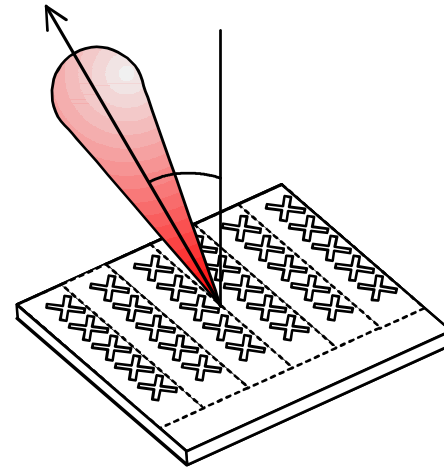


# 導波管スロットアンテナの 数値固有モード基底関数を用いた モーメント法解析



**Tokyo Institute of Technology**  
**Ando and Hirokawa Lab.**  
**Takada Lab.**  
**Takuichi Hirano**

# Outline

---

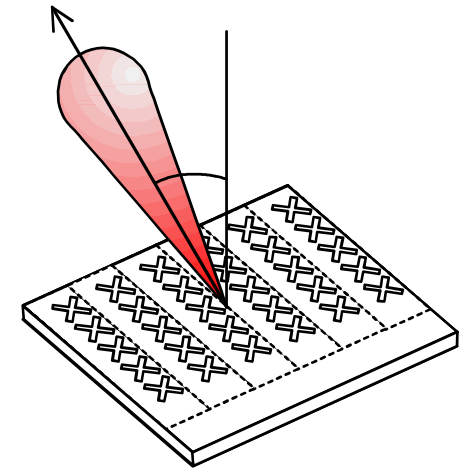
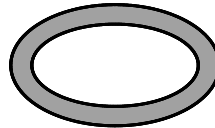
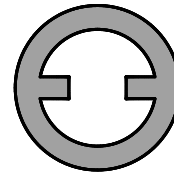
- ◇ **なぜMoM/FEM解析法が必要になったのか？**
- ◇ **MoM/FEM解析法の説明**
- ◇ **導波管クロススロットアレーの設計**

## Objective

# 導波管クロススロットアレーの設計 複雑(任意)な形状を有するスロットの解析

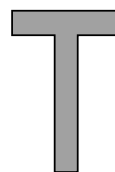
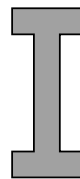
## 円偏波放射素子

クロススロット  
ノッチ付リングスロット  
楕円スロット  
など

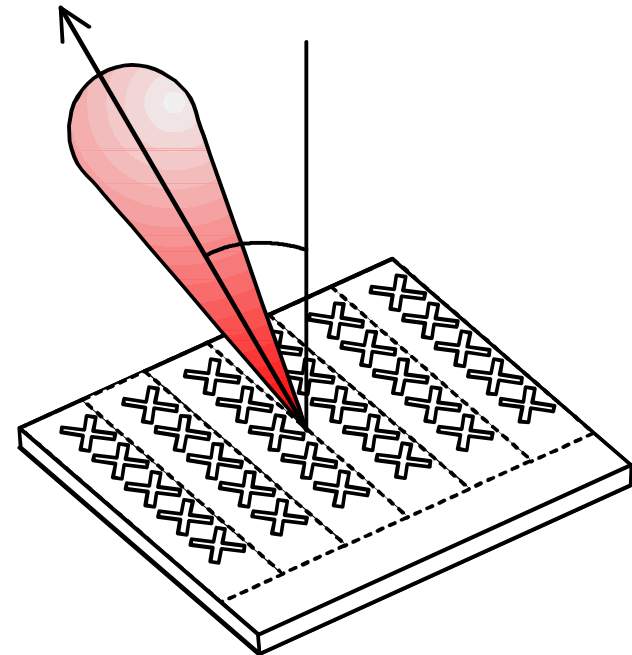
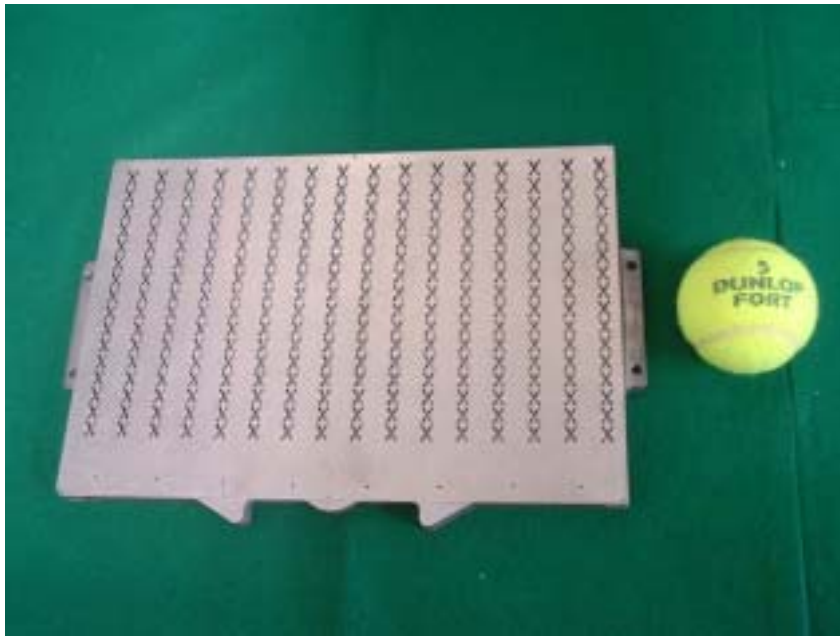


## 円偏波放射素子

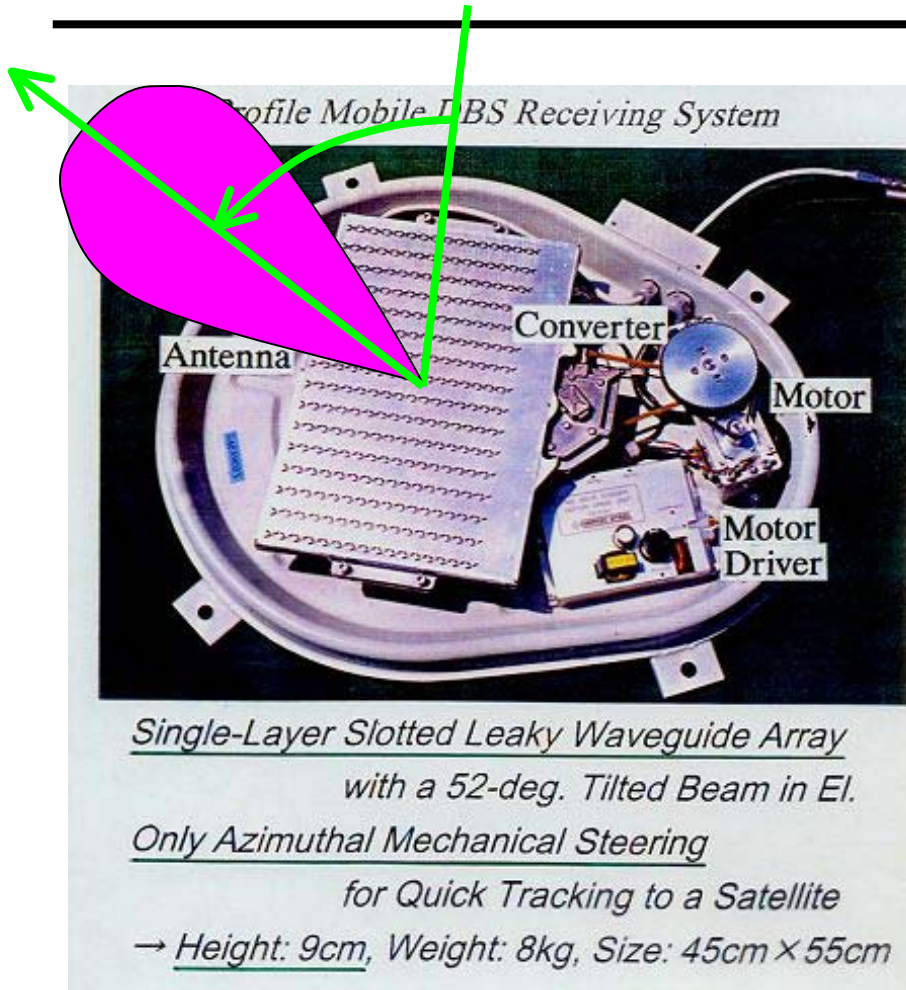
I字型スロット  
T字型スロット



# Waveguide Crossed Slot Array



# Past Work



MotorFan 1994. 11

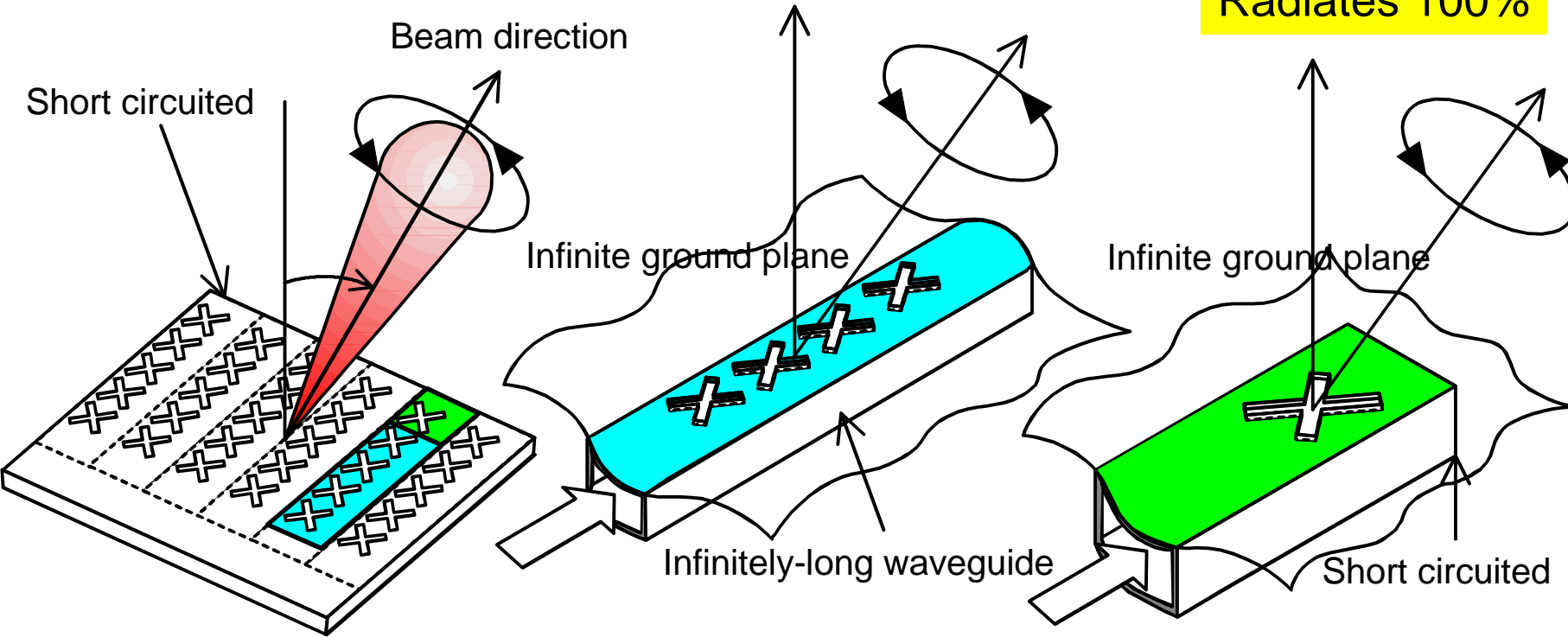
正確ではない

2本の直線スロットの合成として解析

# A Waveguide Crossed Slot Array

Circularly polarized wave

Radiates 100%

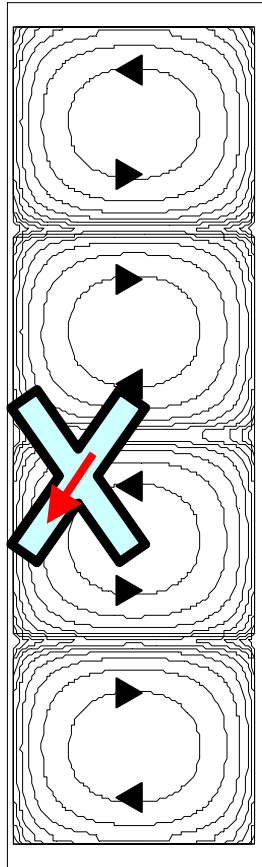


(a) A waveguide crossed-slot array

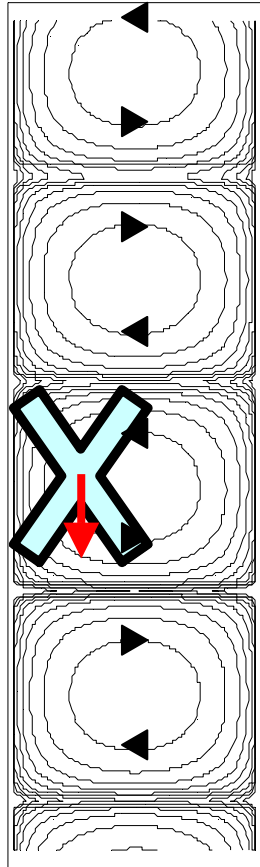
(b) A waveguide crossed-slot linear array

(c) A waveguide matching crossed-slot

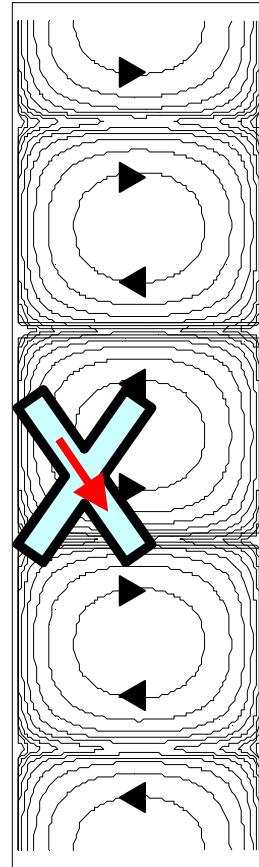
# クロススロットの円偏波放射原理 (直感的説明)



