Robust Design for Block Diagonalization MIMO Systems with CSI Feedback Delay

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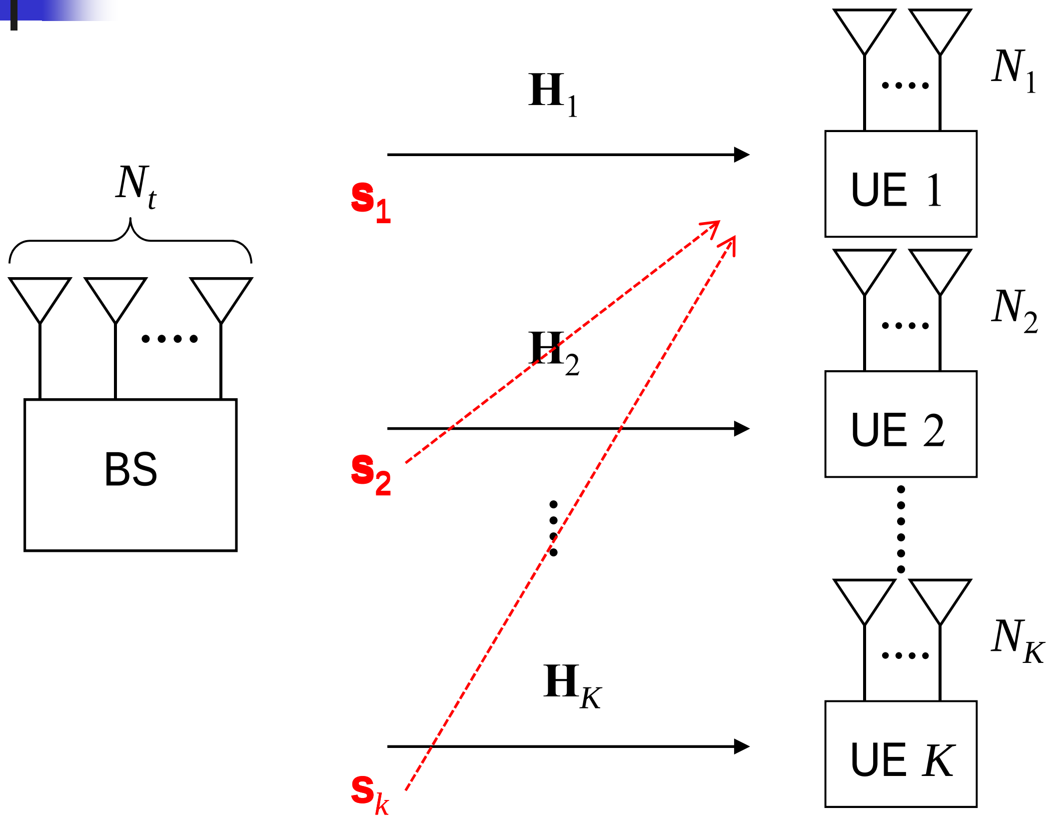
Background

- Multiuser MIMO System for Higher Spatial Diversity Gain.
  - Block Diagonalization (BD): cancel the multiuser interference with designed precoding matrix.
- The problem of BD
  - Base station should know the channel state information (CSI): must feedback CSI.
  - The CSI become outdated due to the time-varying nature of the channels.
- Channel prediction was proposed for single user MIMO, but unpredictable CSI error still remains.

Multiuser MIMO



$$N_t \geq N_r = \sum_{k=1}^{K} N_k$$

BS: base station.
UE: user equipment.
$\mathbf{H}_k$: channel matrix for user k
$\mathbf{S}_k$: data stream for user k

- The purpose:
  - Analysis of multiuser interference.
  - Robust scheme for adaptive modulation.

System Model

- Received signal: $\mathbf{r} = \mathbf{HMs} + \mathbf{n}$

$\mathbf{H}$: total channel matrix
$\mathbf{M}$: total pre-coding matrix
$\mathbf{s}$: data vector for users
$\mathbf{n}$: received noise vector

$$\begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \vdots \\ \mathbf{r}_K \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_K \end{bmatrix} \begin{bmatrix} \mathbf{M}_1 & \mathbf{M}_2 & \cdots & \mathbf{M}_K \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_K \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \vdots \\ \mathbf{n}_K \end{bmatrix}$$

If $\mathbf{H}_i\mathbf{M}_j = 0$ for $i \neq j$.

$$\begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \vdots \\ \mathbf{r}_K \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1\mathbf{M}_1 & & & \mathbf{0} \\ & \mathbf{H}_2\mathbf{M}_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \mathbf{H}_K\mathbf{M}_K \end{bmatrix} \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \\ \vdots \\ \mathbf{s}_K \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \\ \vdots \\ \mathbf{n}_K \end{bmatrix}$$

Block Diagonalization (BD)

