Tokyo Institute of Technology

Scalar-field approach of IE-MEI Method

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Outline

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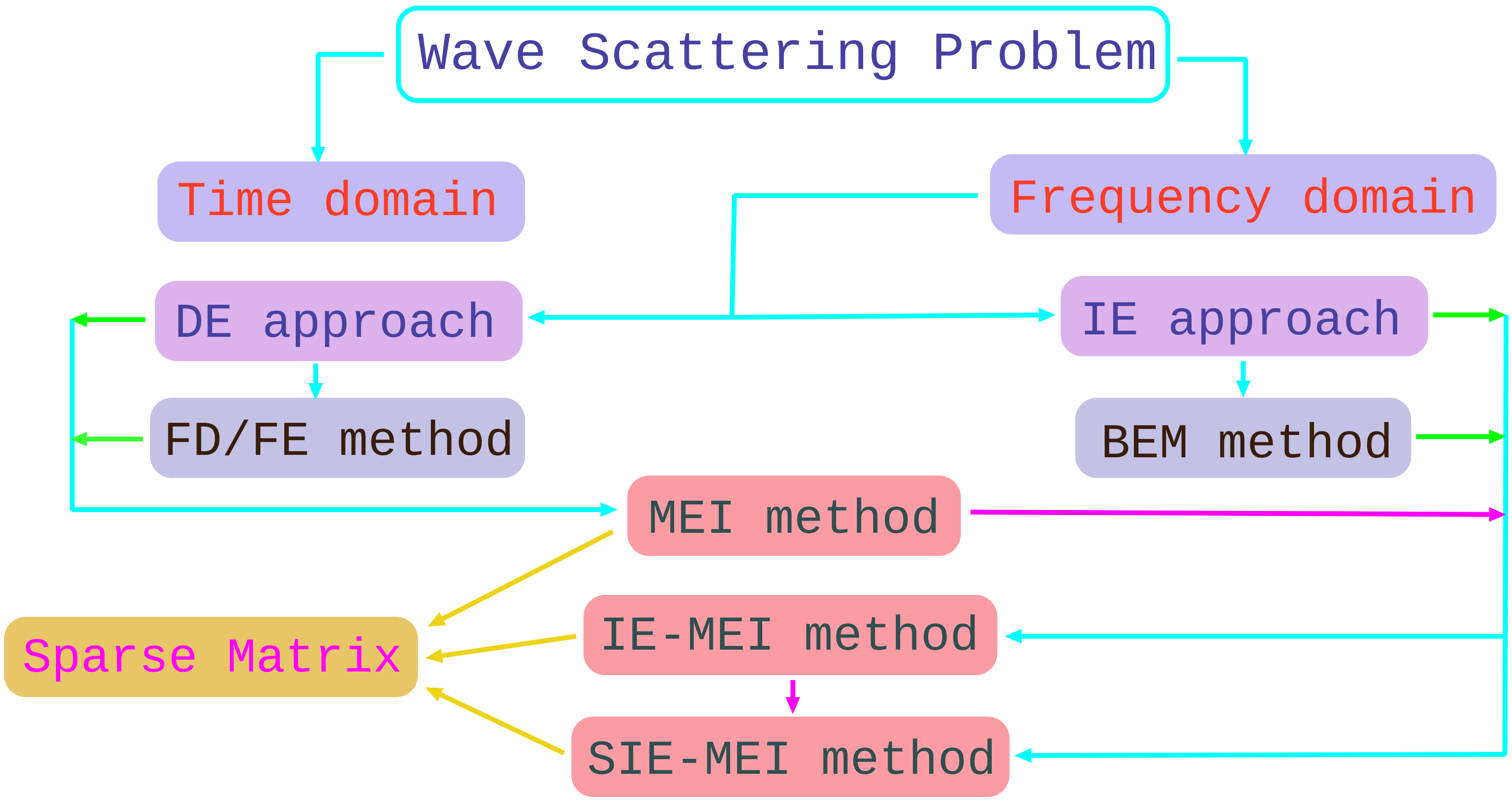
- Motivation of SIE-MEI
- SIE-MEI method
- Numerical Example
- Result
- Remarks & Futuer work