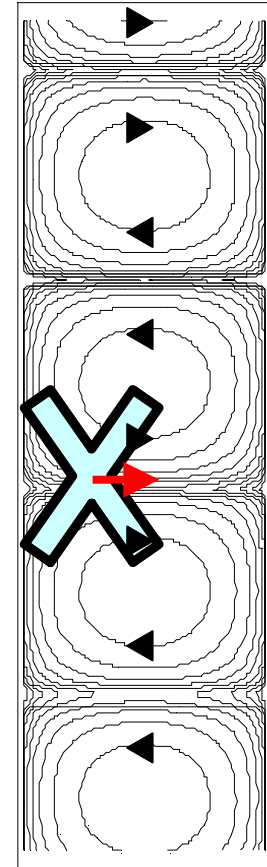
T1



T2



T3

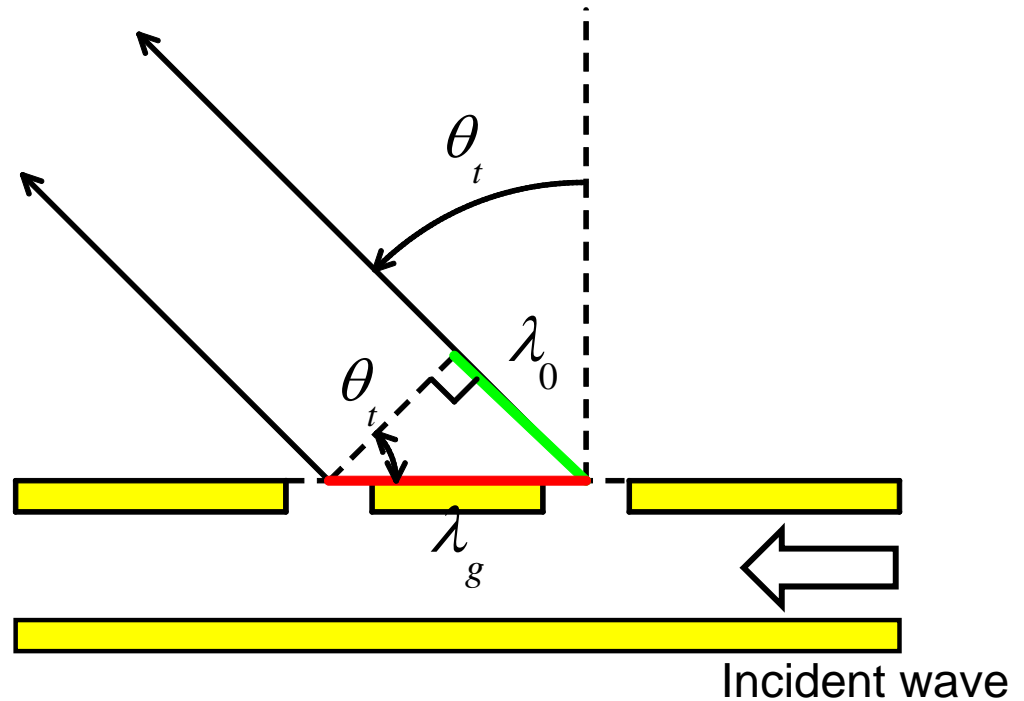


T4

# Principle of Leaky-wave Operation

$$\lambda_g \sin \theta_t = \lambda_0$$

$$\sin \theta_t = \frac{\lambda_0}{\lambda_g}$$

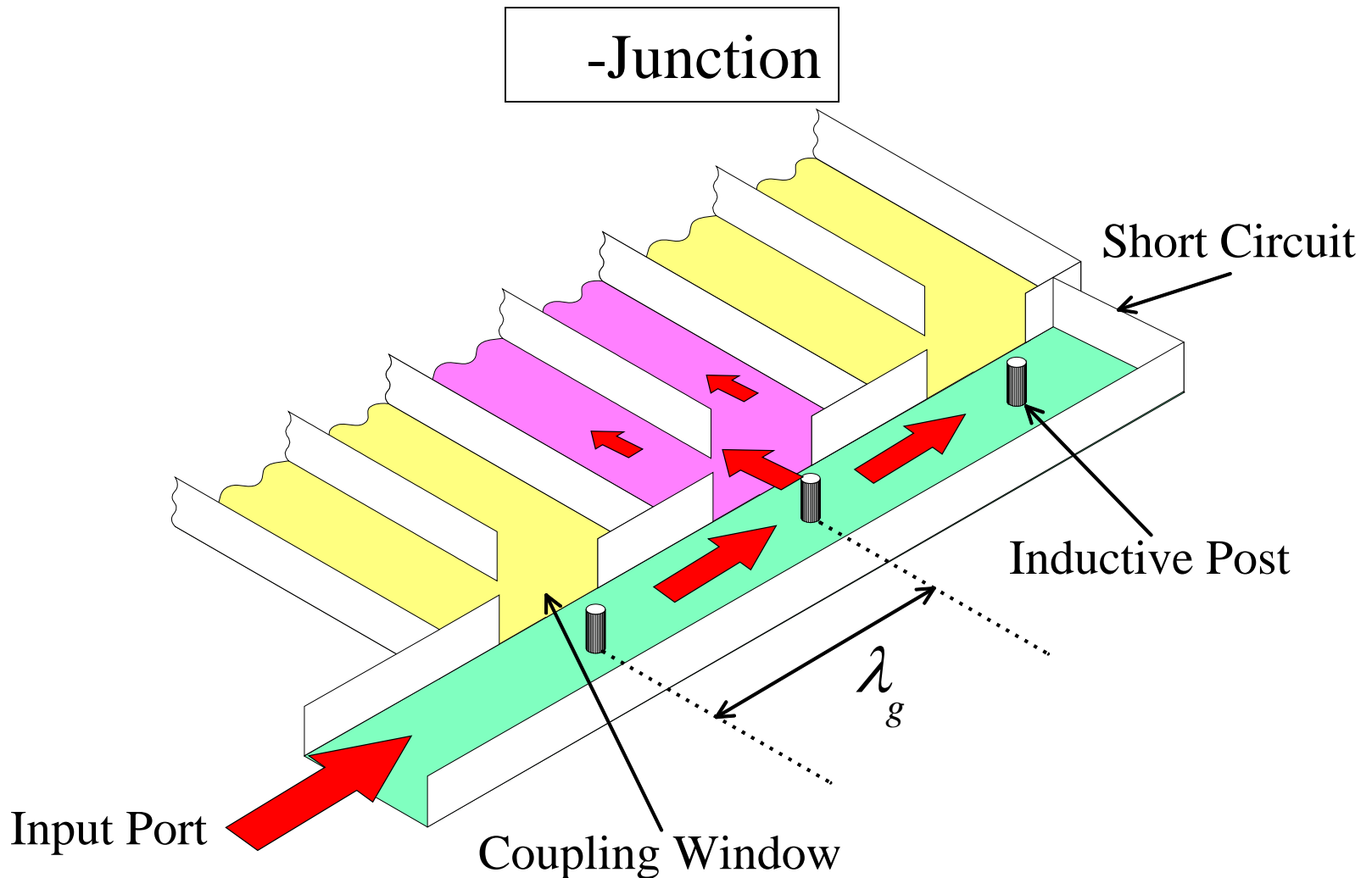


This relation is satisfied when the slot spacing is not equal to one guide wavelength.

**Beam direction is independent of slot spacing**

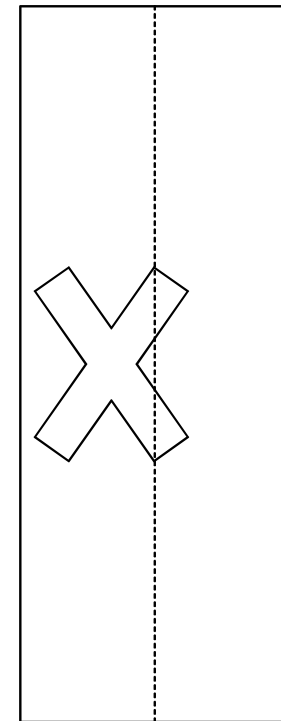
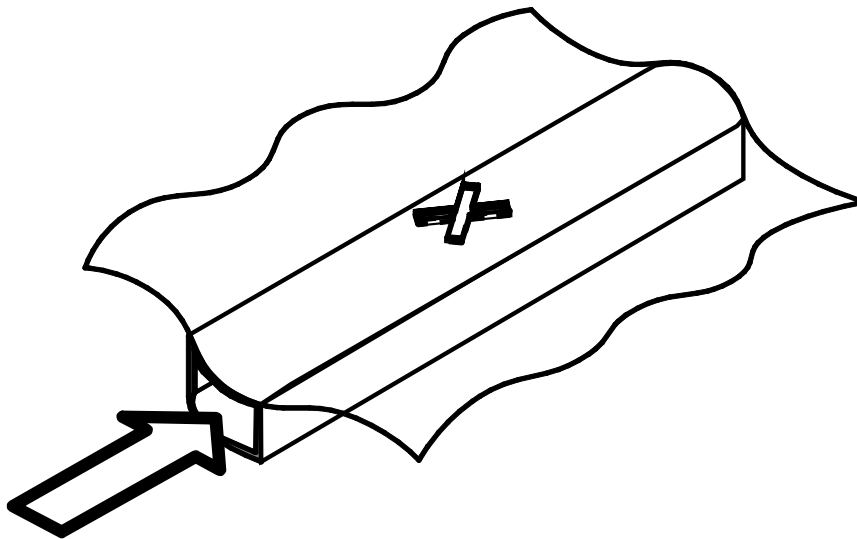


# Feeding Waveguide

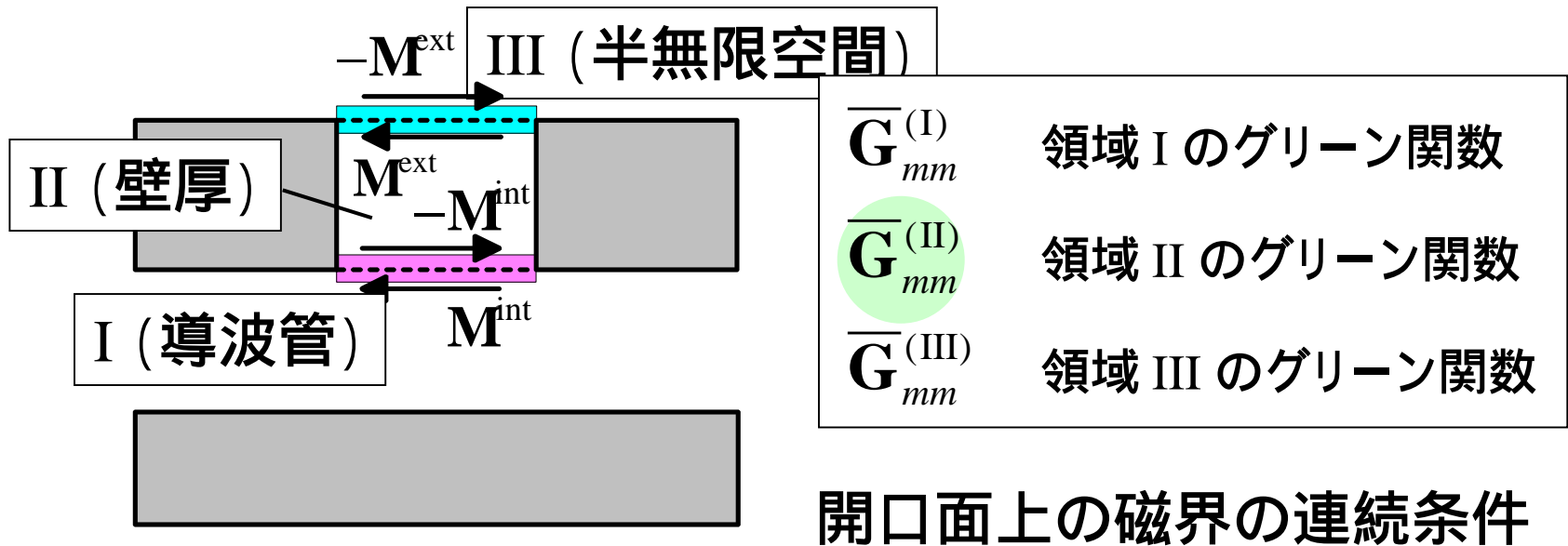


# Analysis of One Slot

## 解析法の確立 & 確認



## Integral Equations

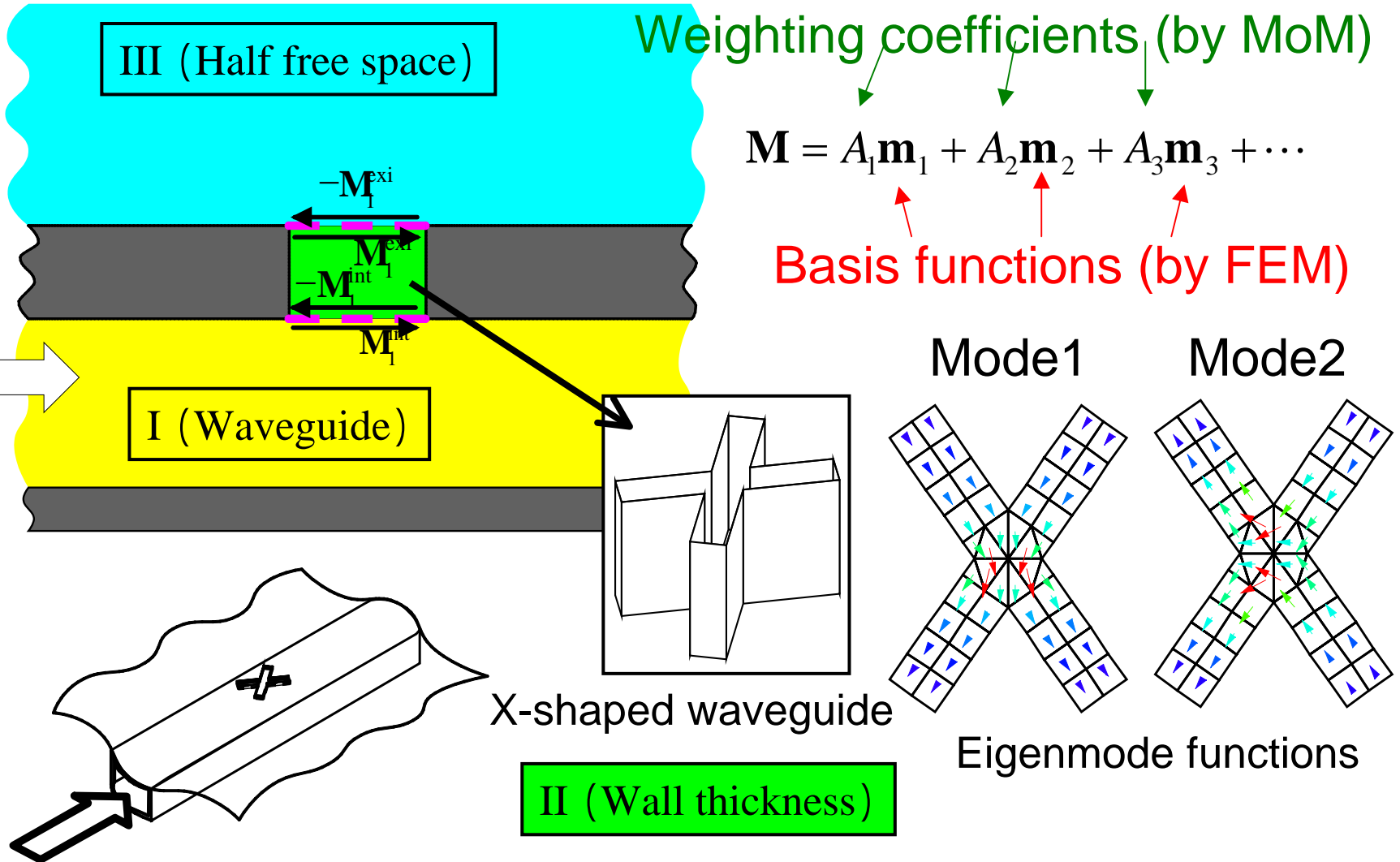


$$\iint_{S_{\text{int}}} \overline{G}_{mm}^{(II)} \cdot (-M^{\text{int}}) dS + \iint_{S_{\text{ext}}} \overline{G}_{mm}^{(II)} \cdot M^{\text{ext}} dS = \iint_{S_{\text{int}}} \overline{G}_{mm}^{(III)} \cdot (-M^{\text{ext}}) dS$$

$$\iint_{S_{\text{int}}} \overline{G}_{mm}^{(I)} \cdot M^{\text{int}} dS = \iint_{S_{\text{int}}} \overline{G}_{mm}^{(II)} \cdot (-M^{\text{int}}) dS + \iint_{S_{\text{ext}}} \overline{G}_{mm}^{(II)} \cdot M^{\text{ext}} dS$$

連立積分方程式

## MoM/FEM Analysis

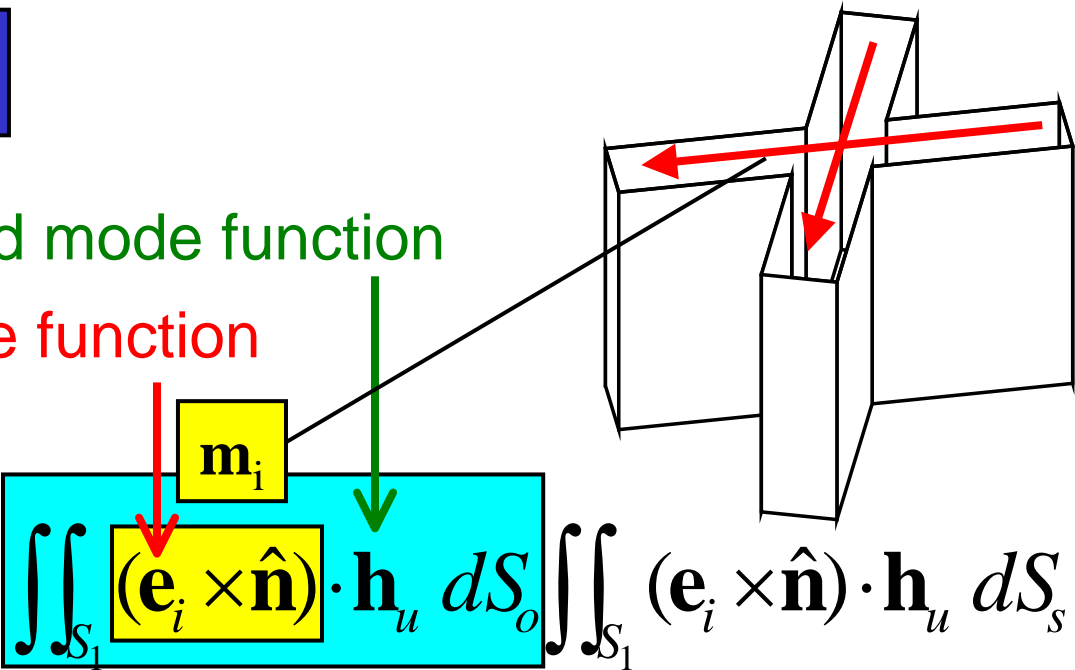


## Effect of the Wall Thickness

## Self-reaction

Magnetic field mode function

Electric field mode function



$$y_{ii} = -\sum_u \coth(\gamma_u t) \iint_{S_1} (\mathbf{e}_i \times \hat{\mathbf{n}}) \cdot \mathbf{h}_u dS_o \iint_{S_1} (\mathbf{e}_i \times \hat{\mathbf{n}}) \cdot \mathbf{h}_u dS_s$$

$$= -\coth(\gamma_i t) \quad (= \delta_{iu})$$

Orthogonality of mode functions

No mode summation

# Edge-based FEM

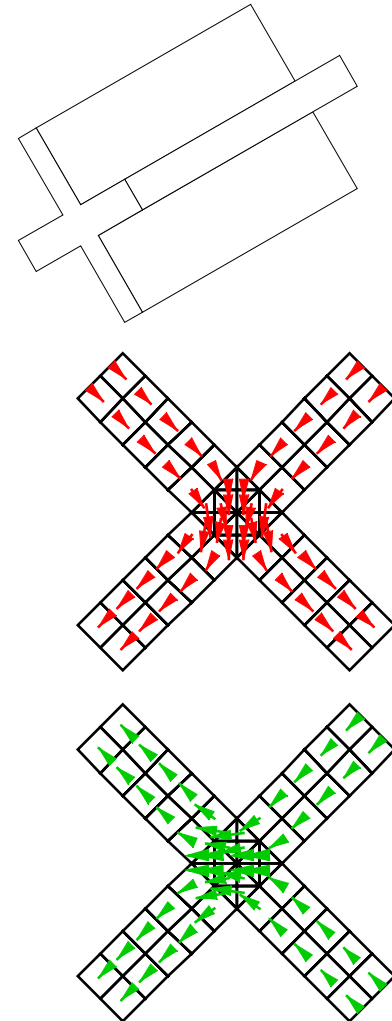
## 固有値問題

$$A\mathbf{x} = \lambda\mathbf{x}$$

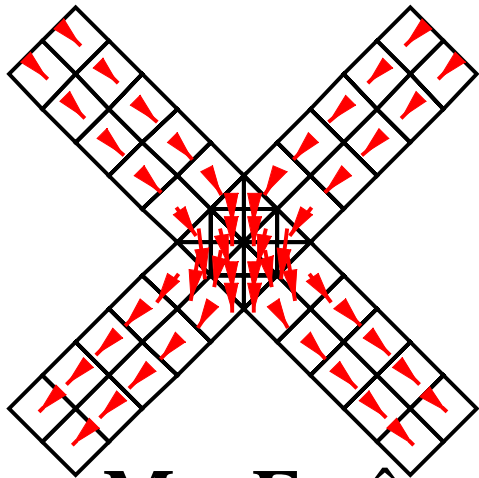
固有値  
カットオフ周波数 $f_{cu}$

固有ベクトル  
固有モード関数の界分布

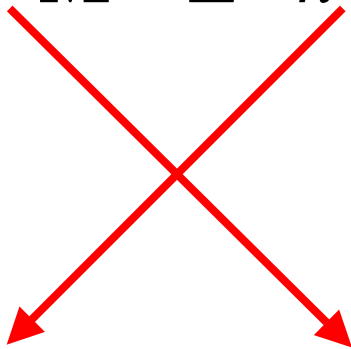
$$\mathbf{E}_u(\mathbf{r}) = \{ \mathbf{e}_{ut}(\mathbf{r}) + \mathbf{e}_{uz}(\mathbf{r}) \} e^{-\gamma_u z}$$