Block Diagonalization I

Aggregate channel matrix beside that of user k:

$$\tilde{\mathbf{H}}_k = \begin{bmatrix} \mathbf{H}_1^T & \cdots & \mathbf{H}_{k-1}^T & \mathbf{H}_{k+1}^T & \cdots & \mathbf{H}_K^T \end{bmatrix}^T$$

Null space:

$$\tilde{\mathbf{H}}_k = \tilde{\mathbf{U}}_k \begin{bmatrix} \tilde{\Sigma}_k & \mathbf{0} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{V}}_k^{(1)} & \tilde{\mathbf{V}}_k^{(0)} \end{bmatrix}^H$$

For null space of $\tilde{\mathbf{H}}_k$

$$\tilde{\mathbf{H}}_k \tilde{\mathbf{V}}_k^{(0)} = \begin{bmatrix} \left(\mathbf{H}_1 \tilde{\mathbf{V}}_k^{(0)}\right)^T & \cdots & \left(\mathbf{H}_{k-1} \tilde{\mathbf{V}}_k^{(0)}\right)^T & \left(\mathbf{H}_{k+1} \tilde{\mathbf{V}}_k^{(0)}\right)^T & \cdots & \left(\mathbf{H}_K \tilde{\mathbf{V}}_k^{(0)}\right)^T \end{bmatrix}^T = \mathbf{0}$$

Decoded signals:

Effective Channel

$$\mathbf{H}'_k = \mathbf{H}_k \tilde{\mathbf{V}}_k^{(0)} = \mathbf{U}'_k \begin{bmatrix} \Sigma'_k & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}'^{(1)}_k & \mathbf{V}'^{(0)}_k \end{bmatrix}^H \Rightarrow \mathbf{M}_k = \tilde{\mathbf{V}}_k^{(0)} \mathbf{V}'^{(1)}_k$$

SVD: singular value decompositions.

Block Diagonalization II

Transmitted signals:

$$\mathbf{x}_k = \mathbf{M}_k \mathbf{s}_k = \tilde{\mathbf{V}}_k^{(0)} \mathbf{V}_k'^{(1)} \mathbf{s}_k$$

$$\sum_j tr\left\{E\left[\mathbf{x}_j \mathbf{x}_j^H\right]\right\} = \sum_j tr\left\{E\left[\mathbf{s}_j \mathbf{s}_j^H\right]\right\} = P_t$$

Received signals:

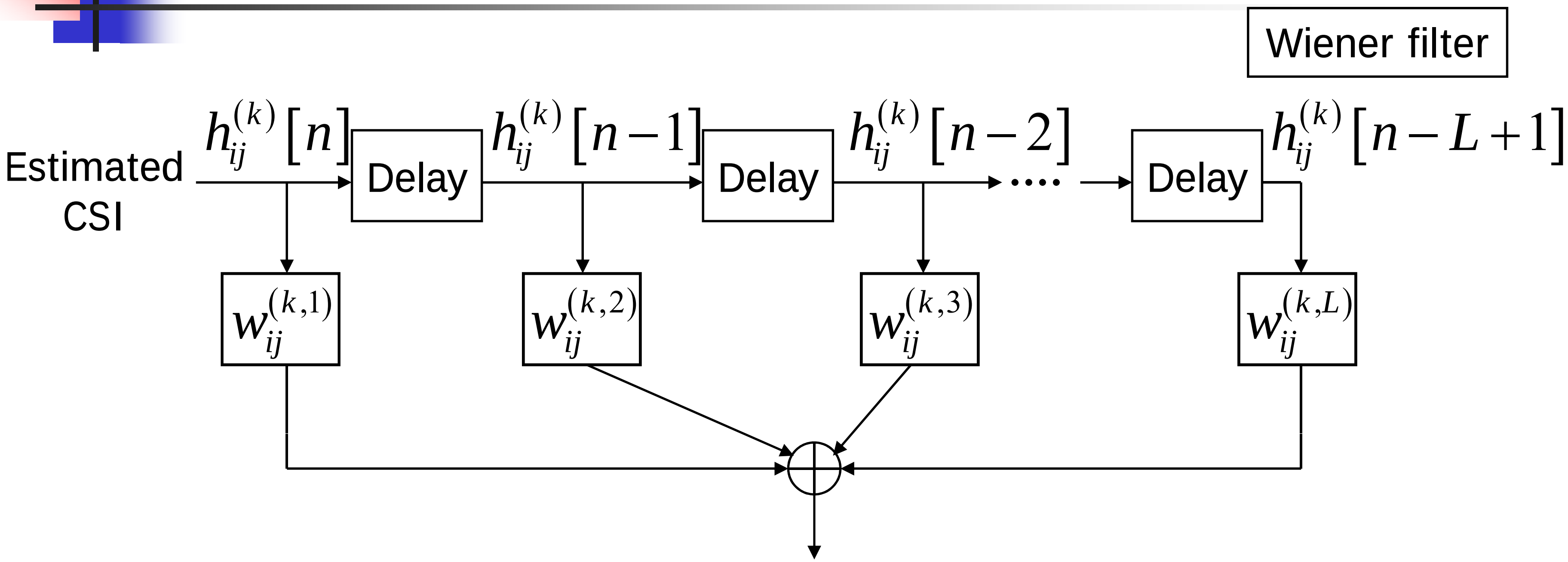
$$\mathbf{r}_k = \mathbf{H}_k \sum_{j=1}^{K} \mathbf{x}_j + \mathbf{n}_k$$