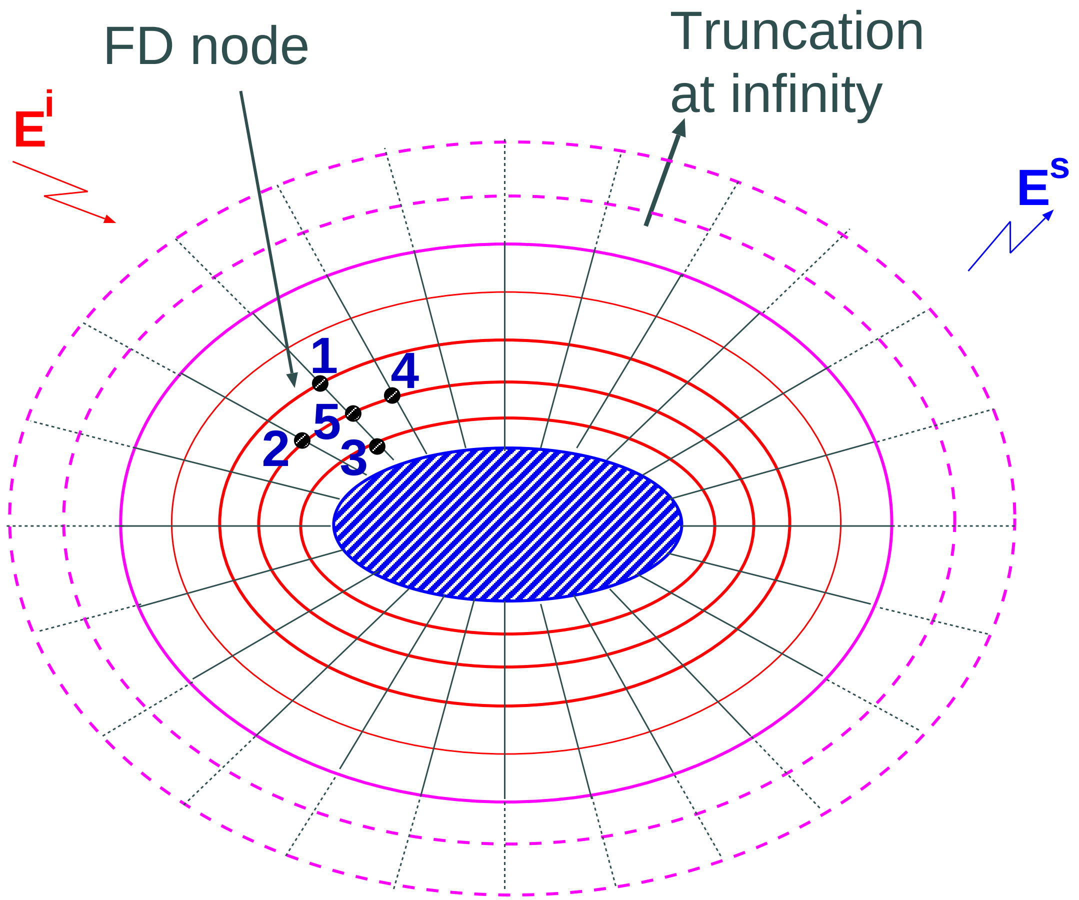
Background


Wave Scattering Problem
Time domain
Frequency domain
DE approach
IE approach
FD/FE method
BEM method
MEI method
IE-MEI method
SIE-MEI method
Sparse Matrix

FD/FE Method : DE Approach

Large Sparse Matrix

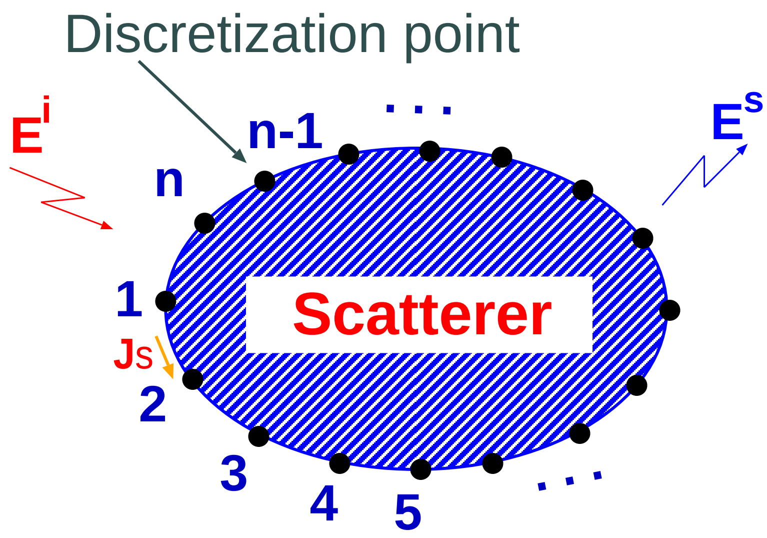
$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & \cdots & a_{1(n-1)} & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & \cdots & 0 & a_{2n} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & 0 & \cdots & a_{n(n-2)} & a_{n(n-1)} & a_{nn} \end{bmatrix}$$



BEM Method : IE Approach

Dense Matrix

$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix}$$



MEI Method : DE Approach

(K.K.Mei *et al.*,1994)

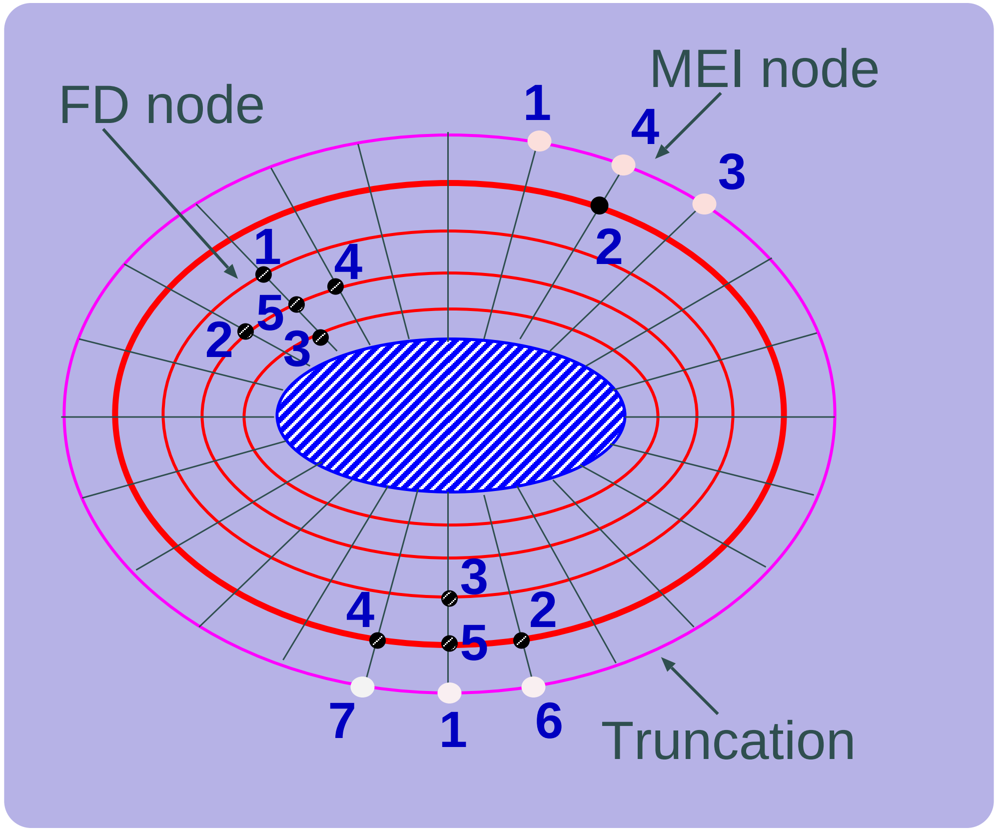
Local linear eqation of MEI node,

$$\sum_{i=1}^{4} C_i \phi_i = 0$$

scattered field

MEI coefficients :
1. location dependent,
2. geometry specific, and
3. invarient to field excitation.