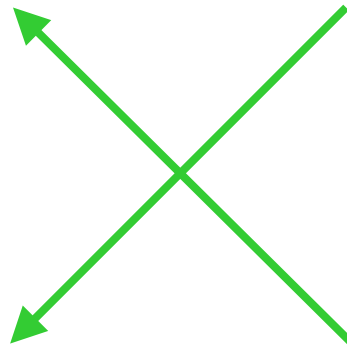
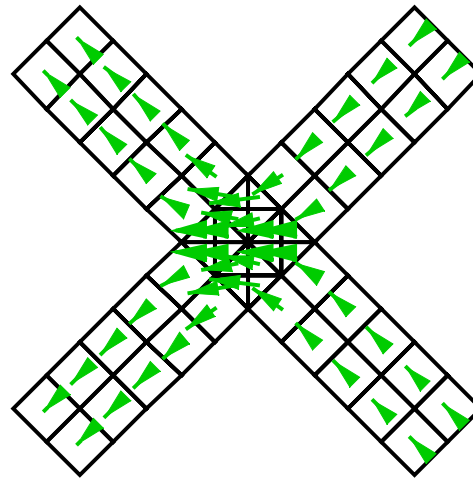
# Eigenmode Basis Functions



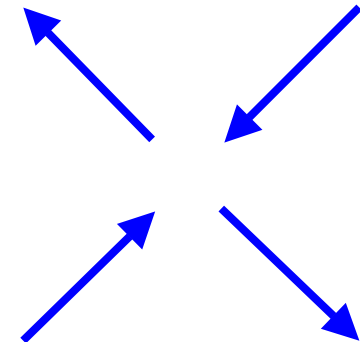
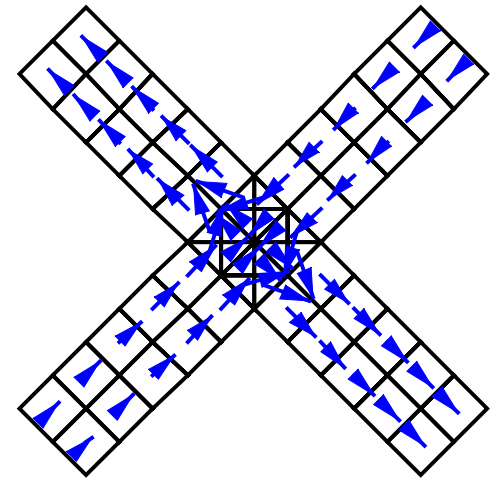
$$\mathbf{M} = \mathbf{E} \times \hat{n}$$



Mode1

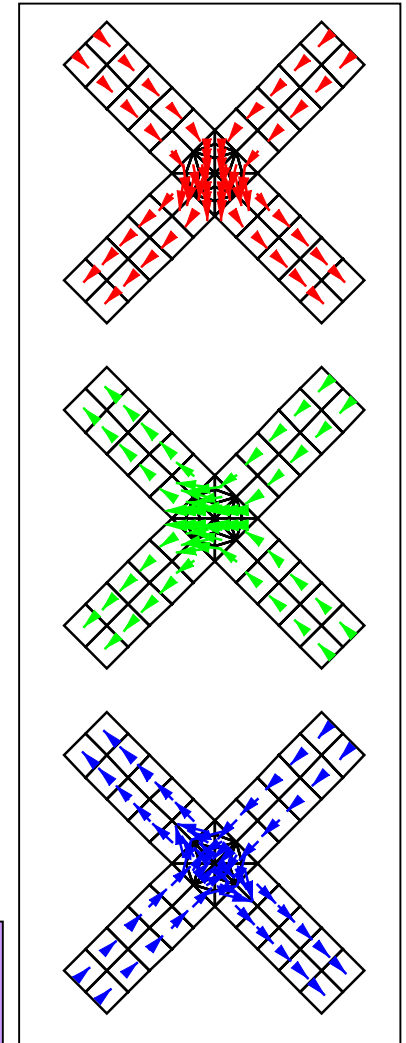
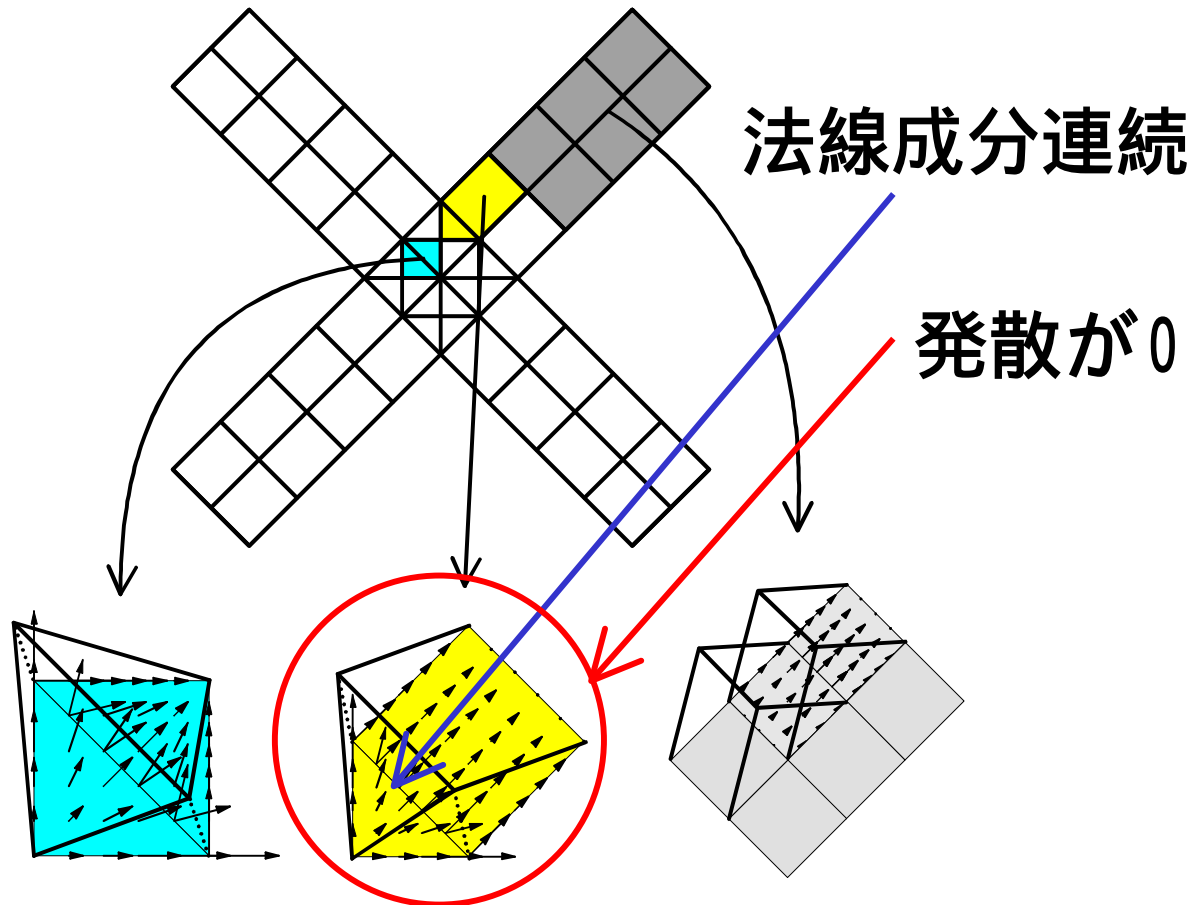


Mode2



Mode3

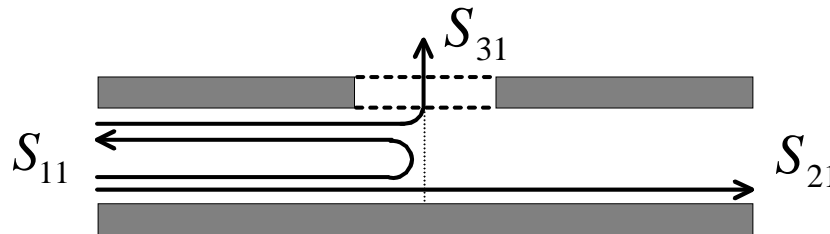
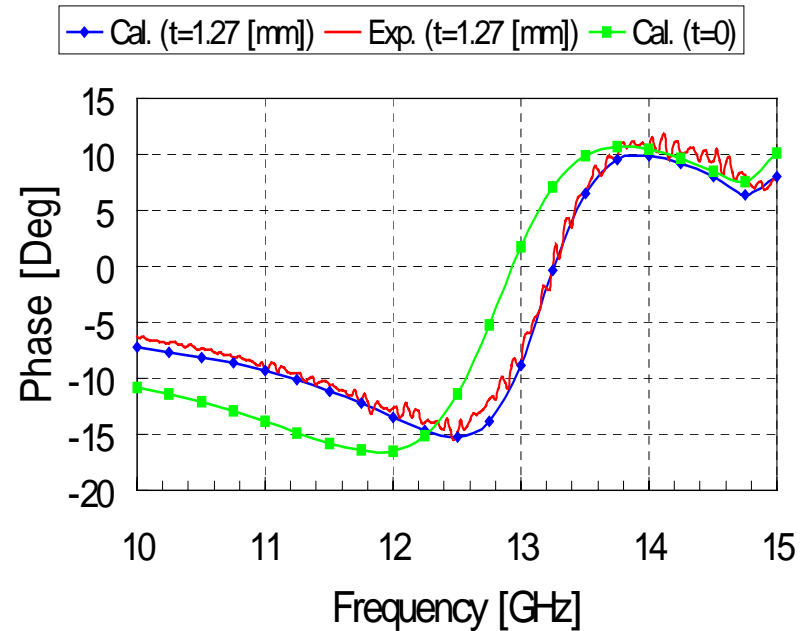
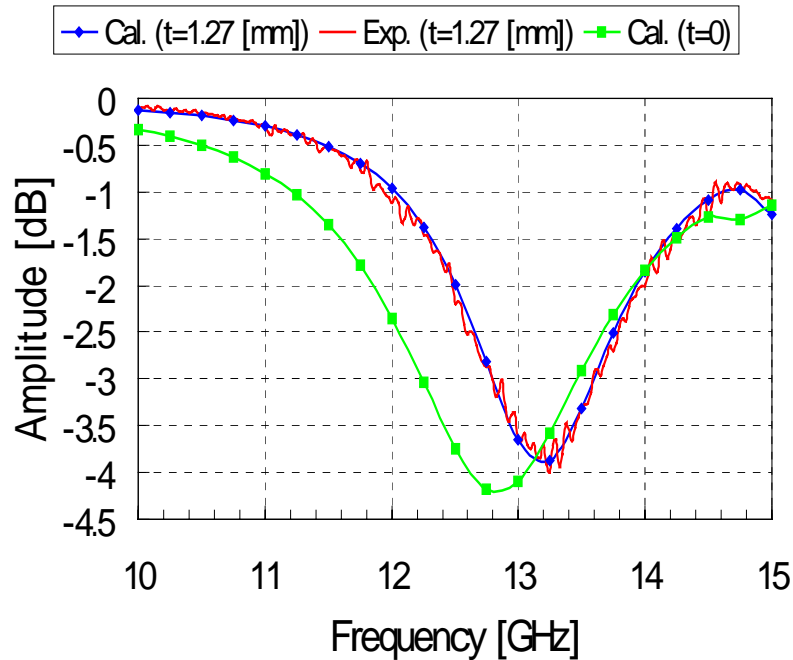
# ルーフトップ関数



Edge-based FEM 基底関数=MoM ルーフトップ関数  
 MoM のリアクションを従来の方法で計算できる

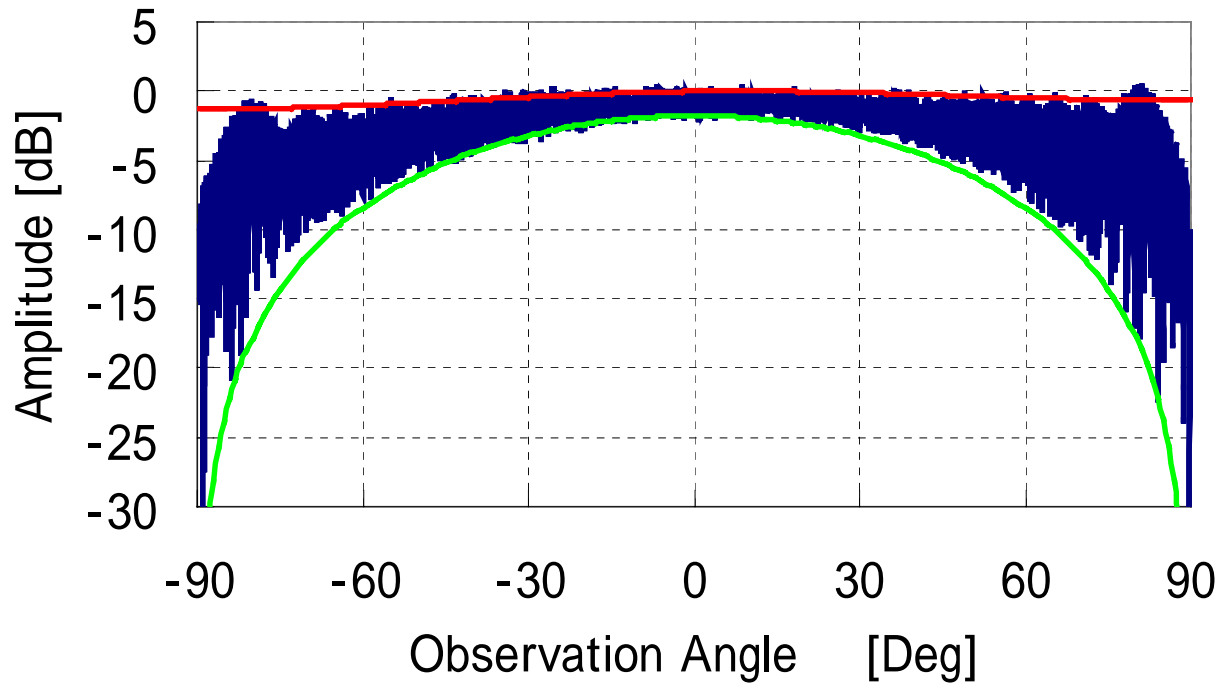


# Frequency vs. S21

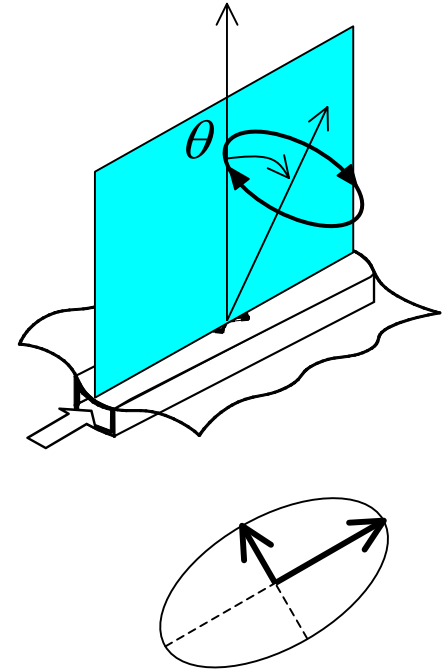


# Radiation Pattern

Radiation Pattern (Observation Angle = 90 ° , 12.0 GHz)



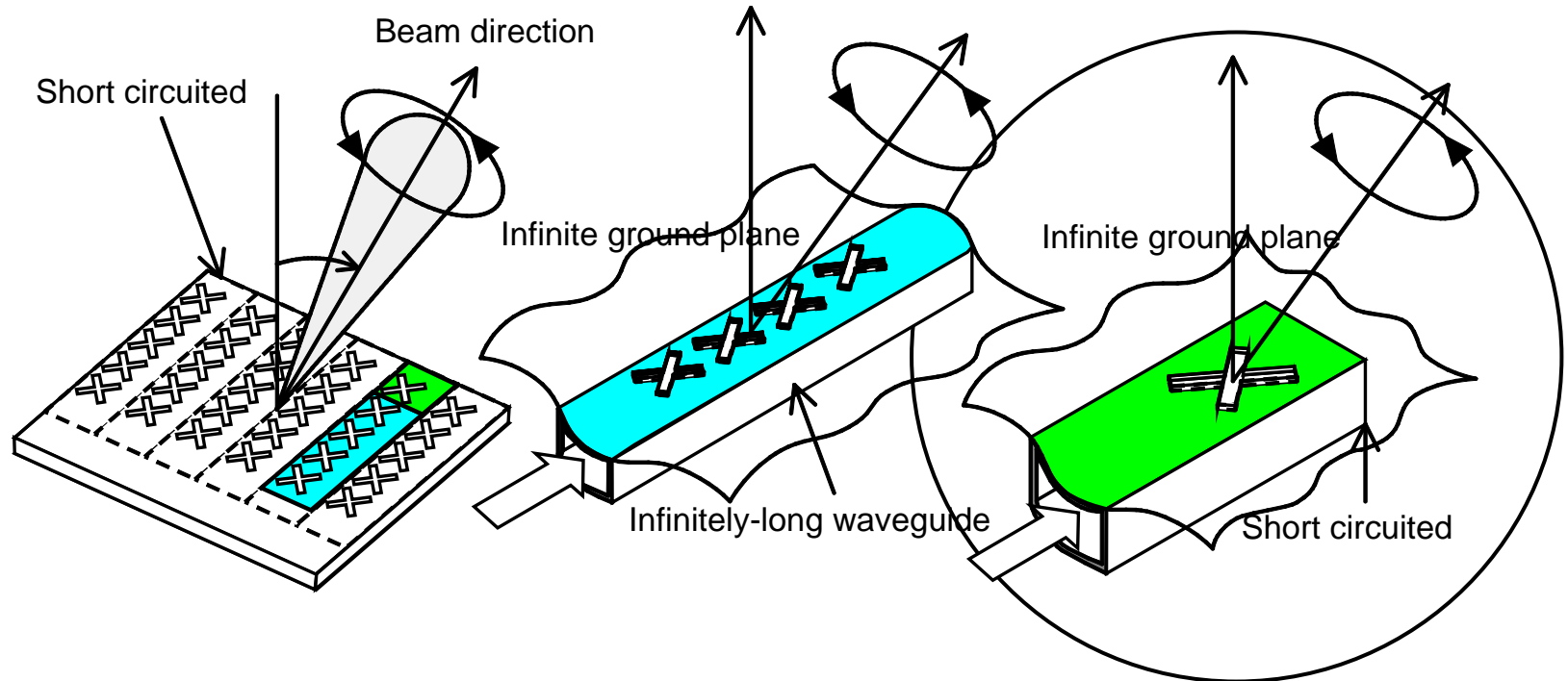
— Spin Linear (Exp.) — E<sub>max</sub> (Cal.) — E<sub>min</sub> (Cal.)