$$= \underbrace{\mathbf{H}_k \tilde{\mathbf{V}}_k^{(0)}}_{\mathbf{H}'_k} \mathbf{V}_k'^{(1)} \mathbf{s}_k + \cancel{\mathbf{H}_k \sum_{j=1, j \neq k}^{K} \underbrace{\tilde{\mathbf{V}}_j^{(0)} \mathbf{V}_j'^{(1)}}_{\mathbf{M}_j} \mathbf{s}_j} + \mathbf{n}_k$$

Decoded signals:

$$\mathbf{y}_k = \mathbf{U}_k'^H \mathbf{r}_k = \mathbf{U}_k'^H \underbrace{\mathbf{H}_k \tilde{\mathbf{V}}_k^{(0)}}_{\mathbf{H}'_k} \mathbf{V}_k'^{(1)} \mathbf{s}_k + \mathbf{U}_k'^H \mathbf{n}_k = \Sigma'_k \mathbf{s}_k + \mathbf{n}'_k$$

Channel Prediction I



$$\hat{h}_{ij}^{(k)}[n+q] = \sum_{l=1}^{L} w_{ij}^{(k,\,l)*} h_{ij}^{(k)}[n-l+1] = \mathbf{w}_{ij}^{(k)H} \mathbf{h}_{ij}^{(k)}[n]$$

$$\mathbf{h}_{ij}^{(k)}[n] = \left[h_{ij}^{(k)}[n] \quad h_{ij}^{(k)}[n-1] \quad \cdots \quad h_{ij}^{(k)}[n-L+1] \right]^T$$

$$\mathbf{w}_{ij}^{(k)H} = \left[w_{ij}^{(k,\,1)*} \quad w_{ij}^{(k,\,2)*} \quad \cdots \quad w_{ij}^{(k,\,L)*} \right]$$

Reference: K. Kobayashi, etc., “MIMO Systems in the Presence of Feedback Delay,” IEICE Technical Report RCS2005-25 (2005-5).

Channel Prediction II

MMSE:

$$\mathbf{w}_{ij}^{(k)} = \begin{bmatrix} w_{ij}^{(k,\,1)} \\ w_{ij}^{(k,\,2)} \\ \vdots \\ w_{ij}^{(k,\,L)} \end{bmatrix} = \mathbf{R}_k^{-1}\mathbf{u}_k$$

MSE:

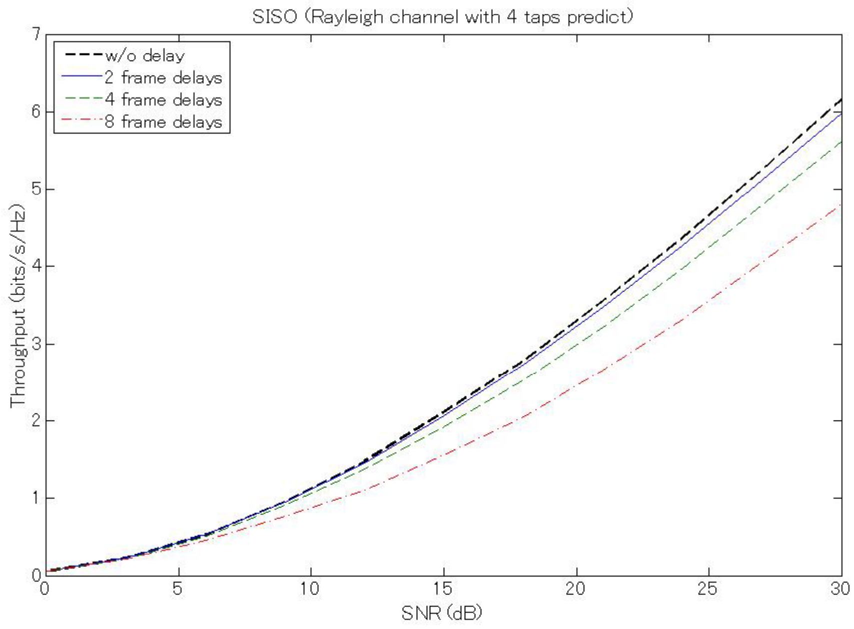
$$\sigma_k^2 = E\left[\left|\varepsilon_{ij}^{(k)}[n]\right|^2\right] = 1 - \mathbf{u}_k^H \mathbf{R}_k^{-1} \mathbf{u}_k$$

