# A MAP Estimation of Rayleigh Fading Channel

-- A Filter Theory of Complex Gaussian Process --

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## Contents



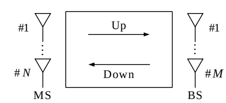


## Background & Motivation

- Recursive Simulation Method for Rayleigh Fading Channel.
  - How to write a computer program ?
- Fading Channel Coefficients should be estimated in SDMA PHS Systems



- Mobile Communication Channel with MIMO Systems
  - Time Variant Linear Reciprocal System




For $(N+M)$-port Circuit, a $(N+M)\times(N+M)$ scattering matrix is defined;

$$S(f,t)=\begin{array}{c} \\ \\ \end{array}\begin{array}{cc} N & M \end{array}$$
$$S(f,t)=\begin{bmatrix} S_{MM} & S_{BM} \\ S_{MB} & S_{BB} \end{bmatrix}\begin{array}{l} N \\ M \end{array}$$



where

$S_{BM}$ : $M\times N$ Transfer Matrix of Up-Link from MS to BS

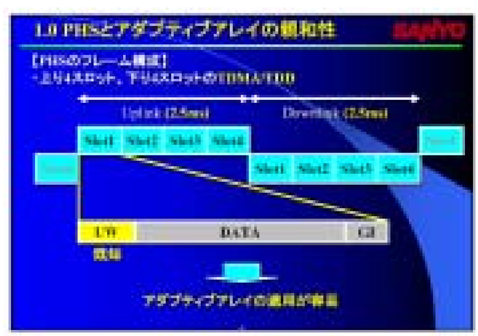
$S_{MB}$ : $N\times M$ Transfer Matrix of Down-Link from BS to MS

By the reciprocity,

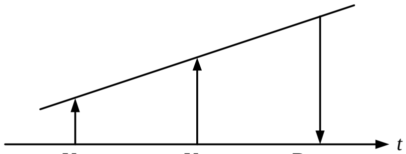
$$S = S^t$$
$$\therefore\quad S_{MB}(f,t) = S_{BM}(f,t)^t$$

Thus, The Down-Link Transfer Characteristics can be determined by the Up-Link one.
The above equality, however, holds only for the same frequency and time.





- Conventionally
  - Linear Extrapolation for Fading coefficient is used.




  - Noise Filtering is not taken into account.



# Complex Gaussian Stochastic Process

1) Rayleigh (or Rice) Fading Coefficient: $X(t)$
2) Random White Gaussian Noise : $Y(t)$

$$\downarrow$$

3) Rayleigh Fading Coefficient contaminated with Noise:

$$Z(t) = X(t) + Y(t)$$



Stationary Gaussian Process can be characterized only by Autocorrelation Function

$$R_{ZZ}(T) = \overline{Z(t)Z(t+\tau)}$$
$$= R_{XX}(\tau) + R_{YY}(\tau)$$

where

$$R_{XX}(\tau) = A\, \mathrm{J}_0(2\pi\ f_D \tau)$$



$A = \overline{|X(t)|^2}$ : Average Fading Level

$\mathrm{J}_0$ : 0th Bessel Function

$f_D$ : Maximum Doppler Frequency $\left(= f\,\frac{v}{c}\right)$

$f$ : Carrier Frequency

$v$ : velocity of MS

$c$ : velocity of Light

$$R_{YY}(\tau) = \begin{cases} N & (\tau = 0) \\ 0 & (\tau \neq 0) \end{cases}$$

$N = \overline{|Y(t)|^2}$ : Average Noise Level



For MAP Estimation, Cross-correlation Function is also needed

$$R_{ZX}(\tau) = \overline{Z(t)X(t+\tau)} = \overline{(X(t)+Y(t))X(t+\tau)}$$
$$= \overline{X(t)X(t+\tau)} = R_{XX}(\tau)$$

$\because$ $X(t)$ and $Y(t)$ are independent.



- MAP (LS) Estimation and Optimal Noise Reduction
  - Wiener-Hopf Equation

Optimal Linear Combination Estimator Vector : $\mathbf{b}$

$$\begin{bmatrix} 1+\frac{N}{A} & \mathrm{J}_0(2\pi\ f_D(t_1-t_0)) & \cdots & \mathrm{J}_0(2\pi\ f_D(t_{n-1}-t_0)) \\ & \ddots & & \\ & & \ddots & \\ & & & 1+\frac{N}{A} \end{bmatrix} [\mathbf{b}] = \begin{bmatrix} \mathrm{J}_0(2\pi\ f_D(t_n-t_0)) \\ \vdots \\ \vdots \\ \mathrm{J}_0(2\pi\ f_D(t_n-t_{n-1})) \end{bmatrix}$$

MAP Estimator for $X(t_n)$

$$X(t_n)_{\mathrm{MAP}} = \mathbf{b}^{\dagger}\mathbf{Z}$$

where

$$\mathbf{Z} = (Z(t_0), \ldots, Z(t_{n\text{-}1}))^t \text{ : Observed Noise Data}$$