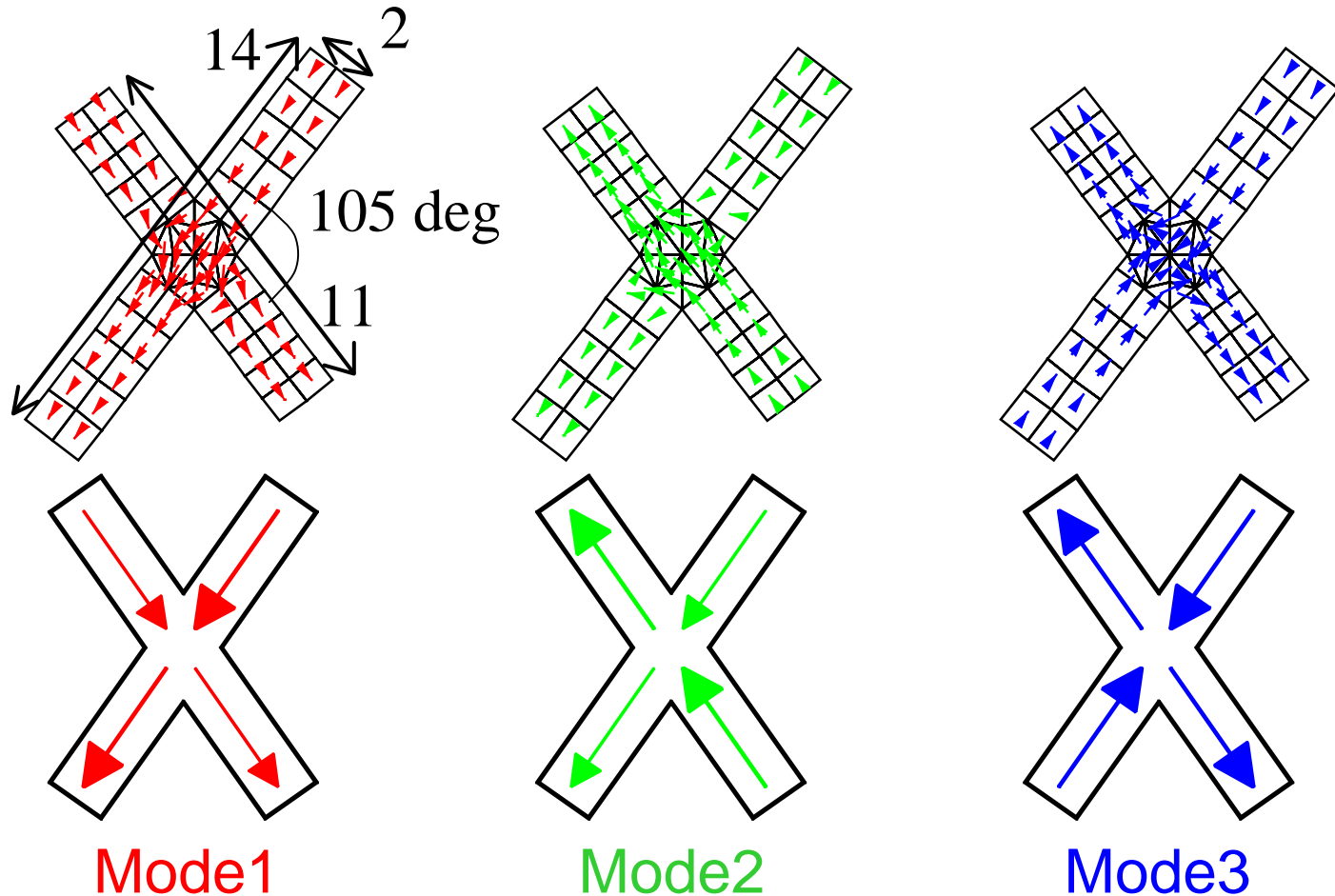
# 解析法の信頼性を確認できた



# Analysis of a Matching Crossed Slot

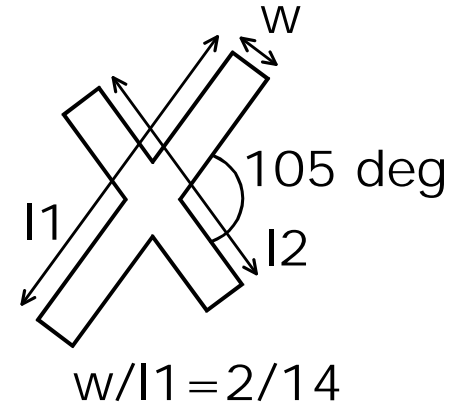
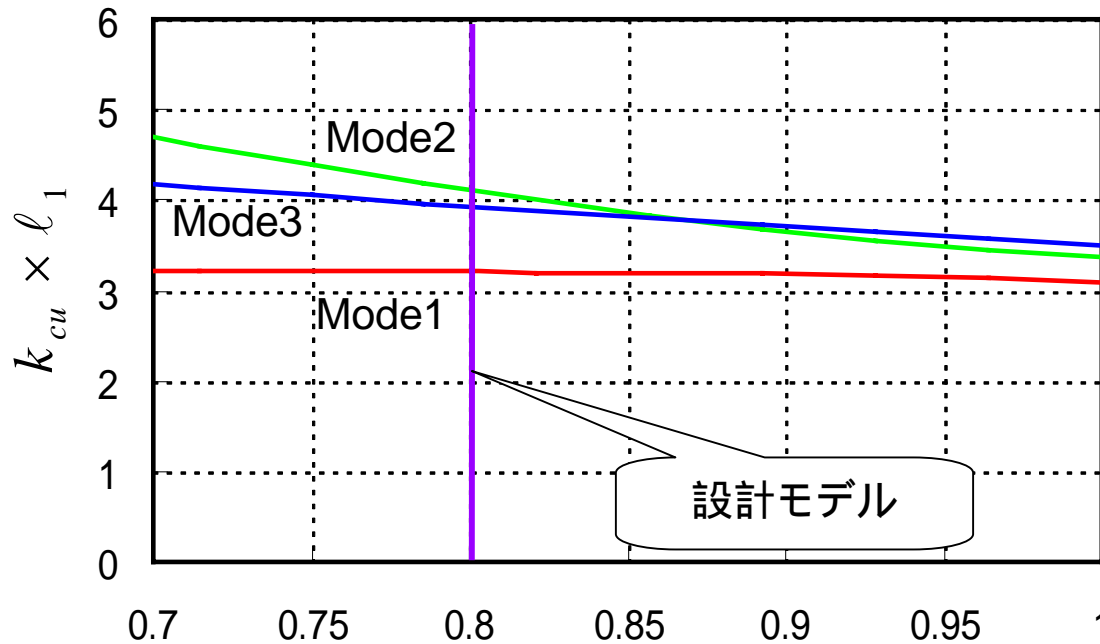


# X字型導波管の固有モード関数 (磁流基底関数)

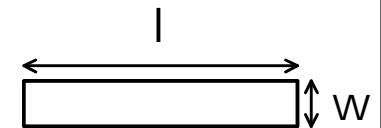


Cutoff Wavenumbers 1:1.3:1.24, Excitation coefs 1.1:1.18:0.00079

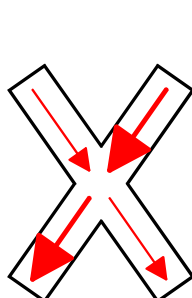
# カットオフ波数の変化



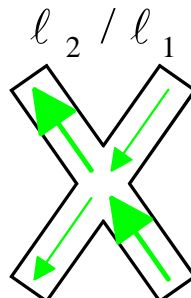
c.f. 方形導波管の  
カットオフ波数



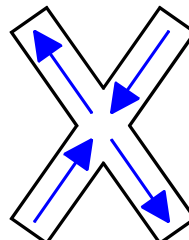
$$k_{cu} = \frac{\pi}{l}$$



Mode1

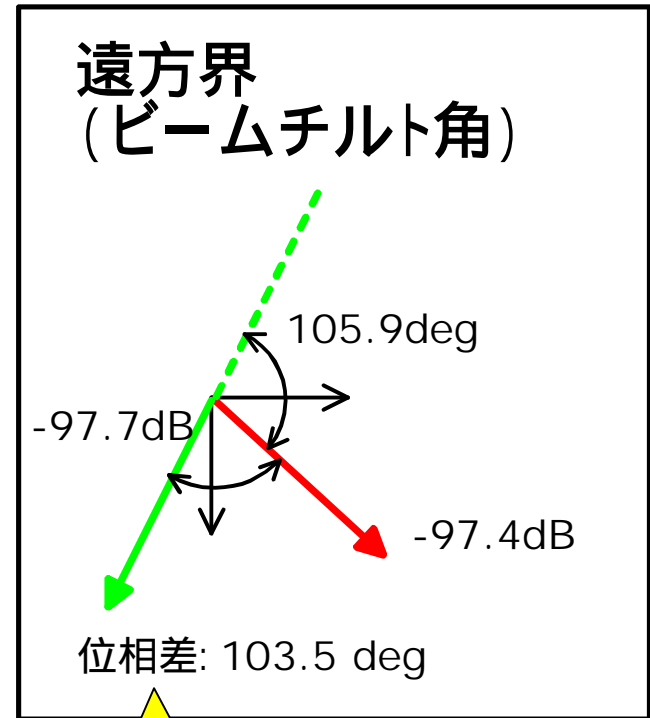
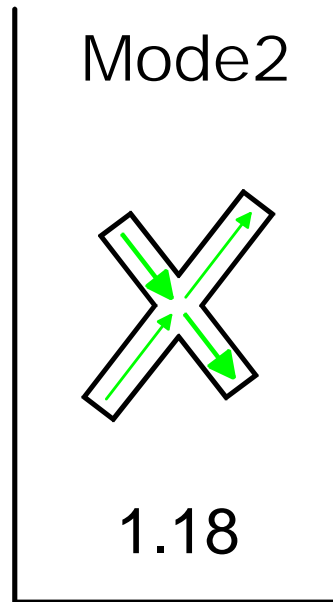
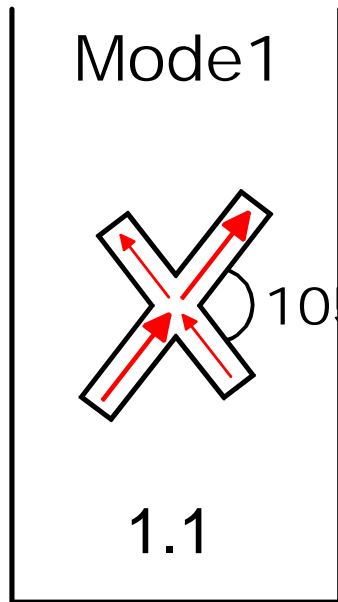


Mode2



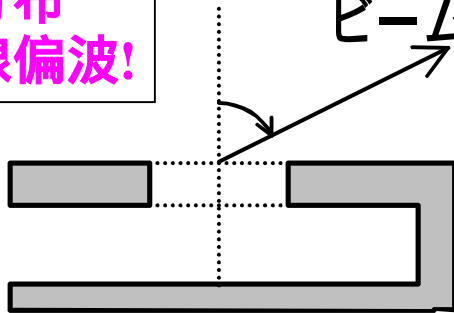
Mode3

# 円偏波放射原理



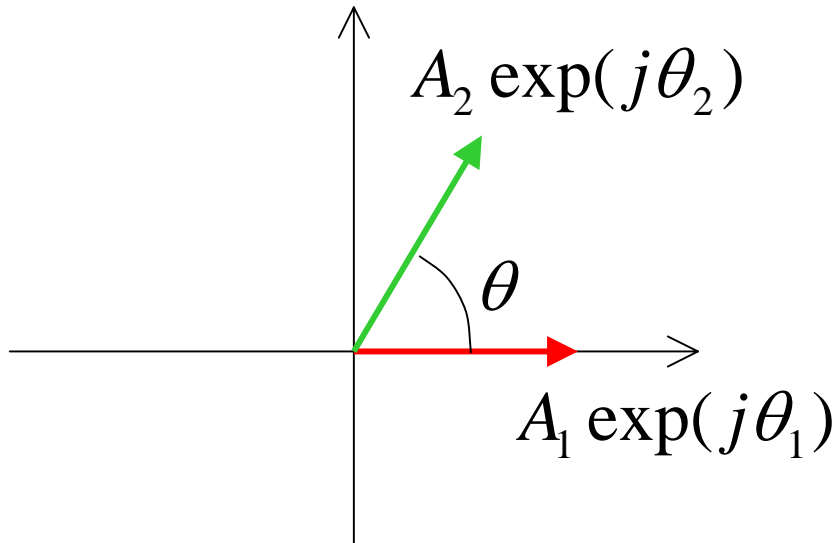
実数分布  
直線偏波!

ビームチルト方向



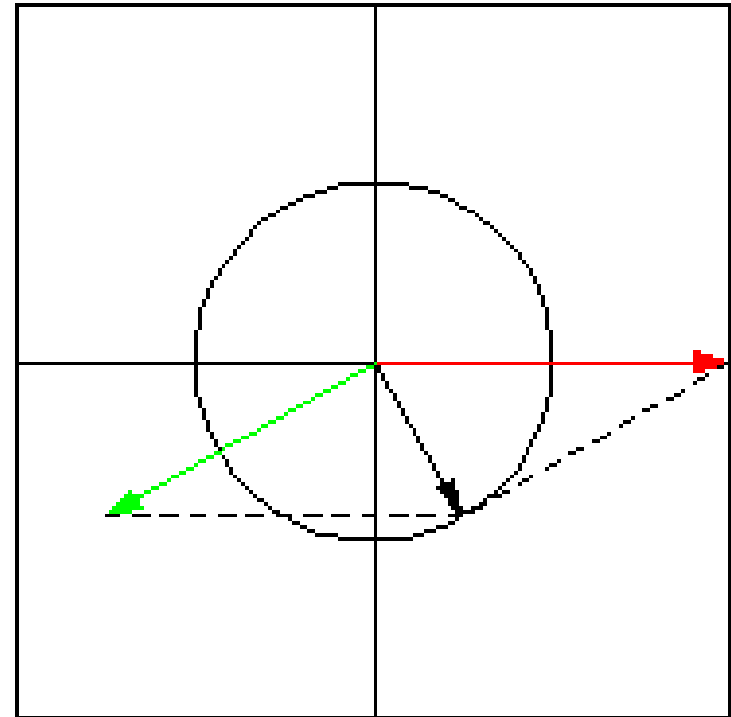
スロット長(共振周波数)  
の差による

## 円偏波放射原理アニメーション



円偏波放射条件

$$\begin{cases} A_1 = A_2 \\ \theta_2 - \theta_1 = \pm\theta + \pi + 2n\pi \end{cases}$$

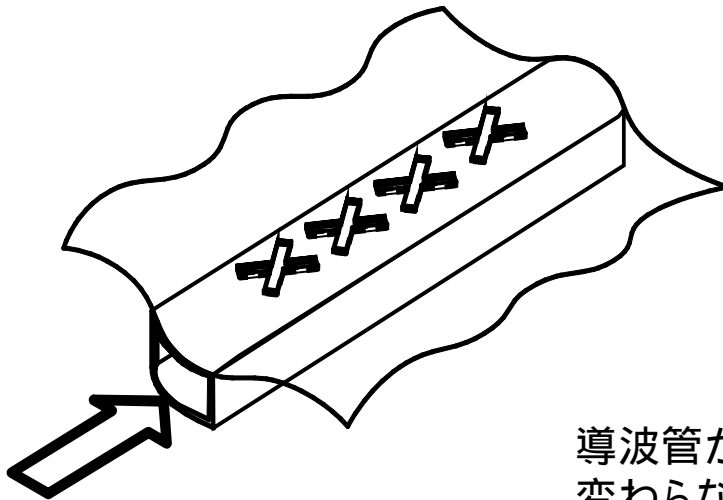


$\theta = 30^\circ$  の例



# Array Design

## 導波管1本



導波管が並んだ2次元アレーの設計と基本的に変わらない。

## MoM/FEM Analysis

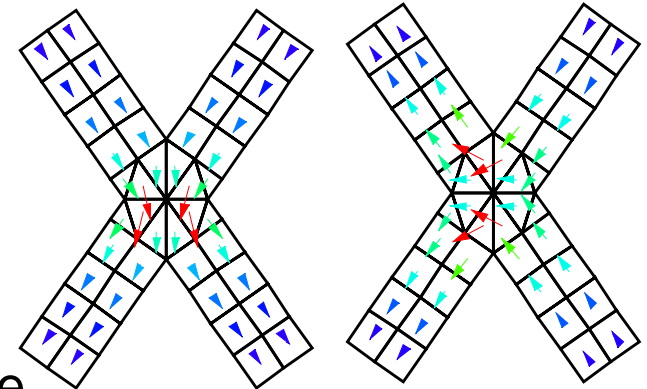
Weighting coefficients (by MoM)

$$\mathbf{M} = A_1 \mathbf{m}_1 + A_2 \mathbf{m}_2 + A_3 \mathbf{m}_3 + \dots$$

Basis functions (by FEM)

Mode1

Mode2



Eigenmode functions

X-shaped waveguide

II (Wall thickness)