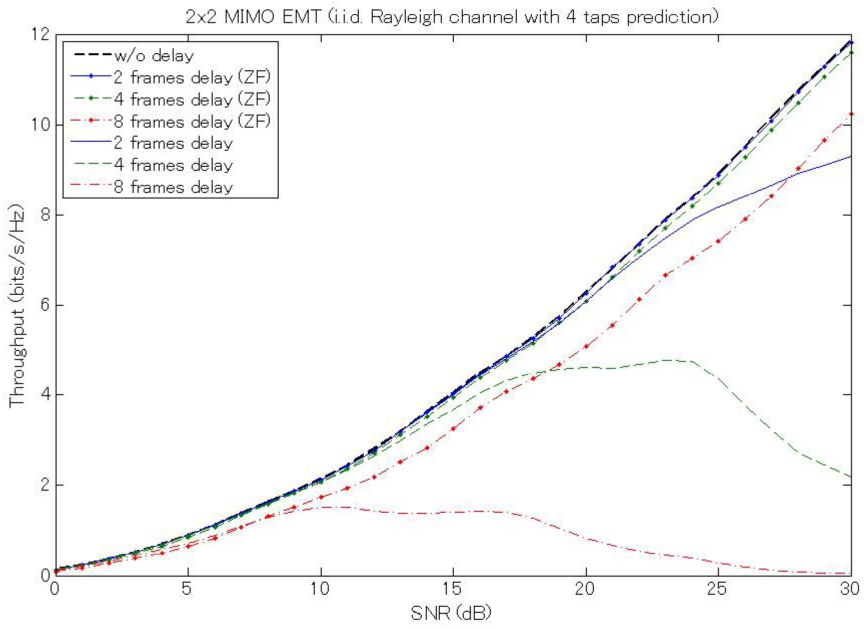
$$\boxed{\varepsilon_{ij}^{(k)}[n] = h_{ij}^{(k)}[n+q] - \hat{h}_{ij}^{(k)}[n+q]}$$

$$\mathbf{R}_k = \begin{bmatrix} E\left[h_{ij}^{(k)*}[n]h_{ij}^{(k)}[n]\right] & E\left[h_{ij}^{(k)*}[n]h_{ij}^{(k)}[n-1]\right] & \cdots & E\left[h_{ij}^{(k)*}[n]h_{ij}^{(k)}[n-l+1]\right] \\ E\left[h_{ij}^{(k)*}[n]h_{ij}^{(k)}[n-1]\right] & E\left[h_{ij}^{(k)*}[n]h_{ij}^{(k)}[n]\right] & \cdots & E\left[h_{ij}^{(k)*}[n]h_{ij}^{(k)}[n-l+2]\right] \\ \vdots & \vdots & \ddots & \vdots \\ E\left[h_{ij}^{(k)*}[n]h_{ij}^{(k)}[n-l+1]\right] & E\left[h_{ij}^{(k)*}[n]h_{ij}^{(k)}[n-l+2]\right] & \cdots & E\left[h_{ij}^{(k)*}[n]h_{ij}^{(k)}[n]\right] \end{bmatrix} \qquad \mathbf{u} = \begin{bmatrix} E\left[h_{ij}^{(k)*}[n]h_{ij}^{(k)}[n-q]\right] \\ E\left[h_{ij}^{(k)*}[n]h_{ij}^{(k)}[n-q-1]\right] \\ \vdots \\ E\left[h_{ij}^{(k)*}[n]h_{ij}^{(k)}[n-q-l+1]\right] \end{bmatrix}$$

Reference: K. Kobayashi, etc., "MIMO Systems in the Presence of Feedback Delay," IEICE Technical Report RCS2005-25 (2005-5).

SISO and MIMO





Multiuser Interference I

Channel error matrix:

$$\mathbf{\Delta}_k \triangleq \bar{\mathbf{H}}_k - \hat{\mathbf{H}}_k$$

$\bar{\mathbf{H}}_k$: real channel matrix.

$\hat{\mathbf{H}}_k$: predicted channel matrix.