MEI postulates



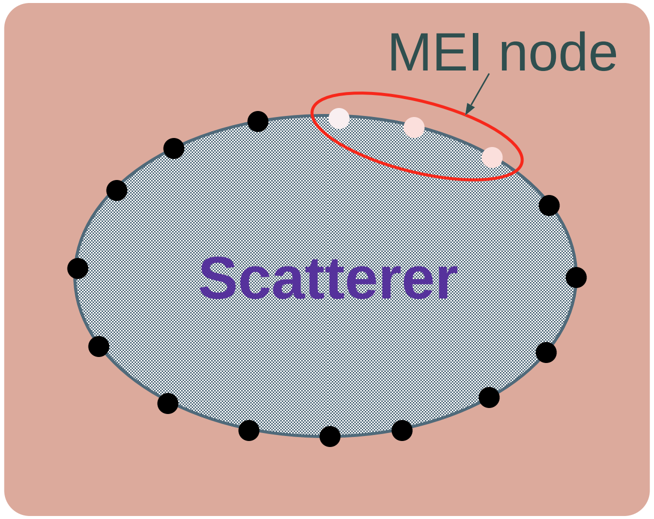
IE-MEI Method : IE Approach

(J.M.Rius *et al.*,1996 & M.Hirose *et al.*,1999)

Surface IE is derived from Reciprocity relation
On surface MEI postulates
Sparse matrix with same number of unknowns as BEM

Savings in Computational time, and Memory needs

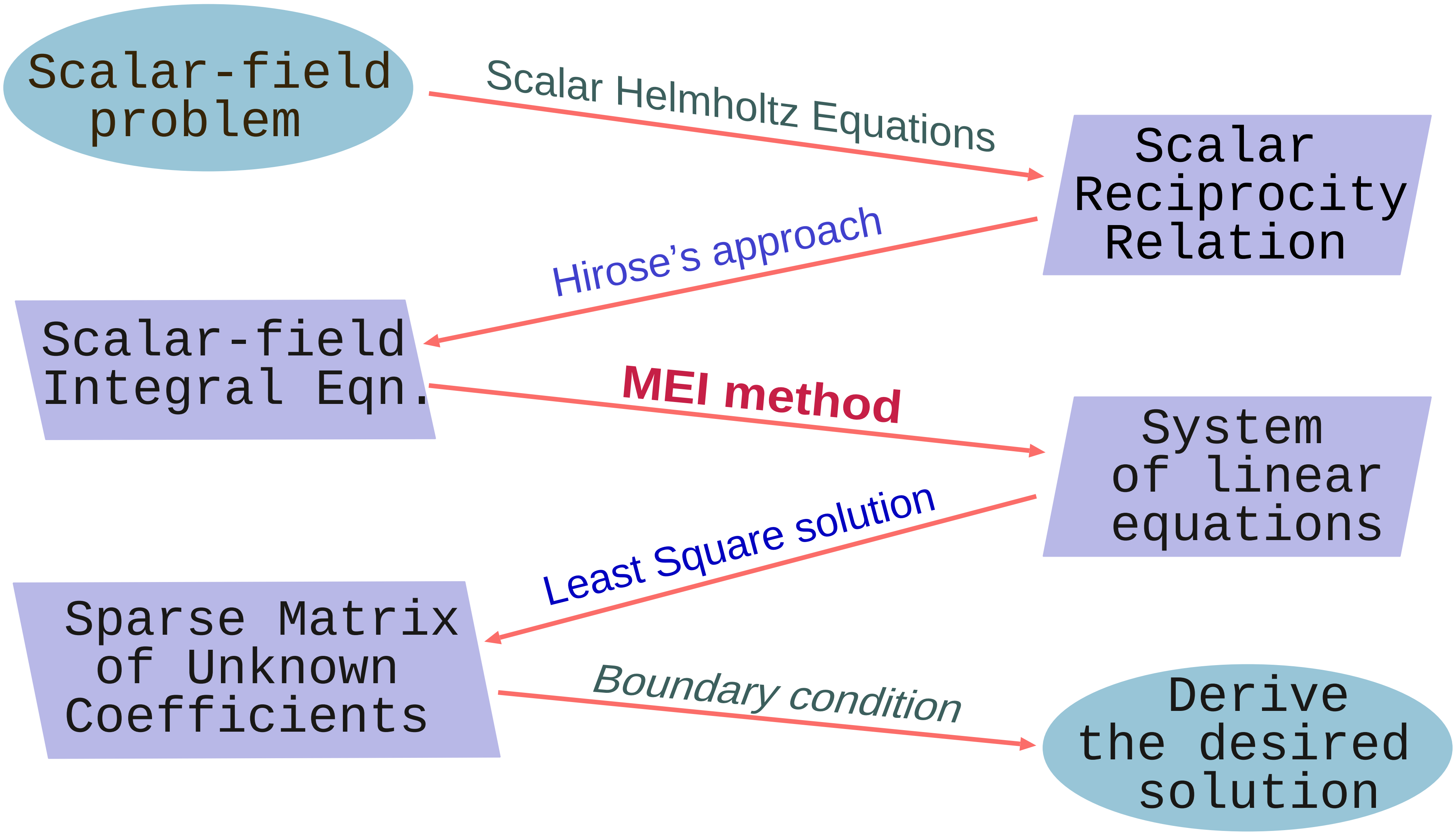
Suitable for arbitary 2D boundaries, but not efficient for 3D boundaries



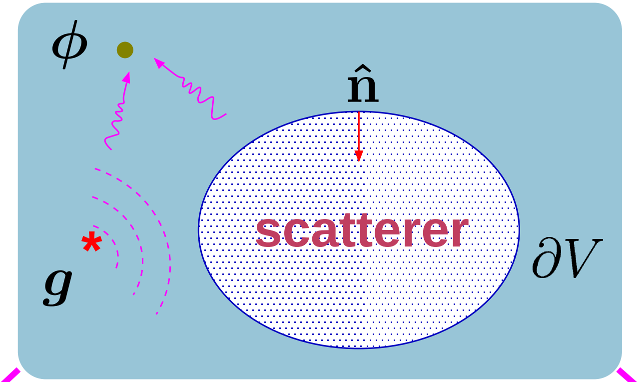
Motivation of SIE-MEI Method

- IE-MEI method has an improvement to store and solve the matrix efficiently
- Not suitable for arbitary shape 3D problem
- Choice of suitable metrons and mesh generation for 3D, yet not established
- To approach this problem introduce scalar-field IE-MEI for the 3D boundaries