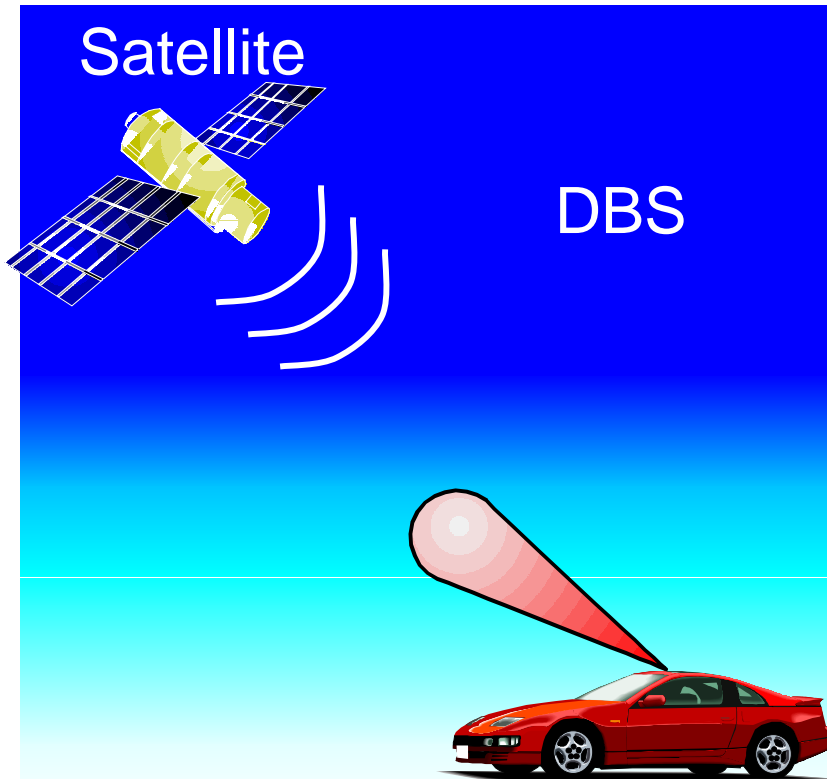
III (Half free space)

slot #1  $-M_1^{exi}$ slot #2  $-M_2^{exi}$ 
 $M_1^{exi}$   
 $-M_1^{int}$   
 $M_1^{int}$ 
 $M_2^{exi}$   
 $-M_2^{int}$   
 $M_2^{int}$ 

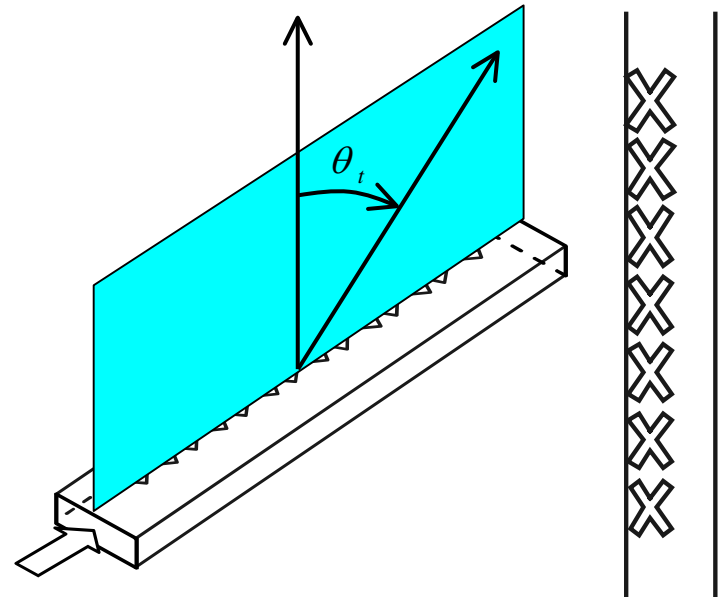
I (Waveguide)

# Setting of the Problem

Frequency: 11.85GHz  
Beam-tilting angle : 50deg



## One waveguide

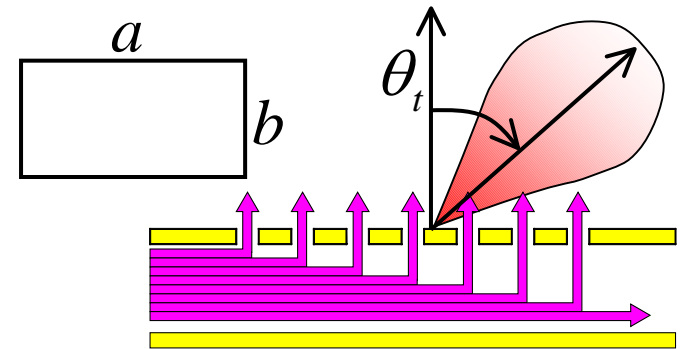


## Purpose

- ✓ Uniform illumination
- ✓ Circularly polarized wave at a beam direction

# Design Procedure

1. Determine the waveguide size and the number of elements

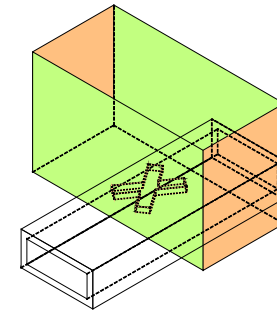


$\theta_t \implies a = 16.0\text{mm}$  **Leaky-wave**

Beam width  $\implies$  Number of slots=8

2. Determine initial slot lengths using infinite periodic model

**Simulate mutual couplings**



3. Finite array analysis

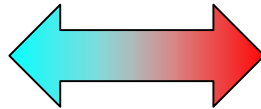
**MoM/FEM**



4. Modify slot parameters

# Power Relation and Slot Coupling

Weak coupling  
(short slot)

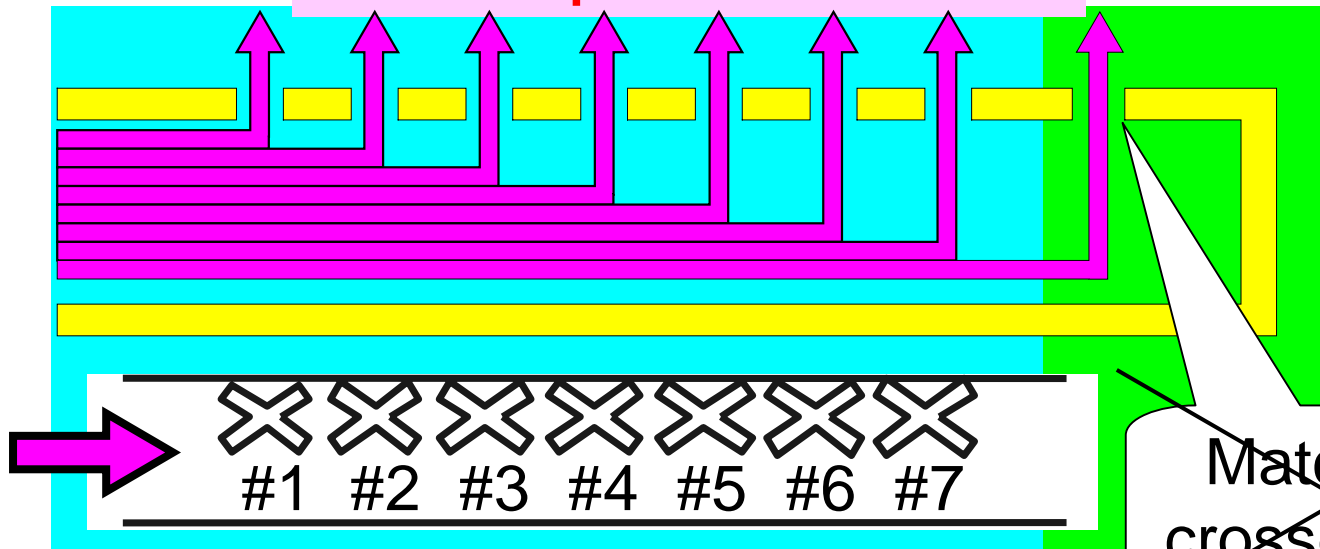
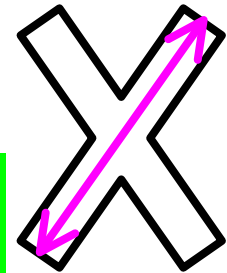


Strong coupling  
(long slot)

Controlled by  
slot length

Slot coupling:  $1/8$   $1/7$   $1/6$   $1/5$   $1/4$   $1/3$   $1/2$  1  
 Radiated power: 12.5% per each slot

**Uniform Aperture Illumination**



~~Matching  
crossed slot~~

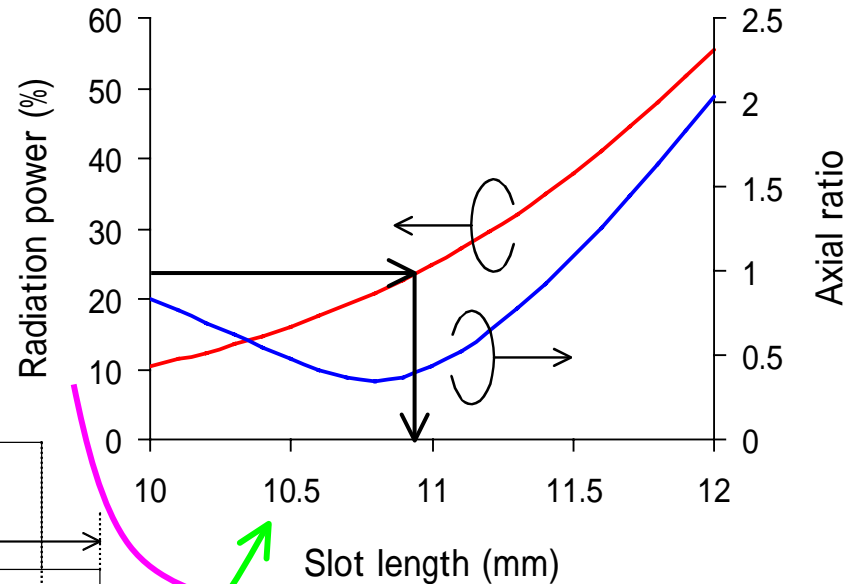
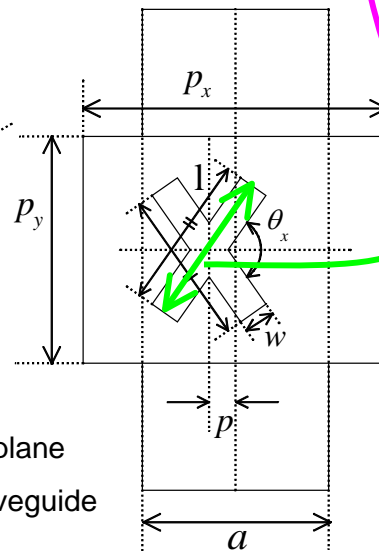
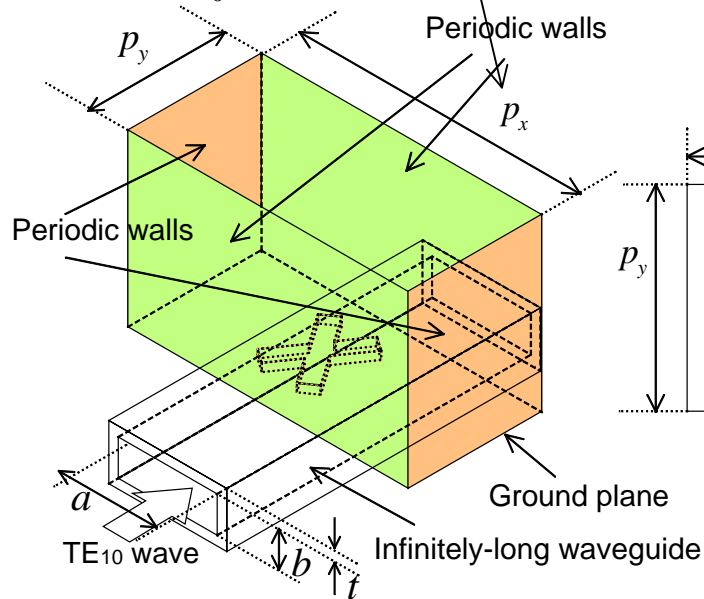
- 12.5% of incident power per each slot
- 87.5% (7 slots), 100% (8 slots)

# Determine Initial Slot Parameters

## Infinite periodic model

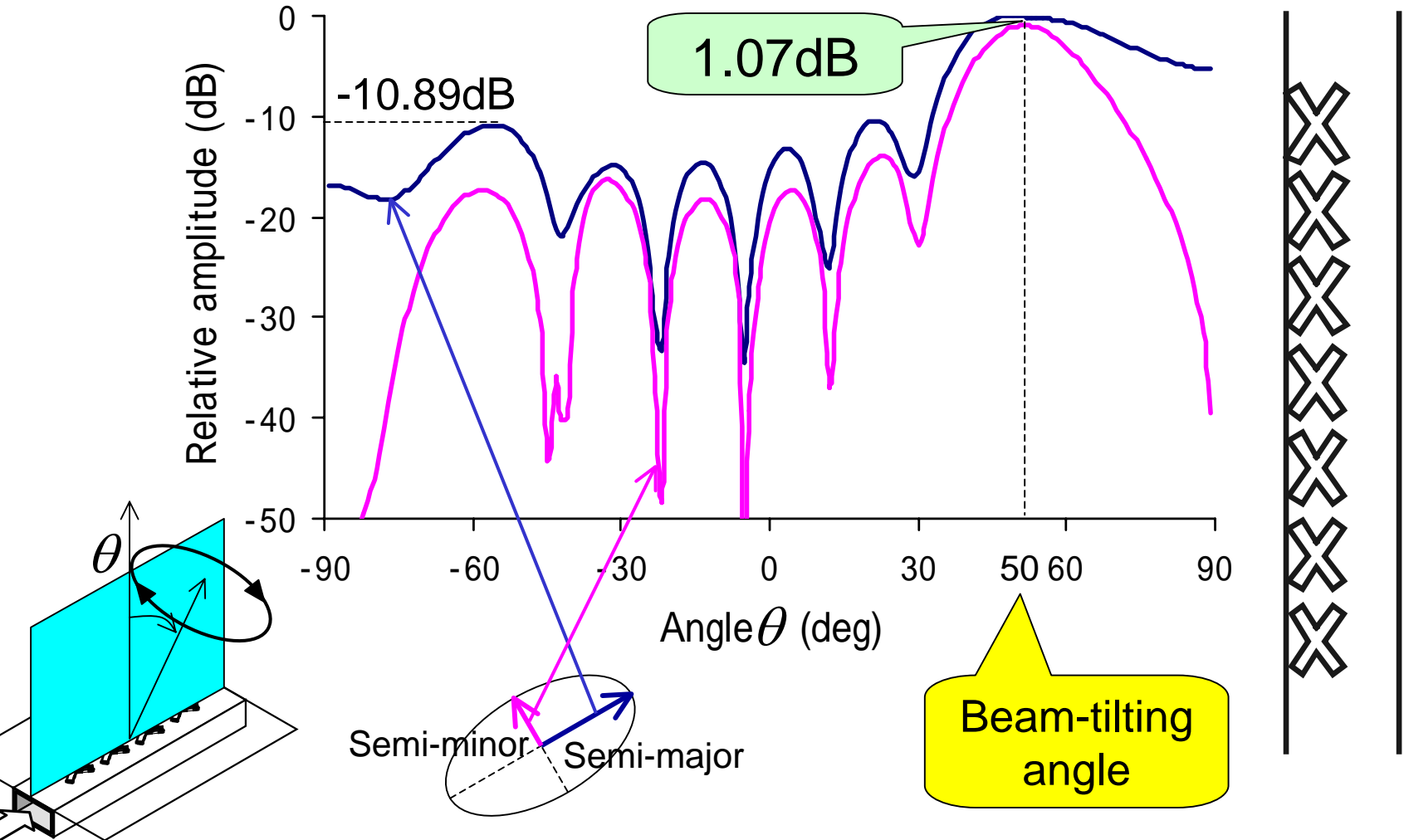
Large in this case  $500.0\text{mm}(20\lambda_0)$

$12.0\text{mm}(0.47\lambda_0)$

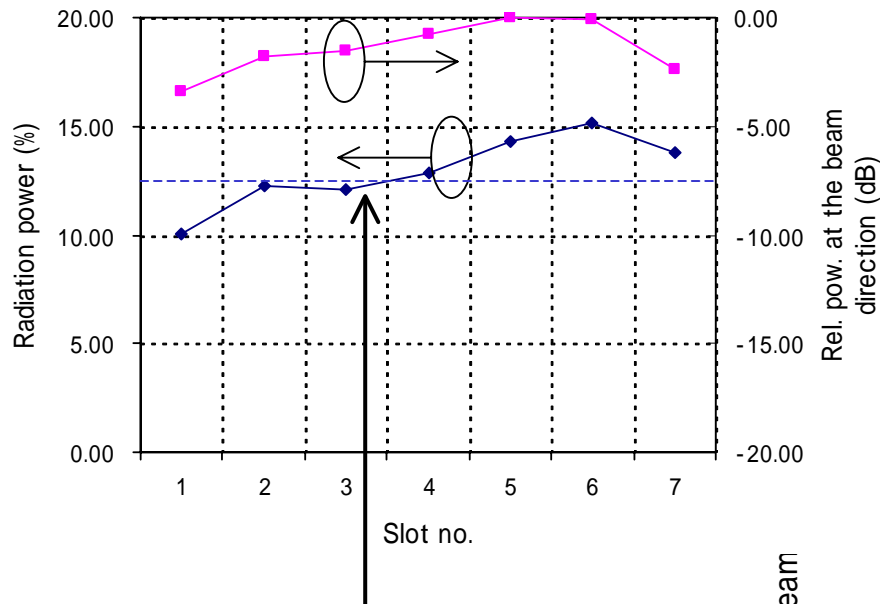


slot no.	coupling (%)	slot length (mm)
1	12.5	10.21
2	14.3	10.37
3	16.7	10.53
4	20.0	10.75
5	25.0	11.01
6	33.3	11.35
7	50.0	11.86
8	100.0	

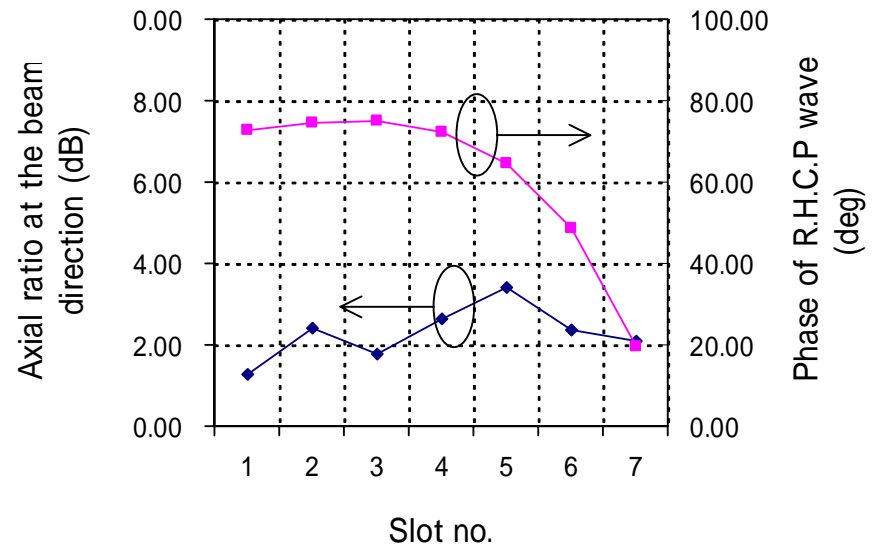
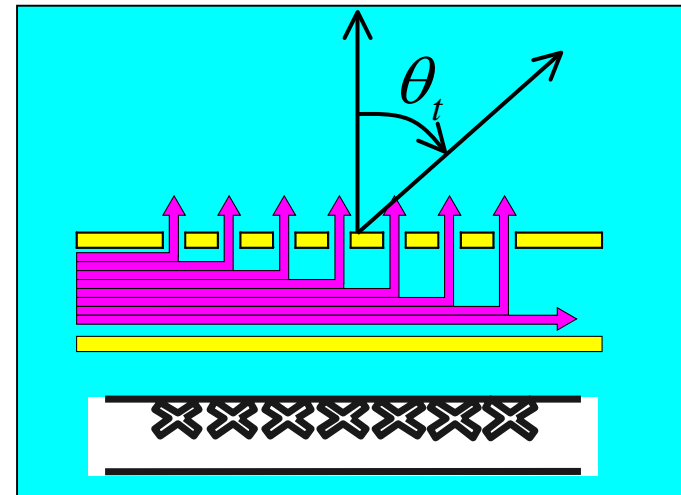
## Radiation Pattern (Init.)