Received signals:

$$\mathbf{r}_k = \bar{\mathbf{H}}_k \mathbf{M}_k \mathbf{s}_k + \left(\hat{\mathbf{H}}_k + \mathbf{\Delta}_k \right) \sum_{j=1,\, j \neq k}^{K} \mathbf{M}_j \mathbf{s}_j + \mathbf{n}_k$$

$$= \bar{\mathbf{H}}_k \mathbf{M}_k \mathbf{s}_k + \mathbf{\Delta}_k \sum_{j=1,\, j \neq k}^{K} \mathbf{x}_j + \mathbf{n}_k$$

$$\mathbf{x}_j = \mathbf{M}_j \mathbf{s}_j$$

$$\mathbf{n}'_k = \mathbf{U}'^{H}_k \mathbf{n}_k$$

Decoded signals:

$$\mathbf{y}_k = \left(\bar{\mathbf{H}}_k \mathbf{M}_k \right)^{-1} \mathbf{r}_k$$

$$\approx \mathbf{s}_k + \Sigma'^{-1}_k \mathbf{U}'^{H}_k \mathbf{\Delta}_k \sum_{j=1,\, j \neq k}^{K} \mathbf{x}_j + \Sigma'^{-1}_k \mathbf{n}'_k$$

Multiuser Interference II

- Decoded signal for user k

$$y_i^{(k)} \approx s_i^{(k)} + \frac{\mathbf{d}_i^{(k)H}}{\sqrt{\lambda_i'^{(k)}}} \sum_{j=1, j\neq k}^{K} \mathbf{x}_j + \frac{n_i'^{(k)}}{\sqrt{\lambda_i'^{(k)}}}$$

- Ergodic average of SINR

$$E[\mathbf{s}_j\mathbf{s}_j^H] = \frac{P_t}{N_r}\mathbf{I}_{N_j} \quad E[\mathbf{x}_j\mathbf{x}_j^H] = \frac{N_j P_t}{N_t N_r}\mathbf{I}_{N_t}$$

$$\mathbf{D}_k \triangleq \mathbf{\Delta}_k^H \mathbf{U}_k' = \begin{bmatrix} \mathbf{d}_1^{(k)} & \mathbf{d}_2^{(k)} & \dots & \mathbf{d}_{N_k}^{(k)} \end{bmatrix}$$

$$E[\mathbf{d}_i^{(k)H}\mathbf{d}_i^{(k)}] = \sigma_k^2 N_t$$

γ : average SNR.

σ_k^2 : channel MSE.

$\lambda_i'^{(k)}$: i'th eigenvalue for effective channel matrix of user k.

$$\begin{aligned} E[\mathrm{SINR}_i^{(k)}] &\approx E\left[\frac{\lambda_i'^{(k)}}{\frac{(N_r-N_k)}{N_t}\mathbf{d}_i^{(k)H}\mathbf{d}_i^{(k)} + \frac{N_r}{\gamma}}\right] \\ &\geq \frac{E\left[\lambda_i'^{(k)}\right]}{\frac{(N_r-N_k)}{N_t}E\left[\mathbf{d}_i^{(k)H}\mathbf{d}_i^{(k)}\right] + \frac{N_r}{\gamma}} \\ &= \frac{E\left[\lambda_i'^{(k)}\right]}{(N_r-N_k)\sigma_k^2 + N_r/\gamma} \end{aligned}$$

Robust Scheme

- The estimated SINR for adaptive modulation:

$$\hat{\text{SINR}}_i^{(k)} \triangleq \frac{\lambda_i'^{(k)}}{\alpha(N_r - N_k)\sigma_k^2 + N_r/\gamma}$$

- If α is set large, the system will be more robust in multiuser interference, but lower transmit data rate.

From the PDF of $\beta \triangleq \mathbf{d}_i^{(k)H}\mathbf{d}_i^{(k)}$

- PDF of the interference part

$$f_{\mathrm{B}}(\beta) = \frac{\beta^{(N_t-1)}}{(N_t-1)!\sigma_k^{2N_t}} \exp\left(-\frac{\beta}{\sigma_k^2}\right)$$

- The most probable β

$$\beta = (N_t - 1)\sigma_k^2$$

- The α becomes

$$\alpha = \frac{(N_t - 1)}{N_t}$$

Throughput vs. Delay (α = 0)