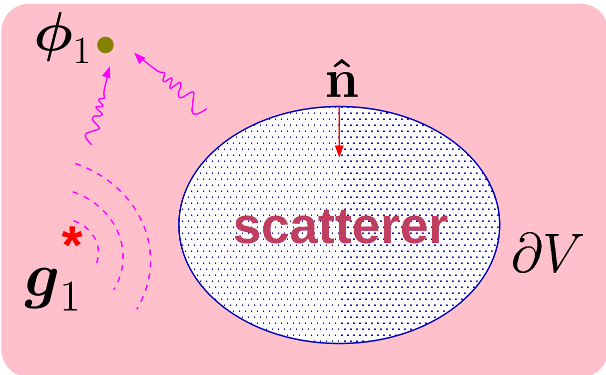
SIE-MEI Method



Scalar-field problem :



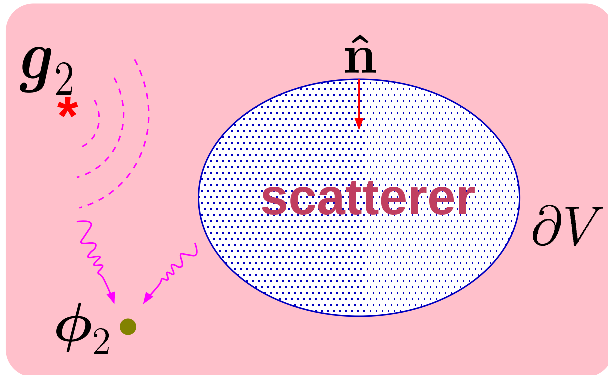
$$L\{\phi(\mathbf{r})\} = -g(\mathbf{r})$$



Scalar Helmholtz Equation

$$\nabla^2\phi_1(\mathbf{r}) + k^2\phi_1(\mathbf{r}) = -g_1(\mathbf{r})$$

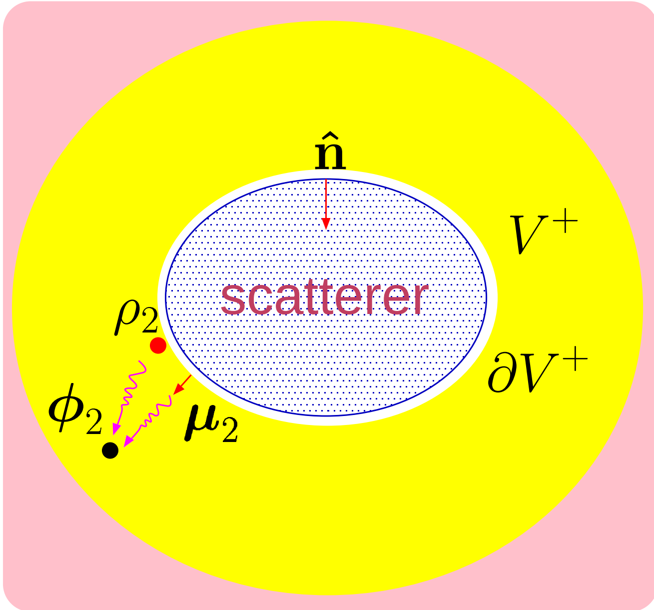
$$\nabla^2\phi_2(\mathbf{r}) + k^2\phi_2(\mathbf{r}) = -g_2(\mathbf{r})$$



$$\int_V (\phi_2(\mathbf{r})g_1(\mathbf{r}) - \phi_1(\mathbf{r})g_2(\mathbf{r}))dV = \oint_{\partial V} \left(\phi_1(\mathbf{r})\frac{\partial\phi_2(\mathbf{r})}{\partial n} - \phi_2(\mathbf{r})\frac{\partial\phi_1(\mathbf{r})}{\partial n}\right) dS$$

Scalar Reciprocity relation

Integral Equation formulation:



scattered field

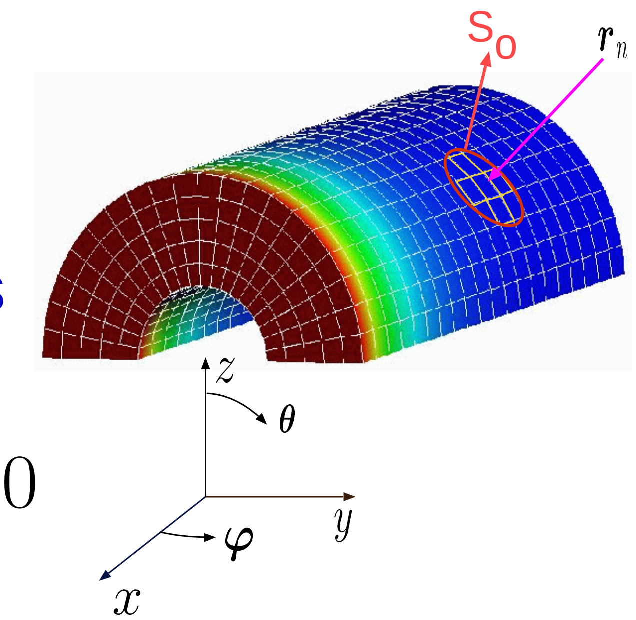
combination of equivalent surface source $\rho_2(\mathbf{r})$ and monopole of dipole moment $\boldsymbol{\mu}_2(\mathbf{r})$

$$\oint_{\partial V^+}\left(\phi_1(\mathbf{r})\frac{\partial \phi_2(\mathbf{r})}{\partial n} - \phi_2(\mathbf{r})\frac{\partial \phi_1(\mathbf{r})}{\partial n}\right)dS = -\int_{V^+}\phi_1(\mathbf{r})\rho_2(\mathbf{r})dV$$

$V^+ \rightarrow V$

$$\oint_{\partial V}\left(\phi_1(\mathbf{r})\tilde{\rho}_2(\mathbf{r}) - \frac{\partial \phi_1(\mathbf{r})}{\partial n}\tilde{\boldsymbol{\mu}}_2(\mathbf{r})\cdot\hat{\mathbf{n}}\right)dS = 0$$

equivalent local sources near the scatterer



Localization & Discretisation:

Localize the IE which satisfies MEI postulates

$$\int_{S_o}\left(\phi_1(\mathbf{r})\tilde{\rho}_2(\mathbf{r}) - \frac{\partial \phi_1(\mathbf{r})}{\partial n}\tilde{\boldsymbol{\mu}}_2(\mathbf{r})\cdot\hat{\mathbf{n}}\right)dS = 0$$

depends on scatterer geometry,
depends on position, and
invarient to incident field

By discretization and expanding the local sources around $\boldsymbol{r}_n$

unknown MEI coefficients

$$\sum_{m=n-\frac{M-1}{2}}^{n+\frac{M-1}{2}}\left[\phi_1(\boldsymbol{r}_m)\tilde{\rho}_{2,n}(\boldsymbol{r}_m) - \frac{\partial \phi_1(\boldsymbol{r}_m)}{\partial n}\tilde{\boldsymbol{\mu}}_{2,n}(\boldsymbol{r}_m)\cdot\hat{\mathbf{n}}\right] = 0$$

Derivation of MEI coefficients :

$$\sum_{m=n-\frac{M-1}{2}}^{n+\frac{M-1}{2}} \left[\phi_1(\boldsymbol{r}_m)\tilde{\rho}_{2,n}(\boldsymbol{r}_m) - \frac{\partial \phi_1(\boldsymbol{r}_m)}{\partial n} \tilde{\boldsymbol{\mu}}_{2,n}(\boldsymbol{r}_m) \cdot \hat{\mathbf{n}} \right] = 0$$

$$\phi_{1,q}(\boldsymbol{r}_m) = \int_s \rho_q(\boldsymbol{r'}) \, G(\boldsymbol{r_m}, \boldsymbol{r'}) \, ds'$$

called Metrons
$q = 1, 2, \cdots, N_{max}$

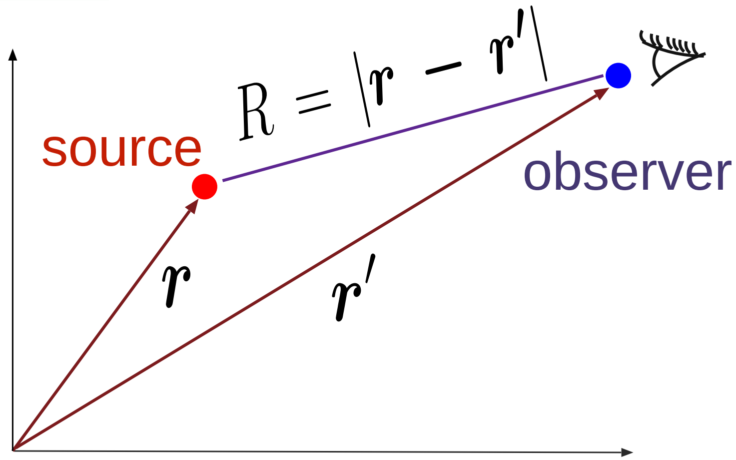
$$G(\boldsymbol{r}, \boldsymbol{r'}) = \frac{e^{-jk|\boldsymbol{r} - \boldsymbol{r'}|}}{4\pi|\boldsymbol{r} - \boldsymbol{r'}|}$$

In matrix form, $[\,\mathbf{A}\ \mathbf{B}\,] \begin{bmatrix} \mathbf{r} \\ \mathbf{m} \end{bmatrix} = 0$

$[\,q \times 2M\,]$ matrix of scattered field

column matrix of $2M$ unknown MEI coefficient

Green's function: $G(\boldsymbol{r}, \boldsymbol{r}')$



Green's function $G(\boldsymbol{r}, \boldsymbol{r}')$, is the response at a observer $\boldsymbol{r}$ from a unit point source at $\boldsymbol{r}'$.

The total field generated by the distributed source can be obtained by the integration of the Green's function weighted by the source distribution.

Free space Green's function for the Scalar Helmholtz equation

2D case

$$G(\boldsymbol{\rho}, \boldsymbol{\rho}') = \frac{1}{4j} H_0^{(2)}(k|\boldsymbol{\rho} - \boldsymbol{\rho}'|)$$

3D case

$$G(\boldsymbol{r}, \boldsymbol{r}') = \frac{e^{-jkR}}{4\pi R} = \frac{e^{-jk|\boldsymbol{r} - \boldsymbol{r}'|}}{4\pi|\boldsymbol{r} - \boldsymbol{r}'|}$$

Local Matrix

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1M} & b_{11} & b_{l2} & \cdots & b_{lM} \\ a_{21} & a_{22} & \cdots & a_{2M} & b_{21} & b_{22} & \cdots & b_{2M} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{q1} & a_{q2} & \cdots & a_{qM} & b_{q1} & b_{q2} & \cdots & b_{qM} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \\ m_1 \\ m_2 \\ \vdots \\ m_M \end{bmatrix} = 0$$

Solve this Matrix equation according to
Least square solution.

Cofficients for particular nodal point n ,

$r_1,\ r_2,\ \cdots\ ,r_M$ and $m_1,\ m_2,\ \cdots\ ,m_M$

Repeat the procedure for each nodal point, $n = 1,\ldots,N$ and get the Sparse matrix **R** and **M**.

Global Matrix

$$\mathbf{R} = \begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3} & 0 & \cdots & r_{1,n-1} & r_{1,n} \\ r_{2,1} & r_{2,2} & r_{2,3} & r_{2,4} & \cdots & 0 & r_{2,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{n,1} & r_{n,2} & 0 & \cdots & r_{n,n-2} & r_{n,n-1} & r_{n,n} \end{bmatrix}$$

For $M = 5$

$$\mathbf{M} = \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} & 0 & \cdots & m_{1,n-1} & m_{1,n} \\ m_{2,1} & m_{2,2} & m_{2,3} & m_{2,4} & \cdots & 0 & m_{2,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ m_{n,1} & m_{n,2} & 0 & \cdots & m_{n,n-2} & m_{n,n-1} & m_{n,n} \end{bmatrix}$$

Scattered field for Soft Surface:

$$\phi_0^{inc}(\boldsymbol{r}_s) + \phi_0^{sc}(\boldsymbol{r}_s) = 0 \qquad \boldsymbol{r}_s \in \partial V$$