## E-field Radiated by Each Slot (Init.)



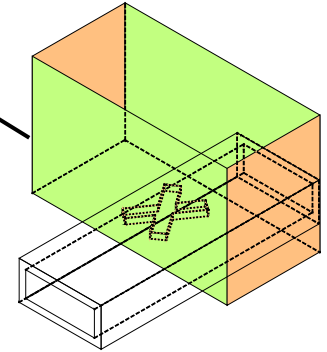
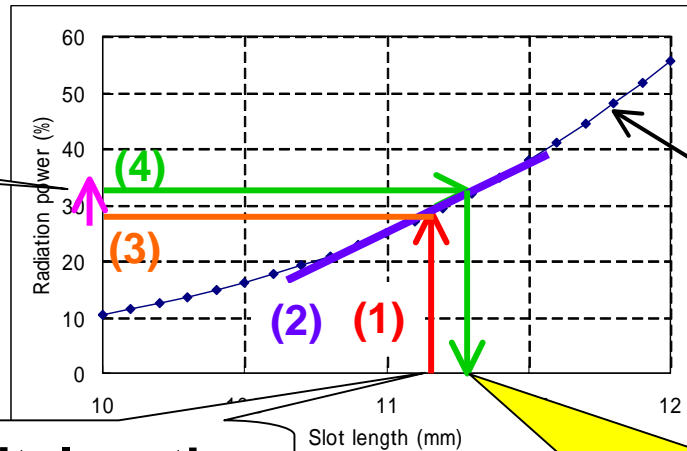
Not uniform enough





# Correction of the Power

Error of the radiation power



Init. length

Corrected slot length

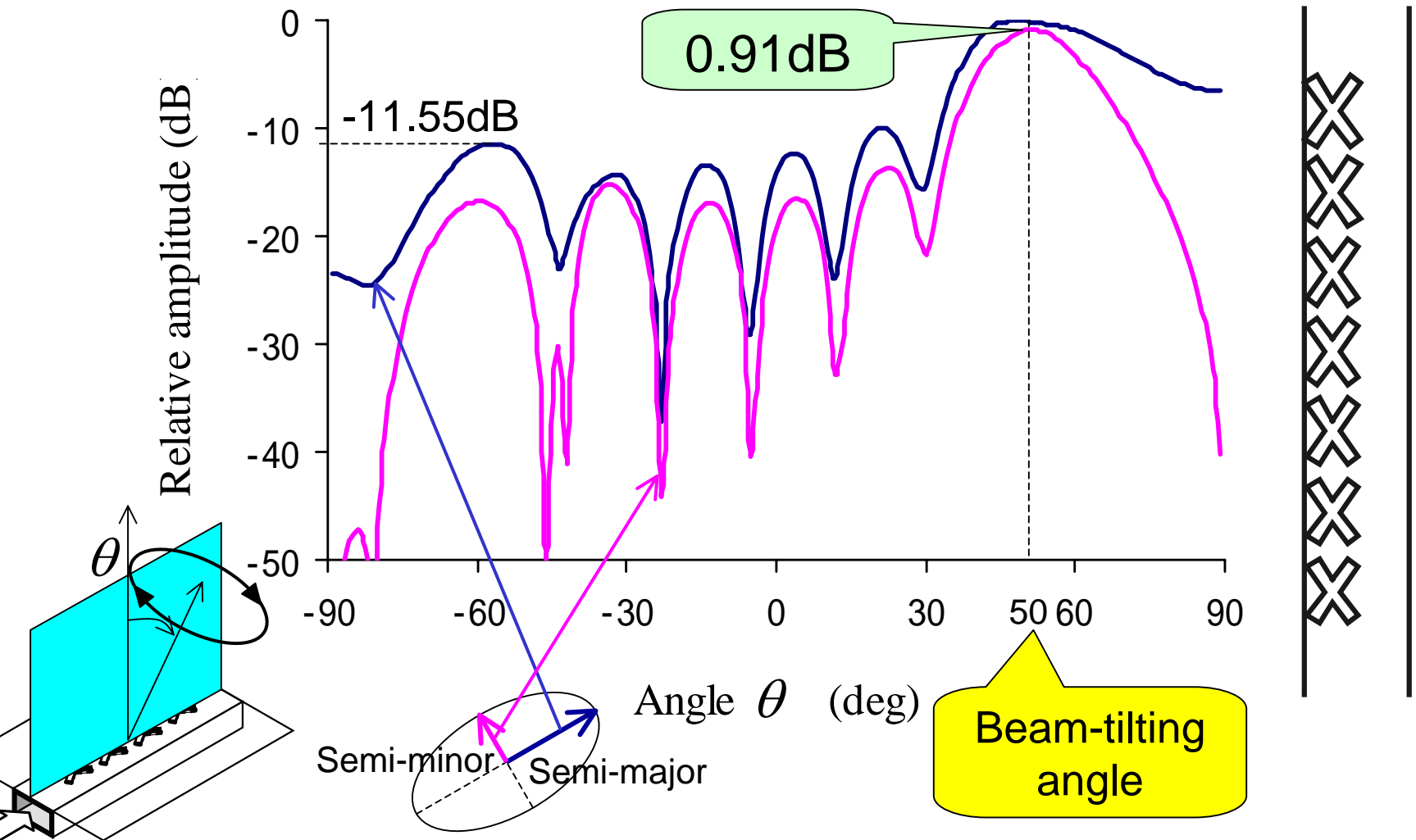
□ At the present slot length (1), differential coefficient (2) is calculated numerically

□ Slot length is corrected by the 1<sup>st</sup> order approximation using the difference between the present (3) and desired power (4) and the differential coefficient.

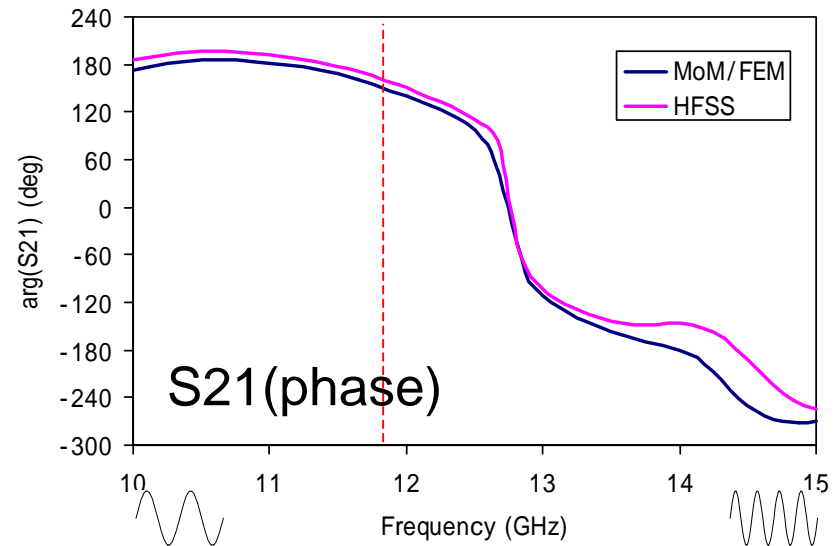
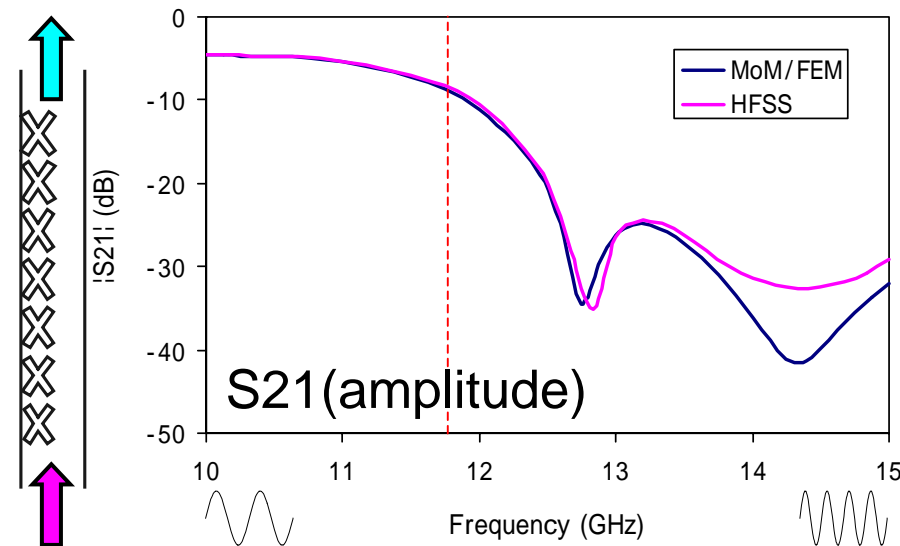
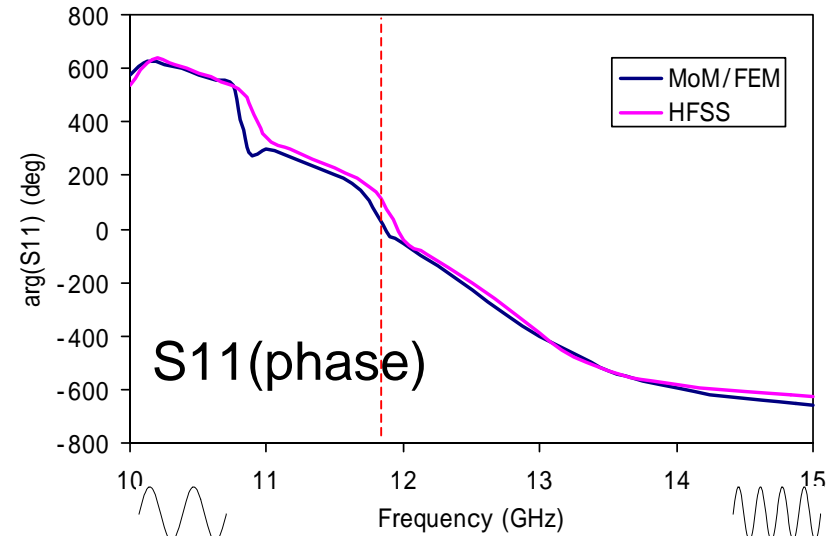
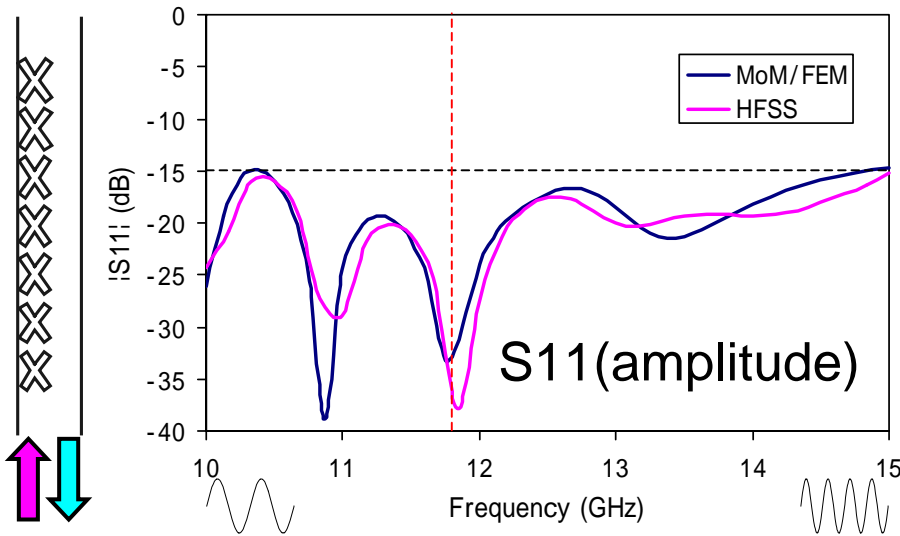
□ Above procedure is applied to all the slots simultaneously



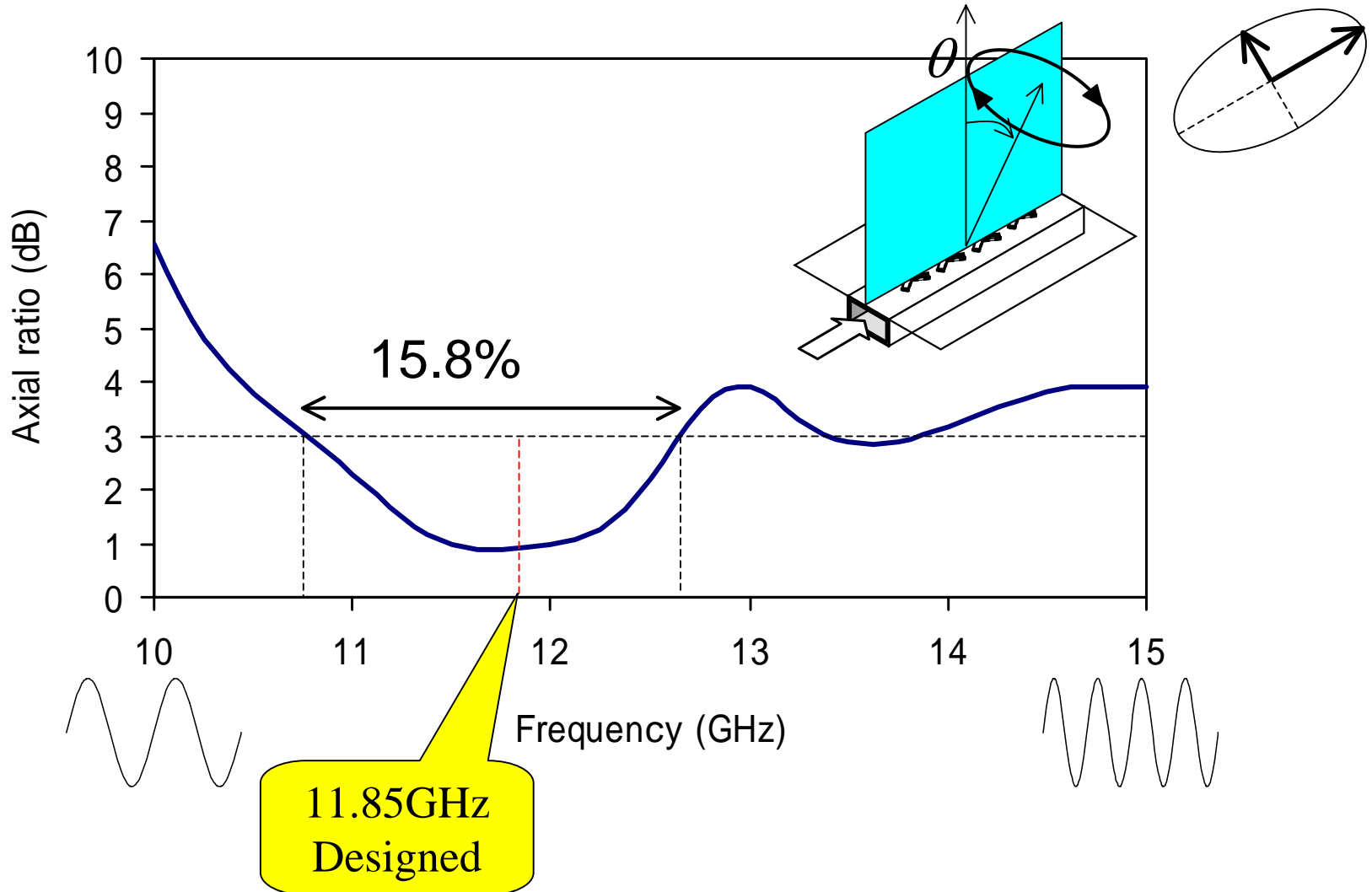
## Radiation Pattern (Corrected, 3 Iterations)