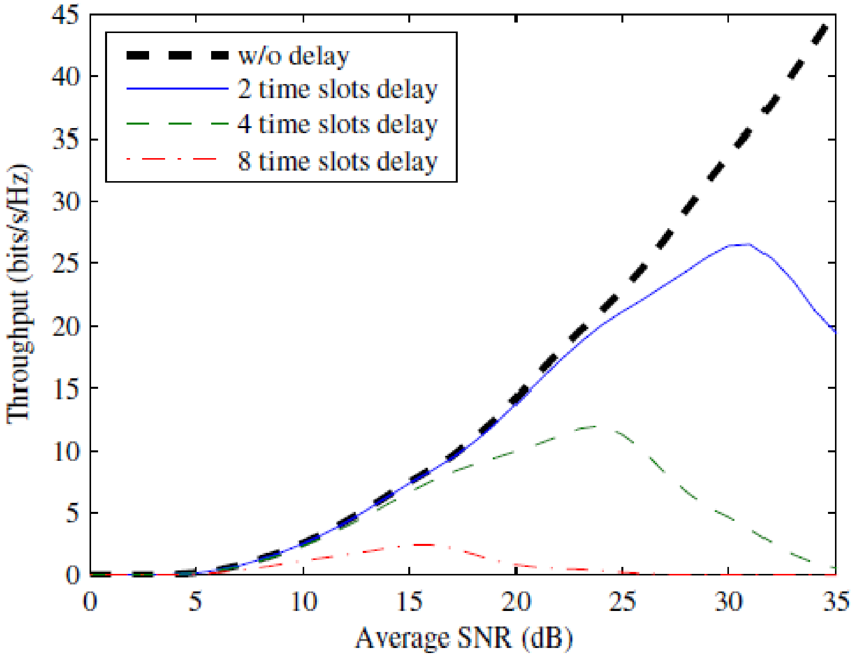


Fig. 1. Average throughput of 4 users BD MIMO Downlink system.

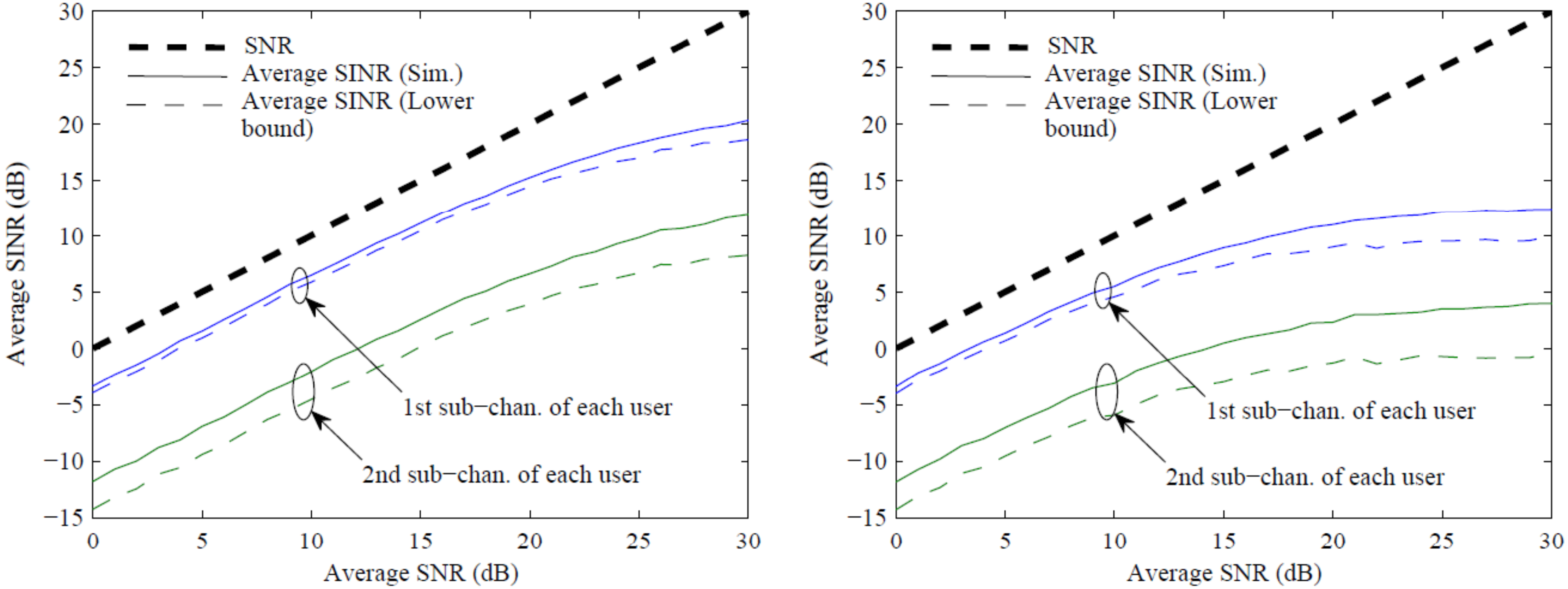
TABLE I
SIMULATION PARAMETERS

Parameter	Value or Setting
No. of Tx, N_t	8
No. of users, K	4
No. of Rx per user, N_k	2
Channel model	Flat i.i.d Rayleigh Channel
Maximum Doppler frequency, $f_d^{(k)}$	67 Hz
Time period per time slot, T_f	0.5 ms
Modulation scheme	Adaptive modulation
Channel prediction filter order, L	4

TABLE II
SINR THRESHOLD OF ADAPTIVE MODULATION

Modulation	SINR Interval
BPSK	$\hat{\text{SINR}}_i^k <$ 9.0 dB
QPSK	9.0 dB $\leq \hat{\text{SINR}}_i^k <$ 14.8 dB
8PSK	14.8 dB $\leq \hat{\text{SINR}}_i^k <$ 17.0 dB
16QAM	17.0 dB $\leq \hat{\text{SINR}}_i^k <$ 23.0 dB
64QAM	23.0 dB $\leq \hat{\text{SINR}}_i^k <$ 29.5 dB
256QAM	29.5 dB $\leq \hat{\text{SINR}}_i^k$

