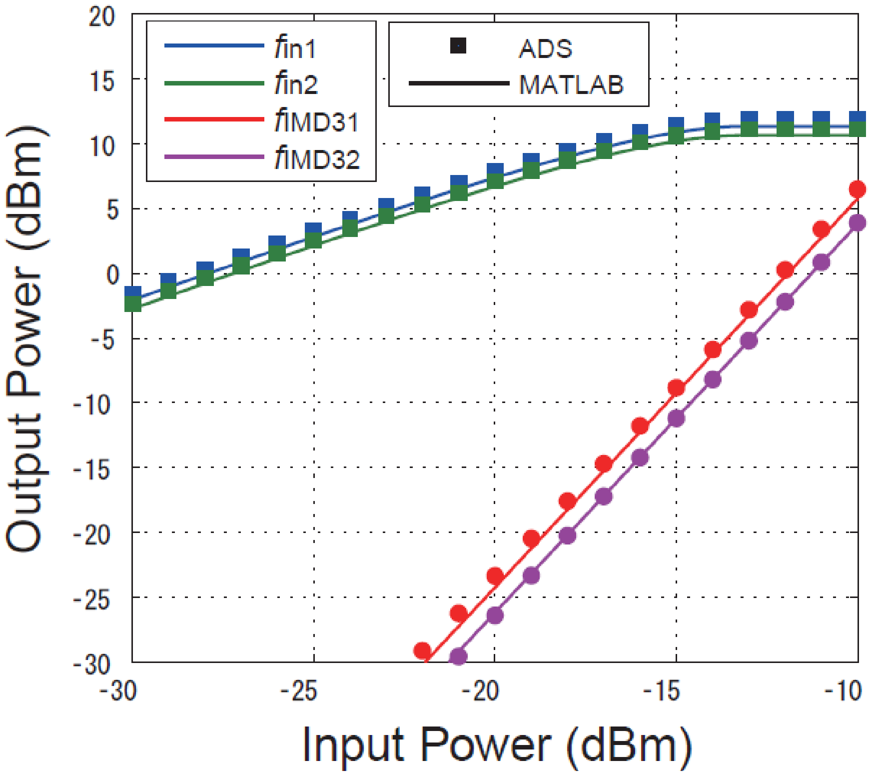
✓From frequency spectrum, small difference is shown between MATLAB and ADS, but <u>these are almost matched at fundamental and IMD3 components</u>.

Simulation

◆NLPTV System : AM-AM Characteristic

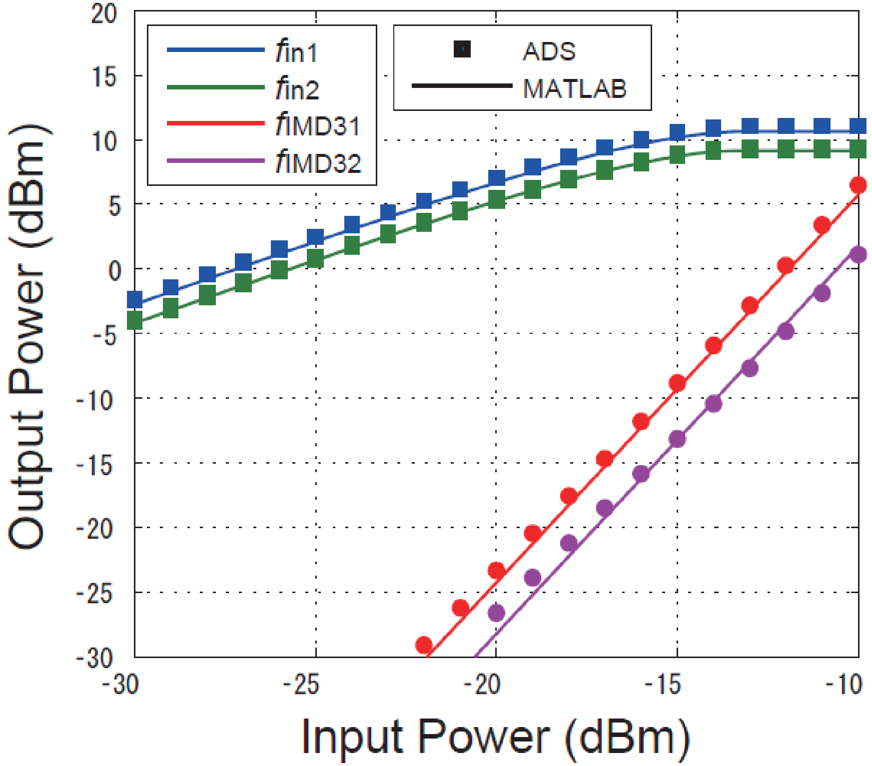
- f_{in1} = 0.2MHz, f_{in2} = 0.3MHz

($f_{IMD31}=2f_{in1}-f_{in2}$=0.1MHz, $f_{IMD32}=2f_{in2}-f_{in1}$=0.4MHz)



- f_{in1} = 0.3MHz, f_{in2} = 0.5MHz

($f_{IMD31}=2f_{in1}-f_{in2}$=0.1MHz, $f_{IMD32}=2f_{in2}-f_{in1}$=0.7MHz)



✓Difference between MATLAB and ADS is less than 0.5dB.
✓IIP3 (3rd Input Intercept Point) is about -4 ~ -3dBm.

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Conclusion and Future Work

◆Conclusion

➢The simple nonlinear periodic time-variant system can be analyzed with F-matrix, Volterra series function and Fourier series expansion.

➢When input frequency is near the clock frequency, this analysis has same result with circuit simulation result.

◆Future Work

➢Improvement of this analysis model (more exact, more complex circuit architecture)

➢Invention of efficient distortion compensation method.

Appendix.1

◆Volterra Inverse

$$F_1(\omega_1)=1$$
$$F_2(\omega_1,\omega_2)=0$$
$$F_3(\omega_1,\omega_2,\omega_3)=0$$

➢If G is inverse function of H, G can be expressed as this.

$$H_1(\omega_1)=G_1^{-1}(\omega_1)$$

$$H_2(\omega_1,\omega_2)=\frac{-G_2(\omega_1,\omega_2)}{G_1(\omega_1+\omega_2)G_1(\omega_1)G_1(\omega_2)}$$

$$H_3(\omega_1,\omega_2,\omega_3)=\left\{-G_3(\omega_1,\omega_2,\omega_3)+\frac{2G_2(\omega_1,\omega_2+\omega_3)G_2(\omega_2,\omega_3)}{G_1(\omega_2+\omega_3)}\right. / G_1(\omega_1+\omega_2+\omega_3)G_1(\omega_1)G_1(\omega_2)G_1(\omega_3)$$