← Boundary condition

$$[\mathbf{R}][\phi_0^{inc}] + [\mathbf{M}]\left[\frac{\partial \phi_0^{sc}}{\partial n}\right] = 0$$

← Boundary condition in Matrix equation

$$\frac{\partial \phi_0(\boldsymbol{r}_n)}{\partial n} = \frac{\partial \phi_0^{inc}(\boldsymbol{r}_n)}{\partial n} + \frac{\partial \phi_0^{sc}(\boldsymbol{r}_n)}{\partial n}$$

← Fictitious Surface sources

$$= \left[\frac{\partial \phi_0^{inc}(\boldsymbol{r}_n)}{\partial n}\right] - [\mathbf{M}]^{-1}[\mathbf{R}][\phi_0^{inc}]$$

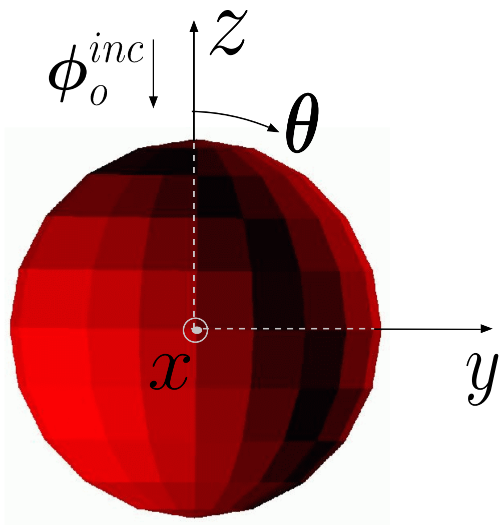
$$\phi_0^{sc}(\boldsymbol{r}) = \sum_{n=1}^{N} \frac{\partial \phi_0(\boldsymbol{r}_n)}{\partial n} G(\boldsymbol{r}, \boldsymbol{r}_n)\,\Delta s$$

← Derivation for Scattered field

Numerical Example

Incident field,

$$\phi_o^{inc}(\boldsymbol{r}) = e^{-jka\cos\theta}$$



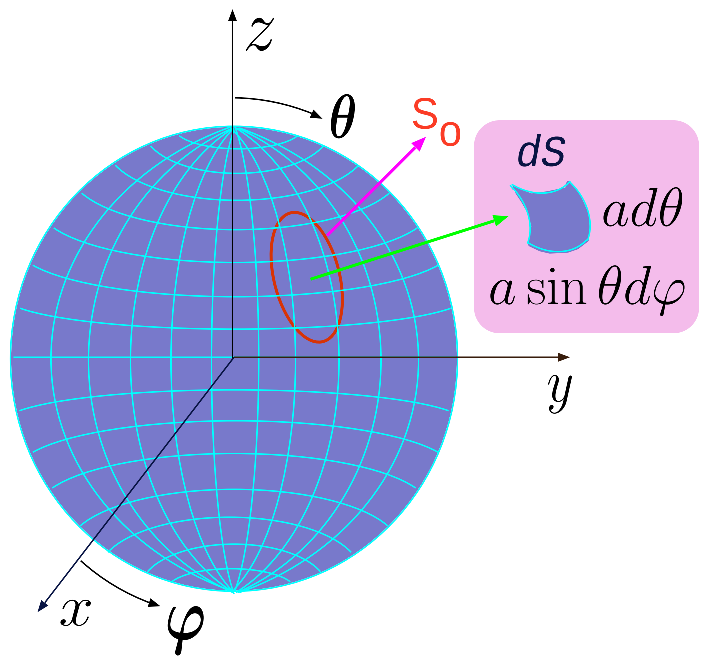
Scattered field,

$$\phi_o^{sc}(\boldsymbol{r}) = \int_s \rho_q(\boldsymbol{r}')\, G(\boldsymbol{r}, \boldsymbol{r}')\, ds'$$

Zonal Harmonics as Metrons,

$$\rho_q = P_q(\cos\theta), \quad q = 1, 2, \cdots, N_{max}$$

Localization and Discretization :



Measuring function varies only in polar direction, θ

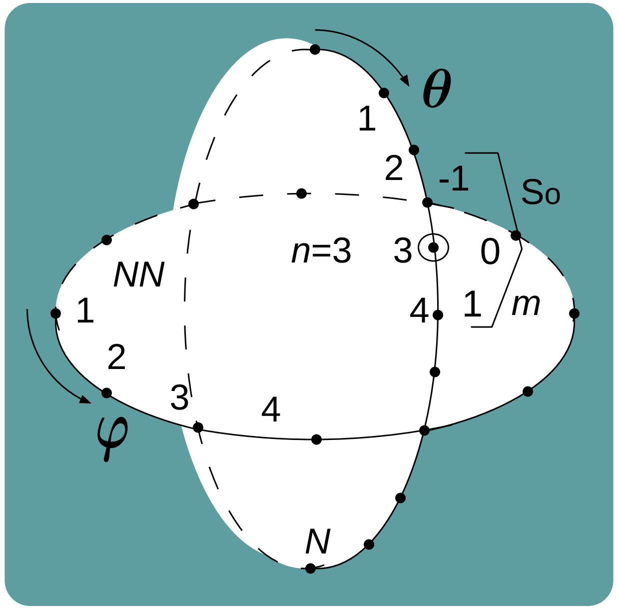
Local sources expanded into, $M = 3$

Use rectangular patches for discretization

Discretisized surface area,

$$dS = a^2 \sin\theta \, d\theta \, d\varphi$$

Segment length, $= \dfrac{1}{10}\lambda$ (wavelength)



Singular value treatment :

When $\boldsymbol{r}$ and $\boldsymbol{r}'$ are on the same patch, i.e., $\boldsymbol{r} \rightarrow \boldsymbol{r}'$