Average SINR



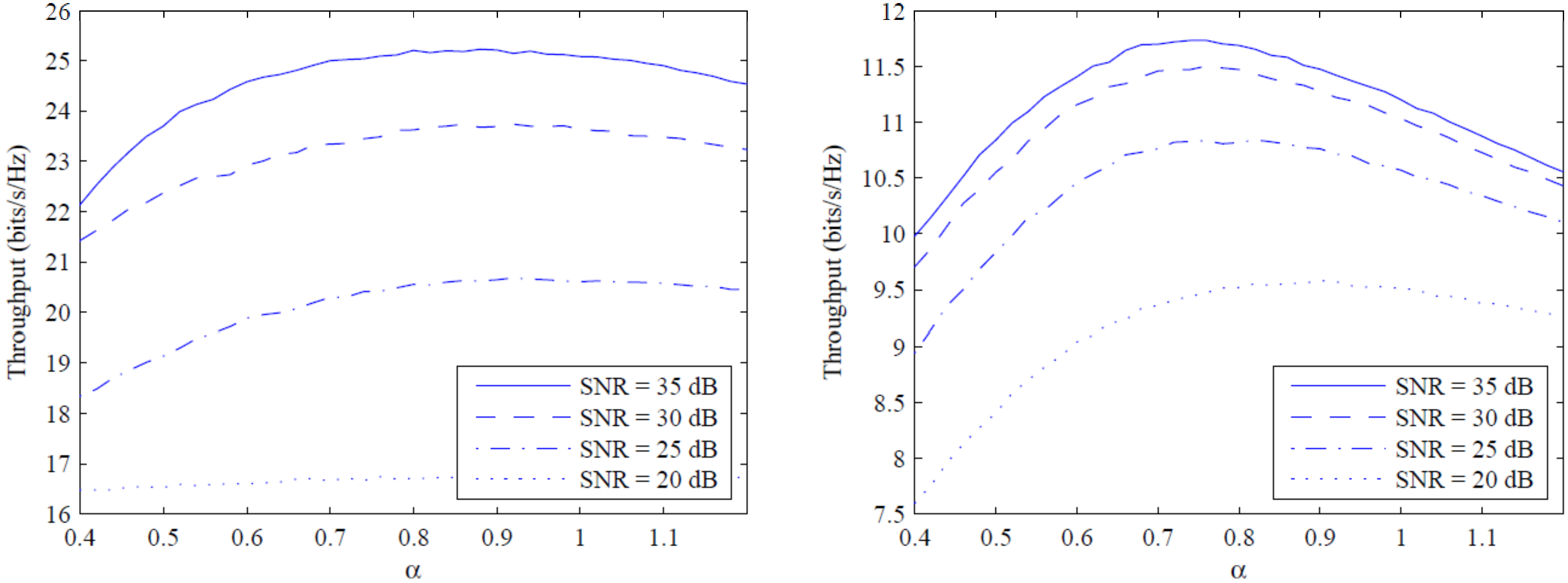
(a) System with 4 time slots feedback delay

(b) System with 8 time slots feedback delay

Fig. 2. Average SINR and the lower bound.

$$E[\mathrm{SINR}_i^{(k)}] \approx E\left[\frac{\lambda_i^{\prime(k)}}{\frac{(N_r - N_k)}{N_t}\mathbf{d}_i^{(k)H}\mathbf{d}_i^{(k)} + \frac{N_r}{\gamma}}\right] \geq \frac{E\left[\lambda_i^{\prime(k)}\right]}{(N_r - N_k)\sigma_k^2 + N_r/\gamma}$$

Throughput vs. α



(a) System with 4 time slots feedback delay

(b) System with 8 time slots feedback delay

Fig. 4 Average throughput vs. α for 3 user BD MIMO Downlink system ($N_t = 8$).

Optimal α

Table 4 Optimal values of α for different setting of systems.

(a) $N_t = 4$

Users no. and delay		Average SNR 35 dB	30 dB	25 dB	20 dB
2 users	4 time slots	0.76	0.77	0.88	-
	8 time slots	0.54	0.64	0.72	0.92

(b) $N_t = 8$

Users no. and delay		Average SNR 35 dB	30 dB	25 dB	20 dB
2 users	4 time slots	0.84	0.86	0.96	-
	8 time slots	0.66	0.70	0.74	0.86
3 users	4 time slots	0.86	0.90	0.92	-
	8 time slots	0.75	0.76	0.78	0.88
4 users	4 time slots	0.84	0.88	0.90	-
	8 time slots	0.76	0.76	0.78	0.84

(c) $N_t = 12$

Users no. and delay		Average SNR 35 dB	30 dB	25 dB	20 dB
2 users	4 time slots	0.82	0.84	0.94	-
	8 time slots	0.72	0.76	0.80	0.94
3 users	4 time slots	0.86	0.88	0.92	-
	8 time slots	0.80	0.80	0.86	0.92
4 users	4 time slots	0.86	0.90	0.94	-
	8 time slots	0.82	0.82	0.83	0.90

Where "-" means it is hard to distinguish the optimal value.