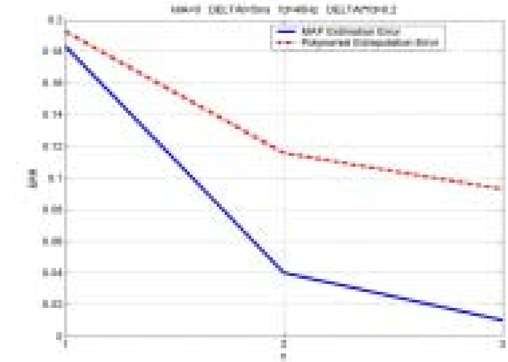
# Numerical Results

(1) Noise Level : $N/A = 0, 0.1, 1$

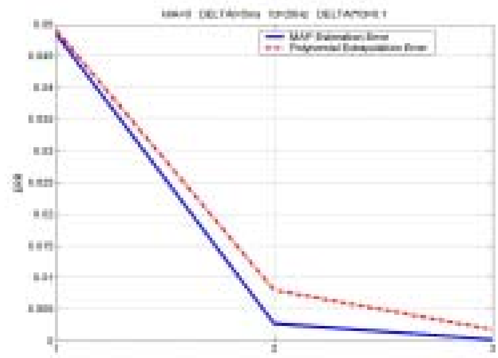
(2) Doppler Frequency : $f_D = 10, 20, 40$[Hz]

(3) No. of Data : $n = 1, 2, 3$

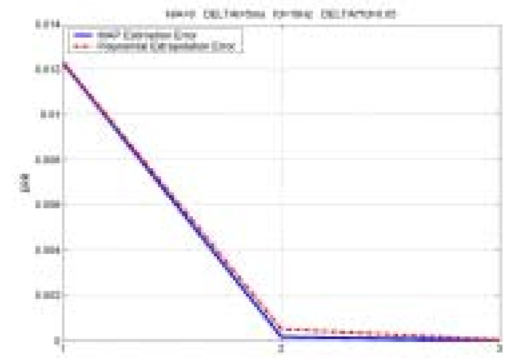




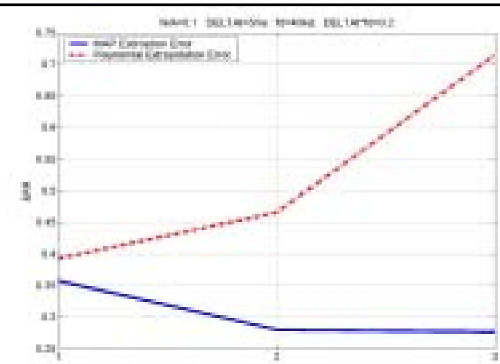


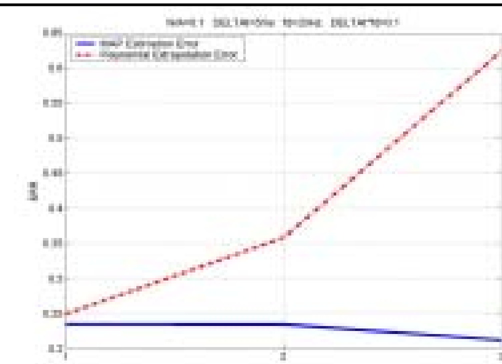


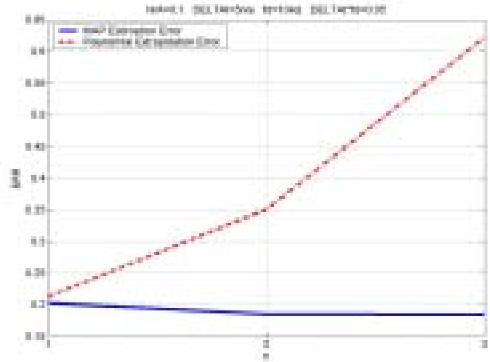





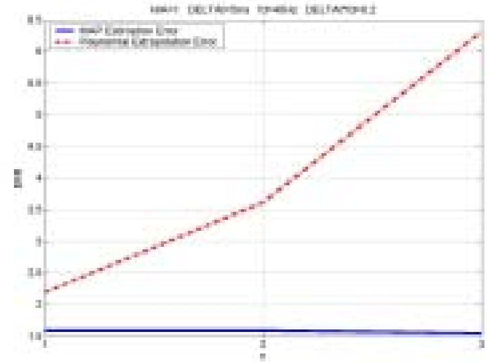





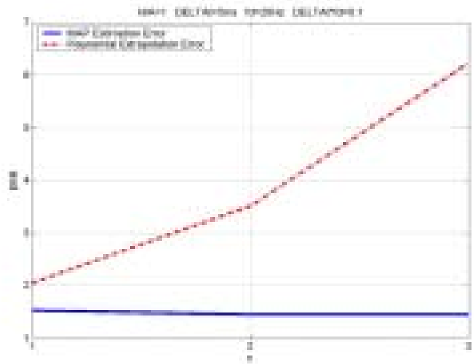





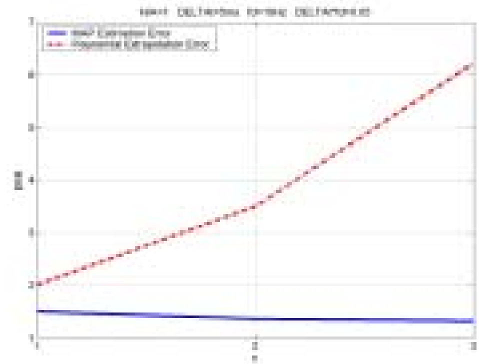














# Conclusion

- Estimation of Fading Coefficient is useful for TDMA/ TDD.
- Conventional Estimation is not satisfactory.
- Estimation Error can be greatly reduced by MAP Estimation.



# Future Works

- Extension to Estimation of Fading Channel Matrix
- Learning (or Estimation) of Model Parameters; $N$, $A$, $f_D$
- Extension to Rice Fading from Rayleigh Fading
- Implementation of MIMO Systems