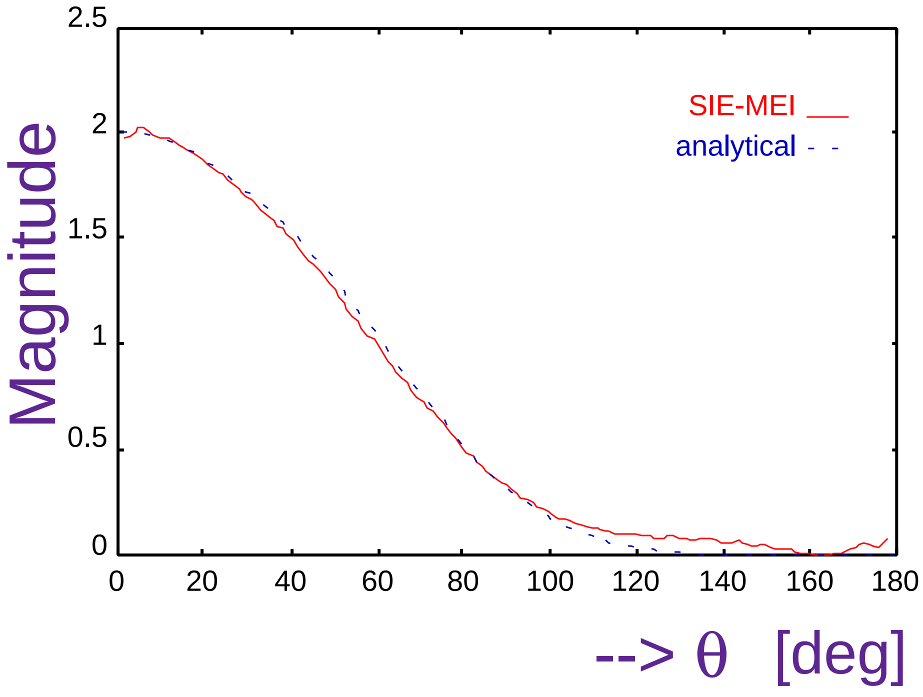
$$G(\boldsymbol{r}, \boldsymbol{r}') = \frac{e^{-jkR}}{4\pi R} = -\frac{jk}{4\pi} h_o^{(2)}(kR), \qquad R = |\boldsymbol{r} - \boldsymbol{r}'|$$

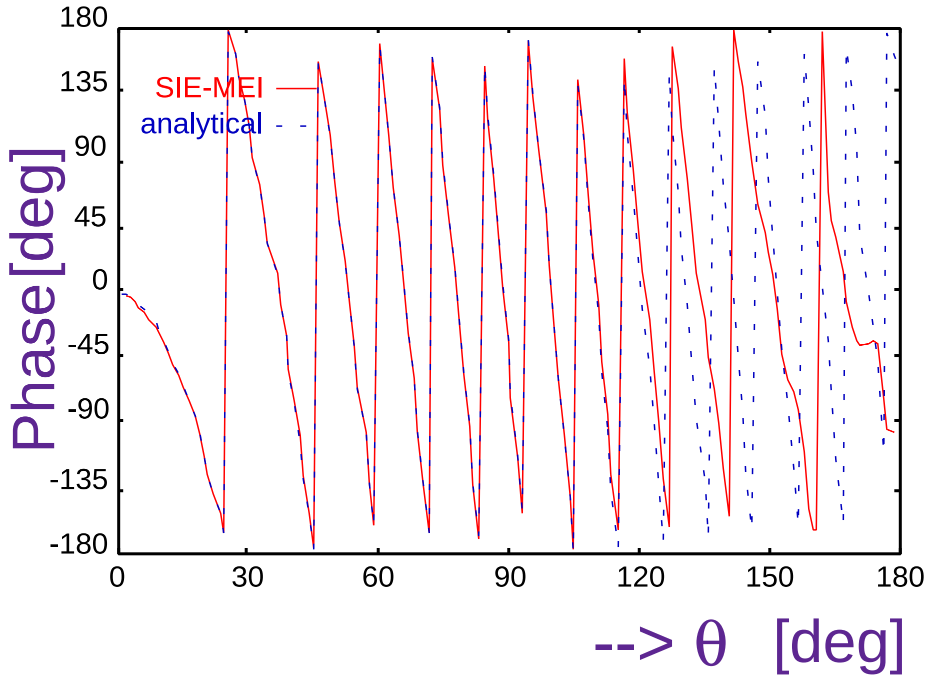
$$\int_{\Delta S_s} h_o^{(2)}(kR_s)ds' = a^2 \sin\theta \Delta\theta \Delta\varphi + j\frac{2a\Delta\theta}{k} ln\left[\mathbf{v} + (\mathbf{v}^2 + 1)^{\frac{1}{2}}\right]$$
$$+j\frac{2a\sin\theta\Delta\varphi}{k} ln\left[\mathbf{v}^{-1} + (\mathbf{v}^{-2} + 1)^{\frac{1}{2}}\right]$$

where, $\mathbf{v} = \dfrac{\sin\theta\Delta\varphi}{\Delta\theta}$

Result

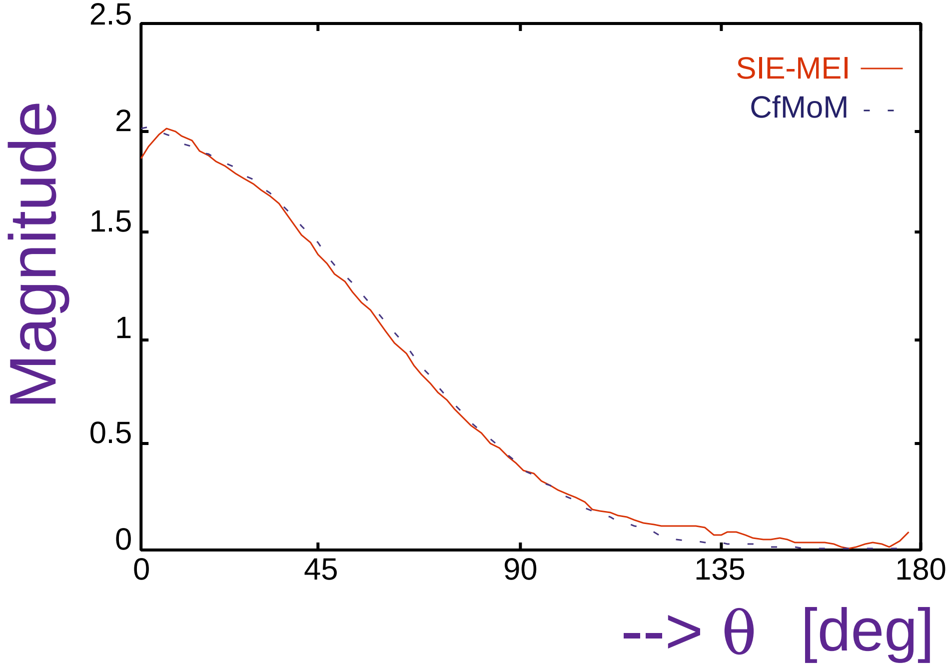
Plot of Normal derivative of the field on the Sphere by using SIE-MEI