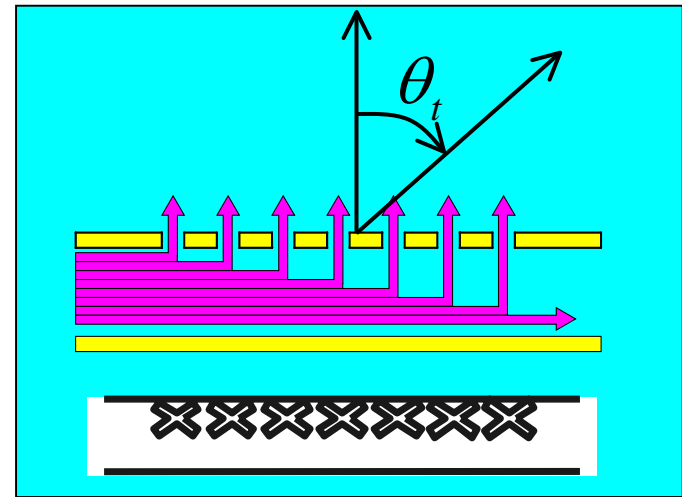
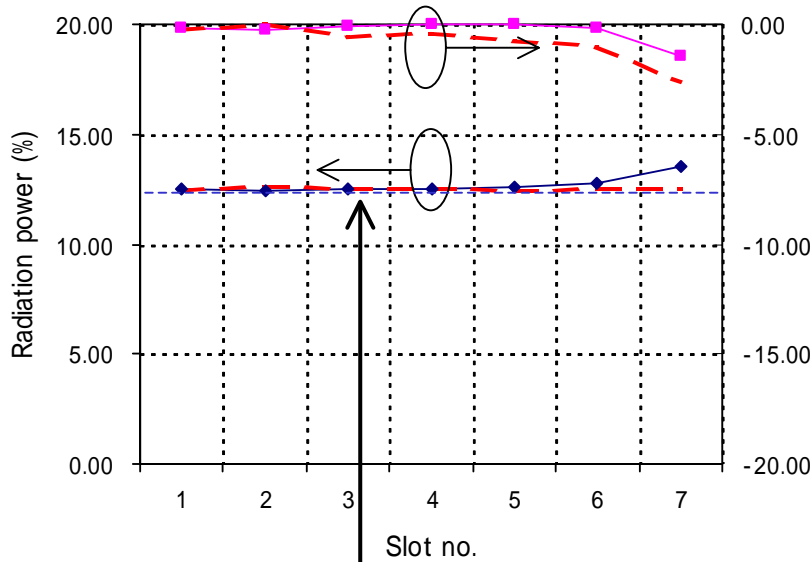
## Freq. vs. S11 and S21 (Corrected, 3 Iterations)



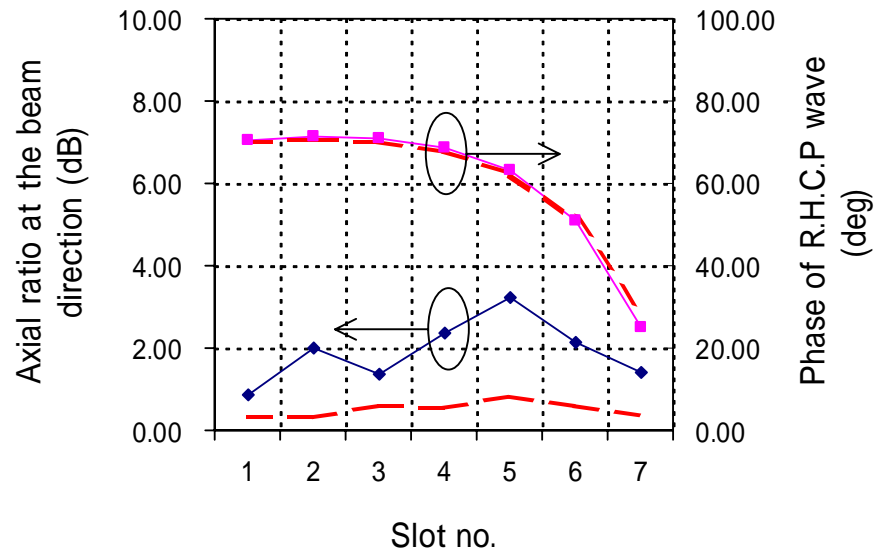
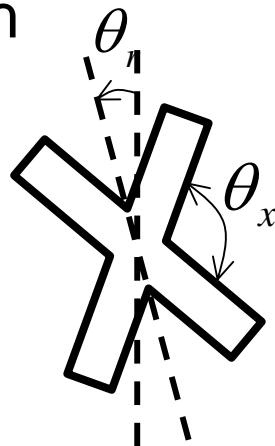
## Freq. vs. Axial Ratio (Corrected, 3 Iterations)



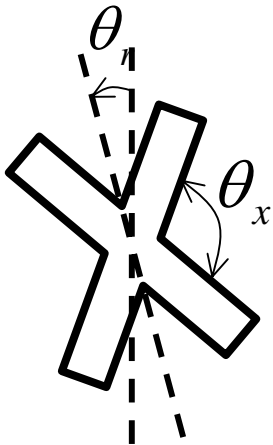
# E-field Radiated by Each Slot (Corrected, 3 Iterations)



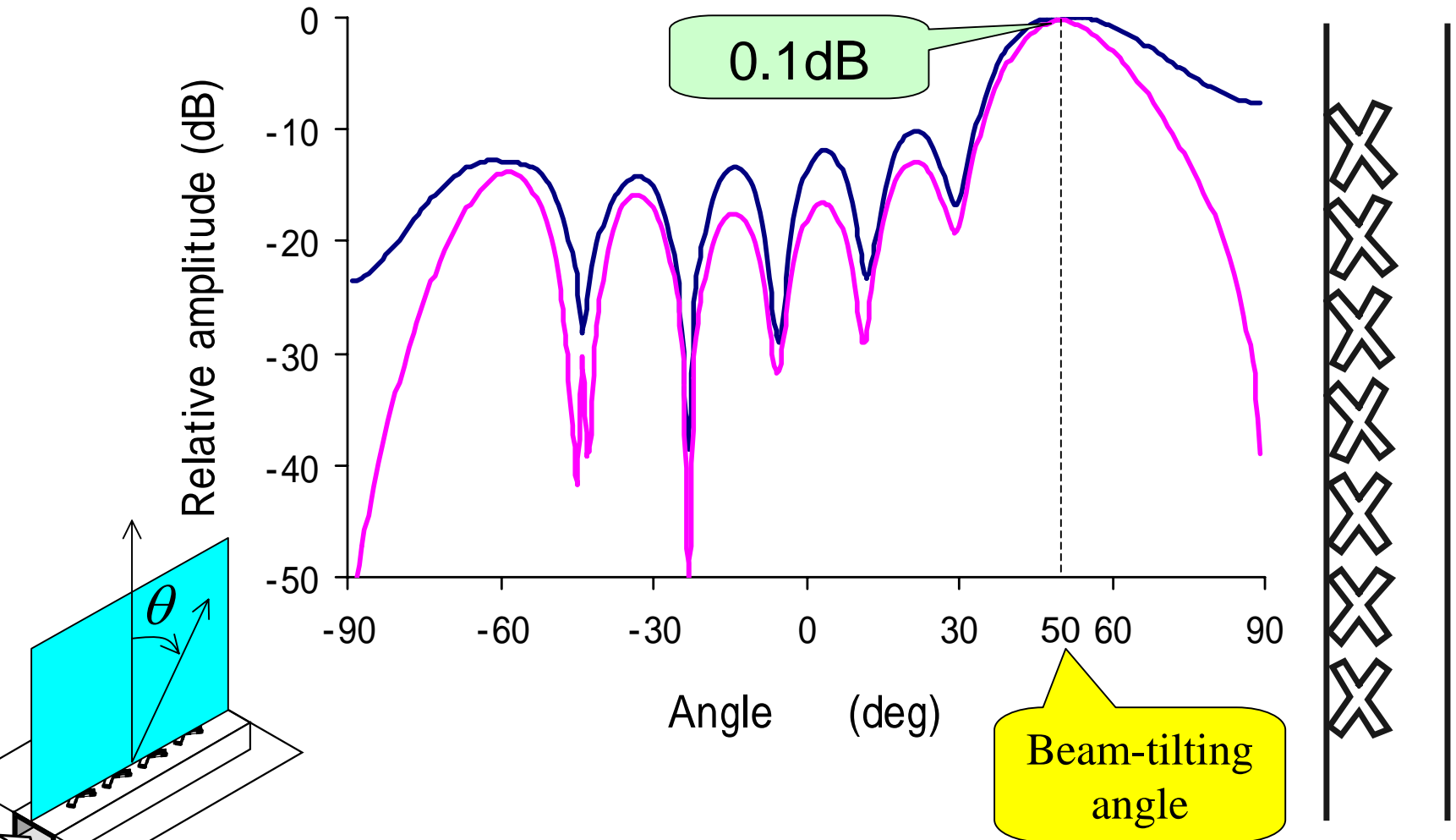
Uniform enough



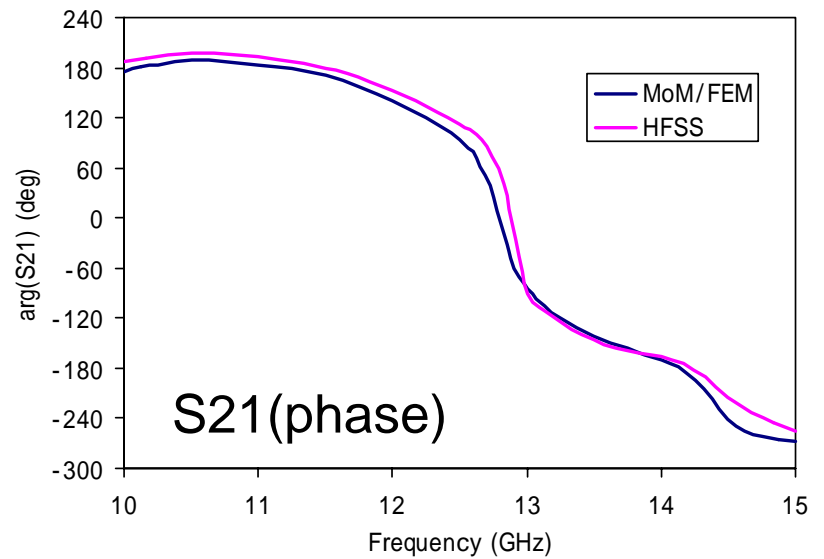
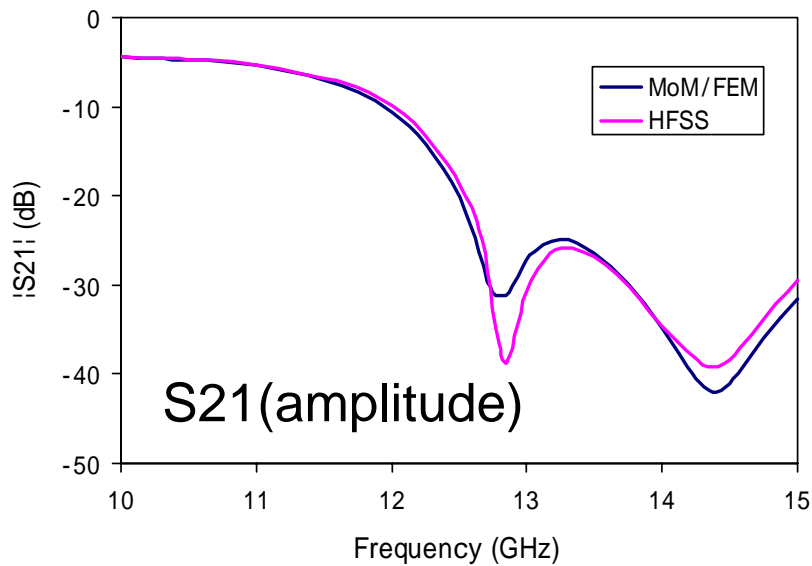
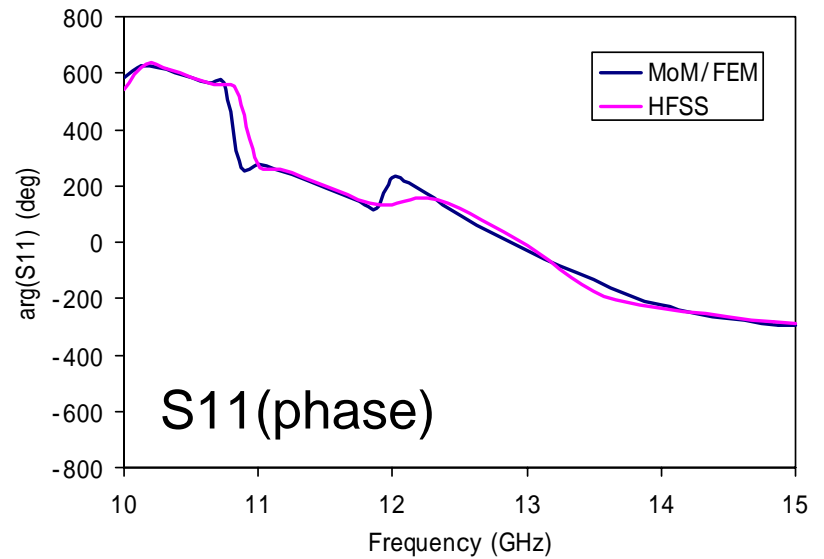
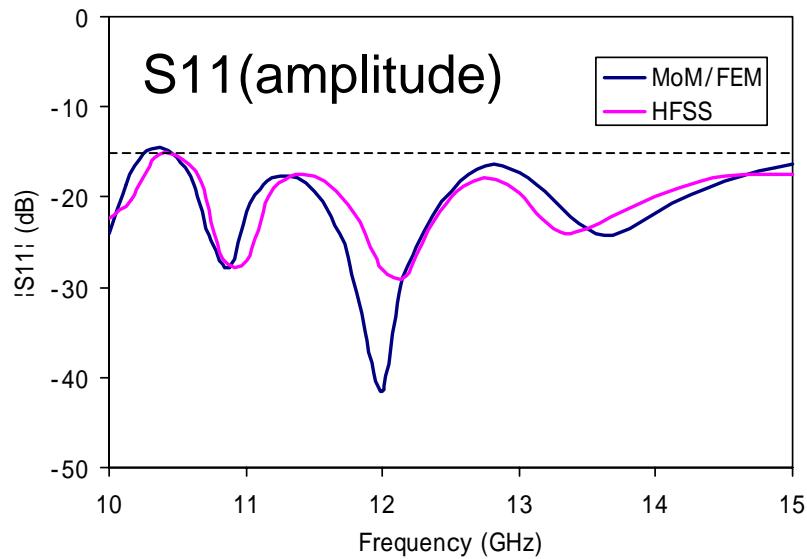
# Correction (Method 2)