Throughput vs. SNR

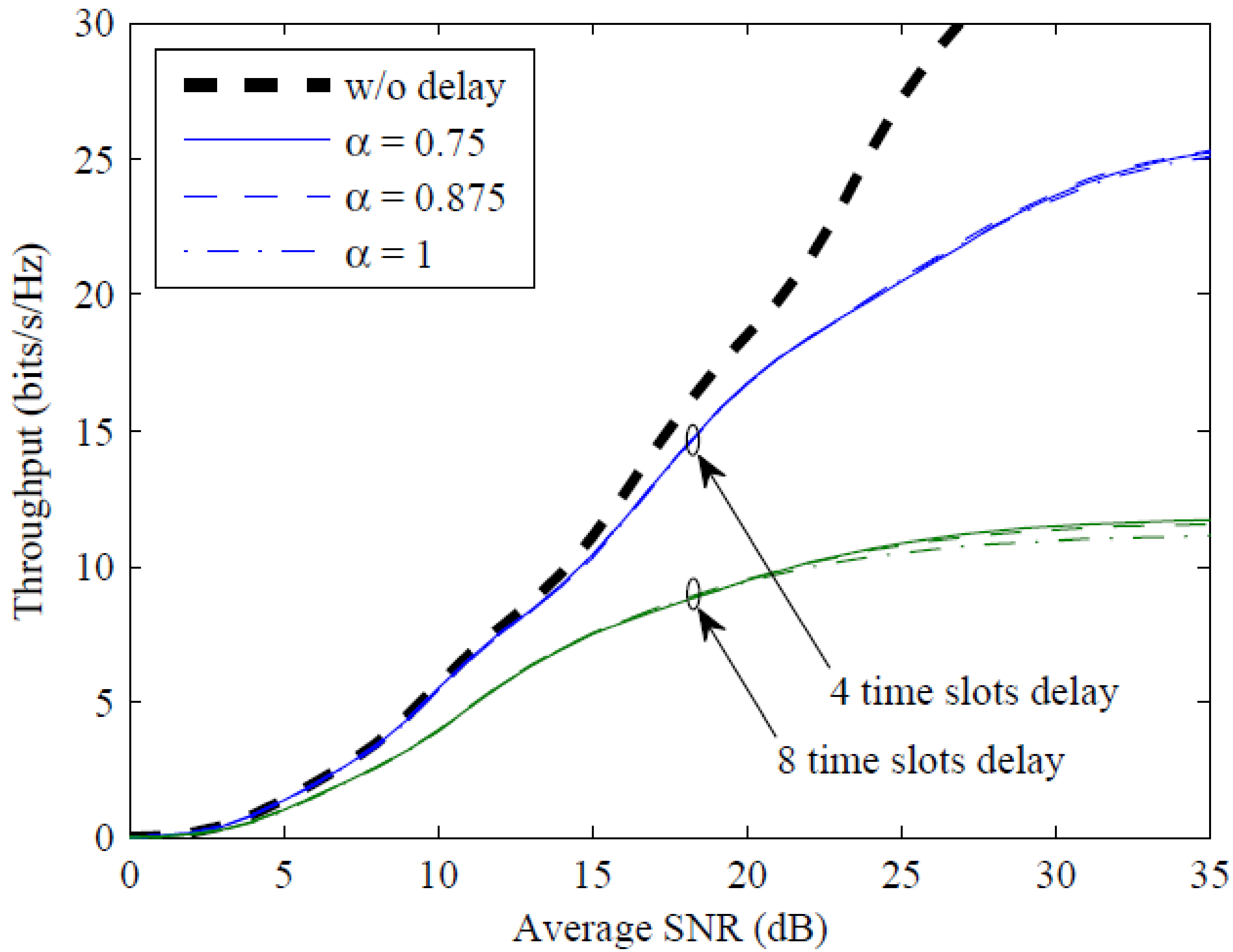


Fig. 6 Average system throughput results with different α for 3 user BD MIMO system ($N_t = 8$).

Conclusion

- In BD MIMO downlink system with channel prediction,
  - we analyzed the multiuser interference caused by feedback delay
  - we proposed a robust scheme for adaptive modulation
- Future work
  - Improving of the robustness
  - User scheduling for multiuser MIMO DL system when $N_r > N_t$.