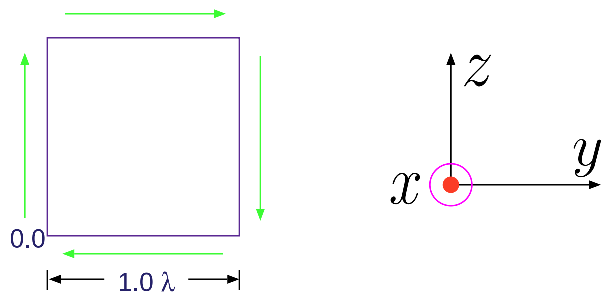
Radius = 5 wavelength (λ)

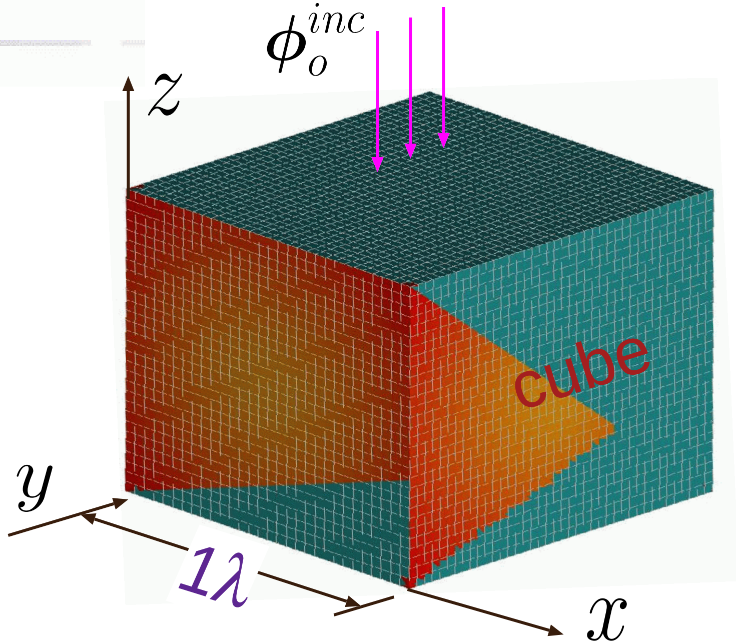
Plot of Near Scattered field



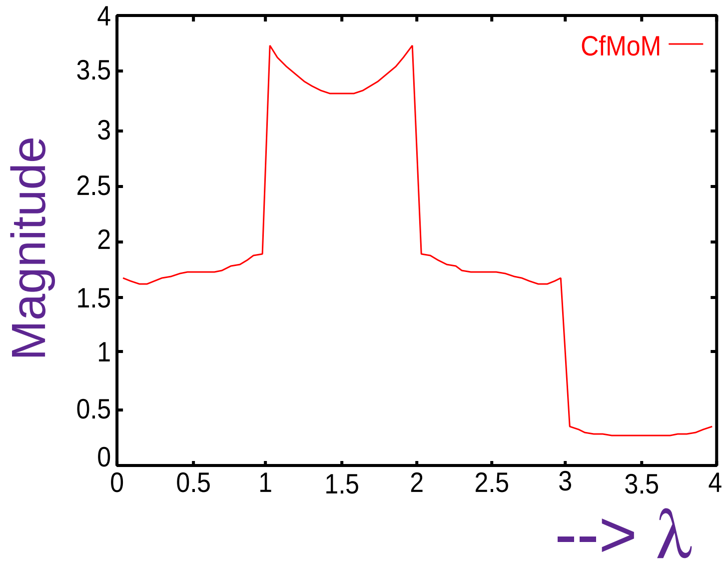
Radius = 5 wavelength (λ)

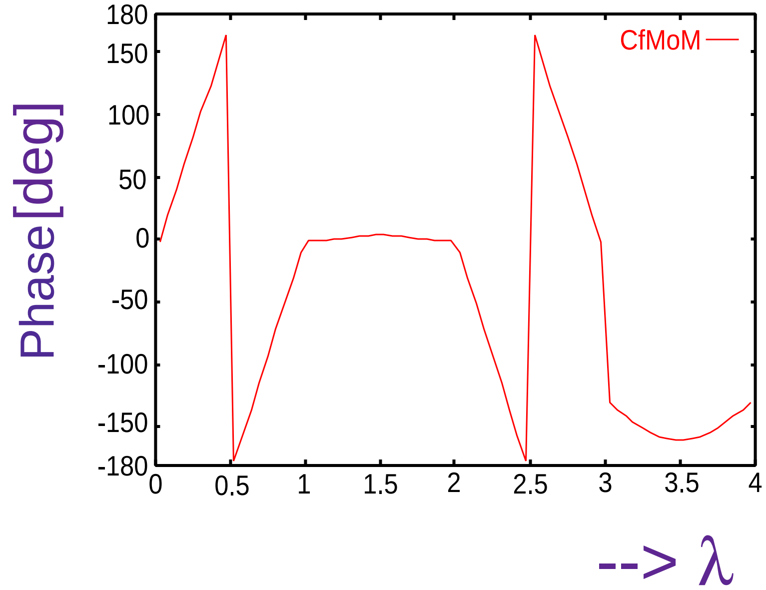
Present Work





Fictitious Surface Source distribution by using CfMoM





Remarks & Future Work

- SIE-MEI is derived from IE-MEI method; Savings in CPU time and memory requirement
- Successfully implemented to uniform shape 3D scalar field (Acoustic) problem
- Numerical results has an excellent agreement with the analytical solution
- In arbitary shape 3D problem, SIE-MEI method can be applied without any significant difference