# Radiation Pattern

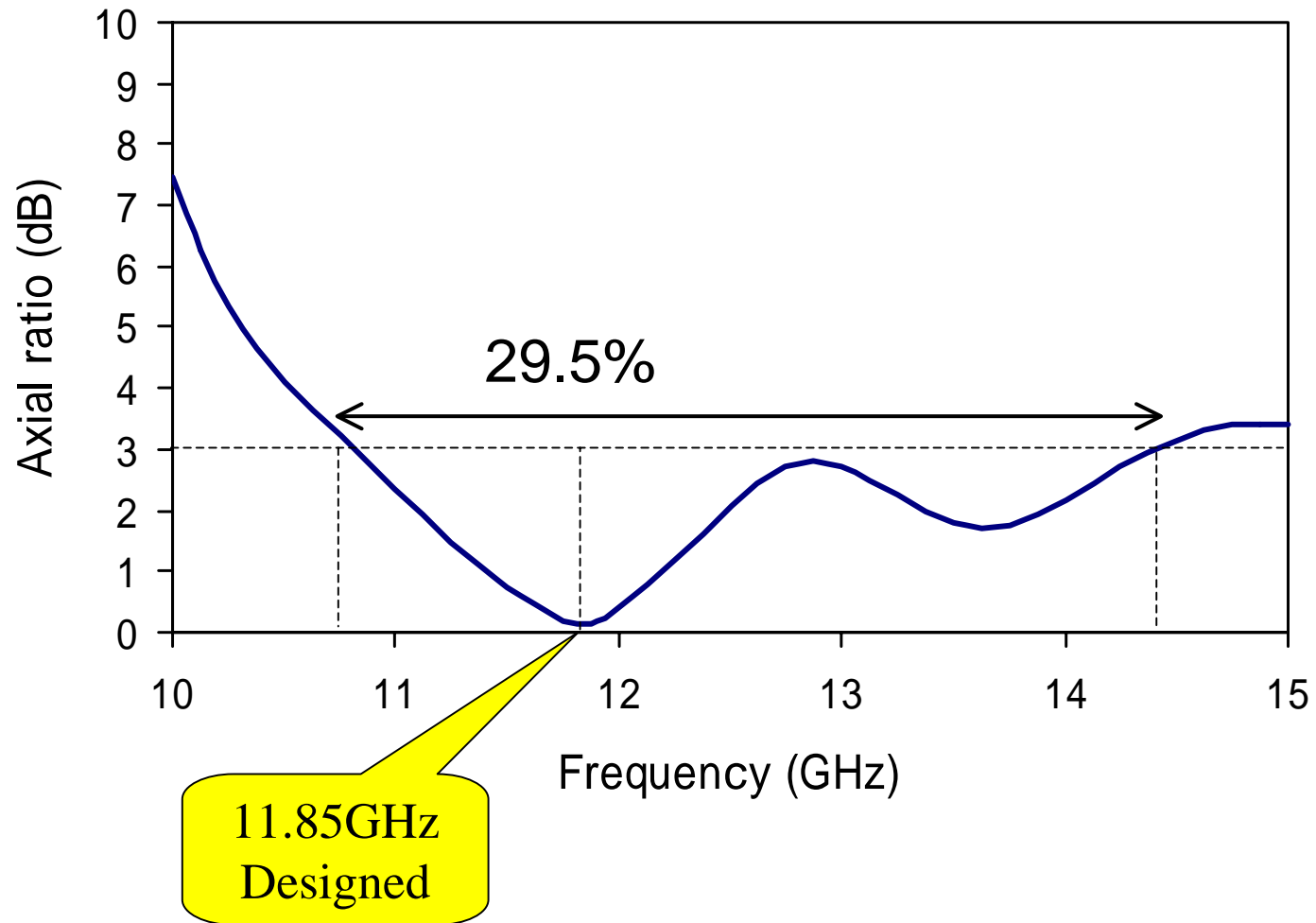


# S11 and S21

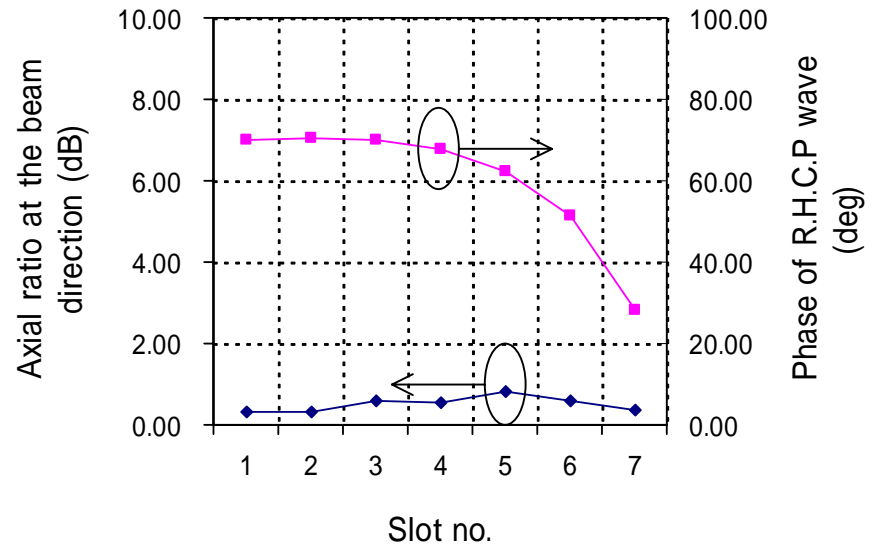
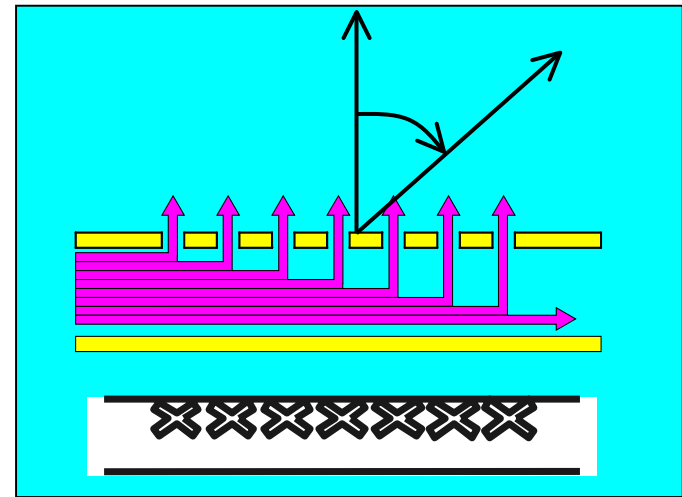
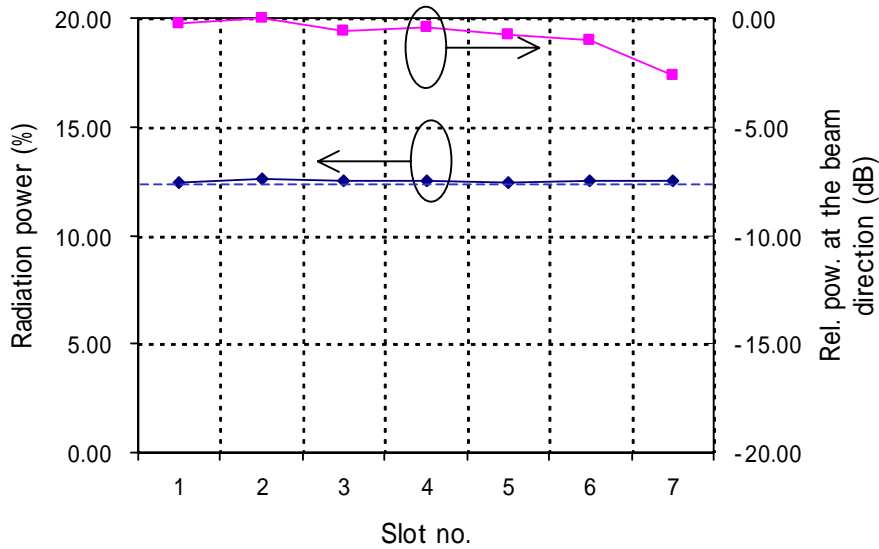




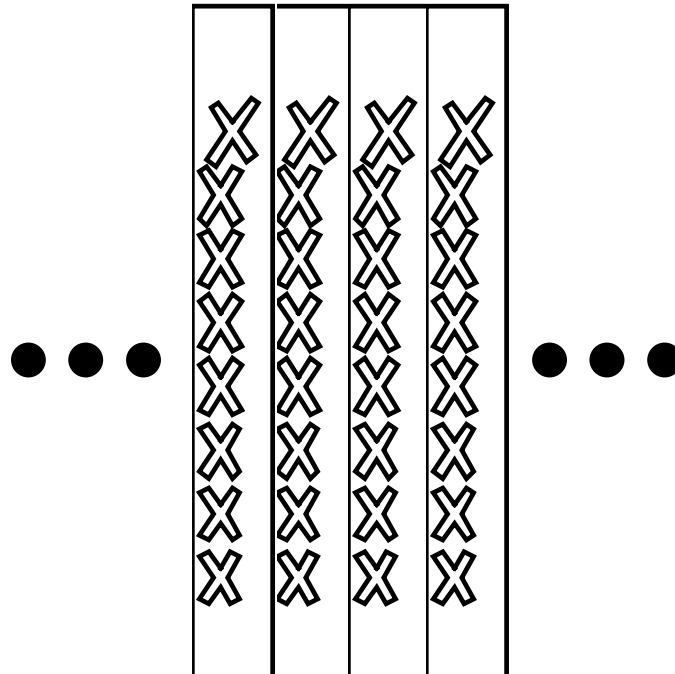
# Axial Ratio



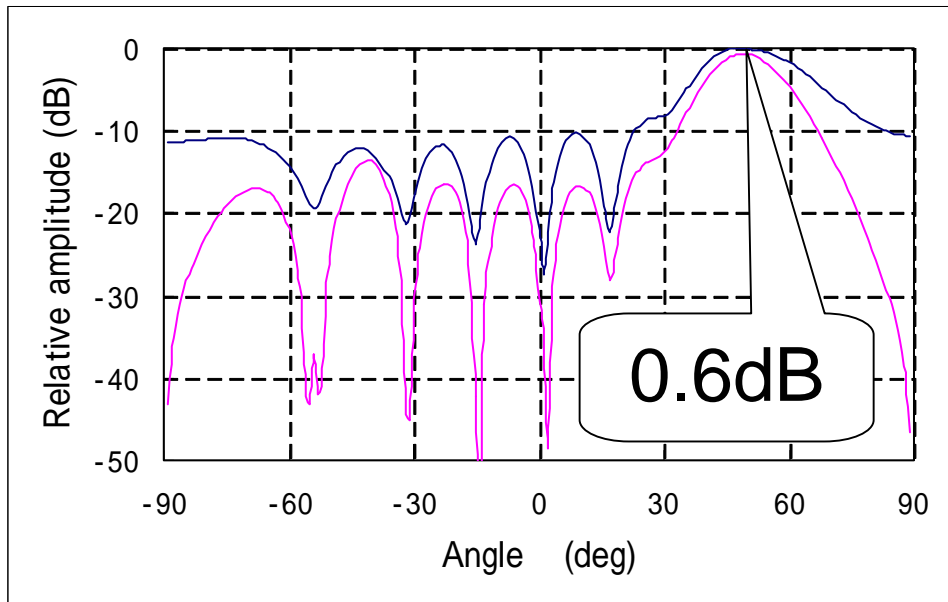
# E-field Radiated by Each Slot



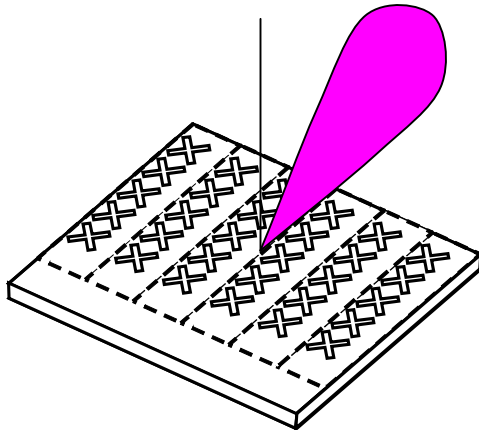
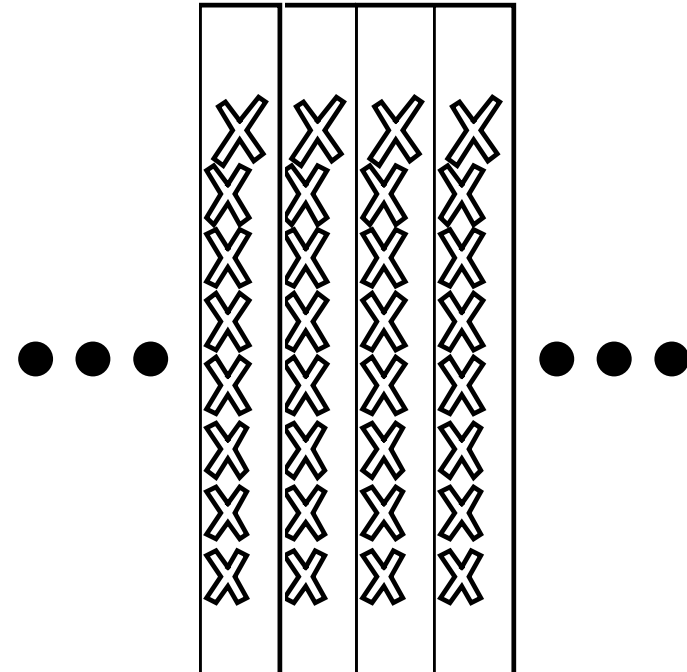
# Array Design 2



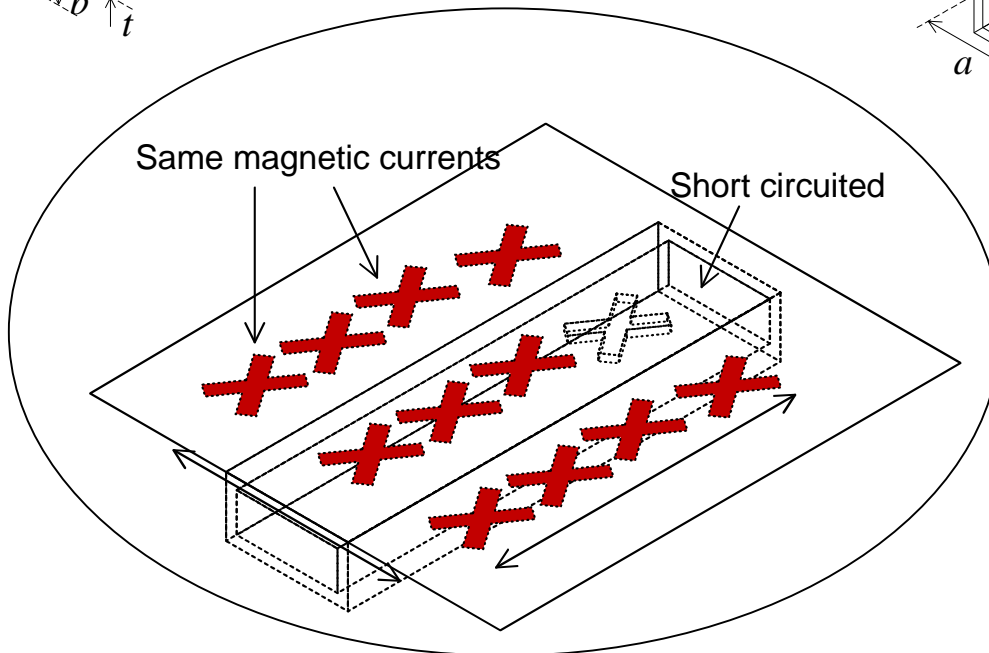
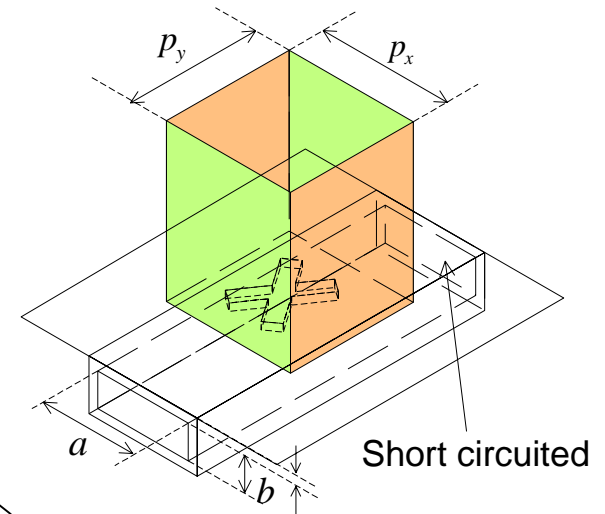
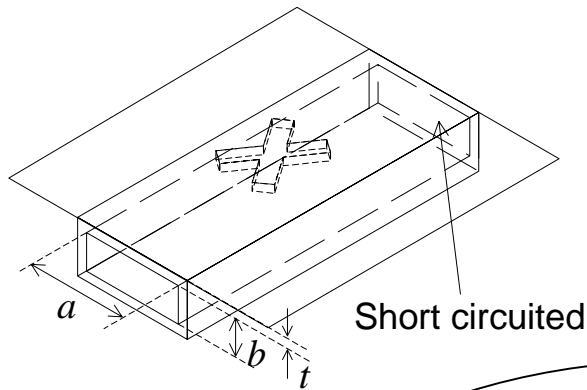
# W.G. Crossed Slot Array



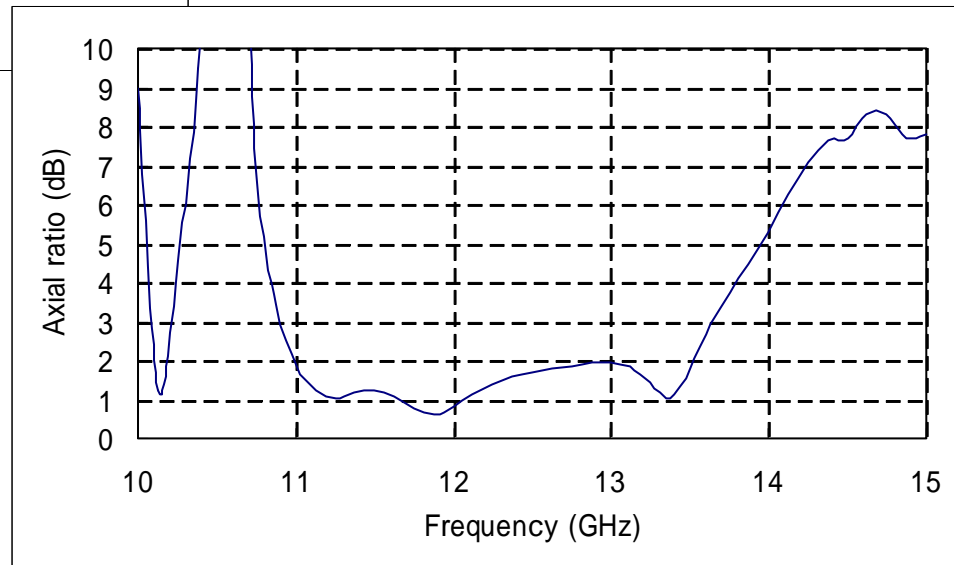
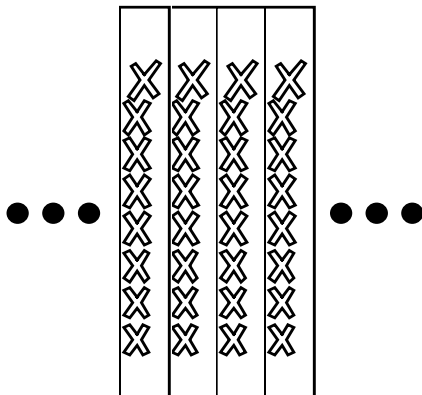
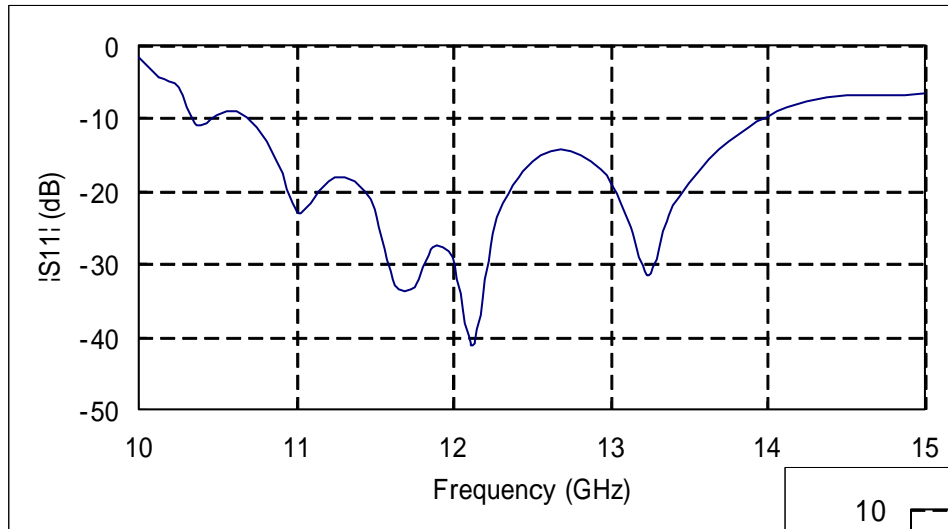
99.8% Radiates



# Design Model for a Matching Crossed Slot



# Frequency Characteristics



# Conclusion

MoM/FEM解析法を1素子で確認した

(任意形状スロットの解析も可能)

MoM/FEM解析法で終端の整合クロススロットを設計した

MoM/FEM解析法を用いて一次元導波管

クロススロットアレーの設計を行った

リングスロット、楕円スロットの解析を行った

## MoM/FEM解析法の利点

速い！

壁厚の影響の評価が簡単！

正確！

円偏波の動作原理(物理現象)が理解できる！

