Power allocation for Block Diagonalization Multi-user MIMO downlink with fair user scheduling and unequal average SNR users

Therdkiat A. (Kiak)
Araki-Sakaguchi Laboratory
MCRG group seminar
12 July 2012

TOKYO TECH
Pursuing Excellence

AGENDA

- Background
- System model
  - Block Diagonalization(BD)
- Proportional fair User scheduling
- Power allocation
  - Equal power allocation
  - Water filling
  - Proposed scheme
- Simulation results
- Conclusion and future work

Background

- MIMO system
  - Improve throughput and spectral efficiency
  - Capacity of the system increases with the number of transmit and receive antennas
- Multi-user MIMO system
  - Serving multiple MIMO users by using the benefit of spatial multiplexing instead of TDMA, FDMA or CDMA

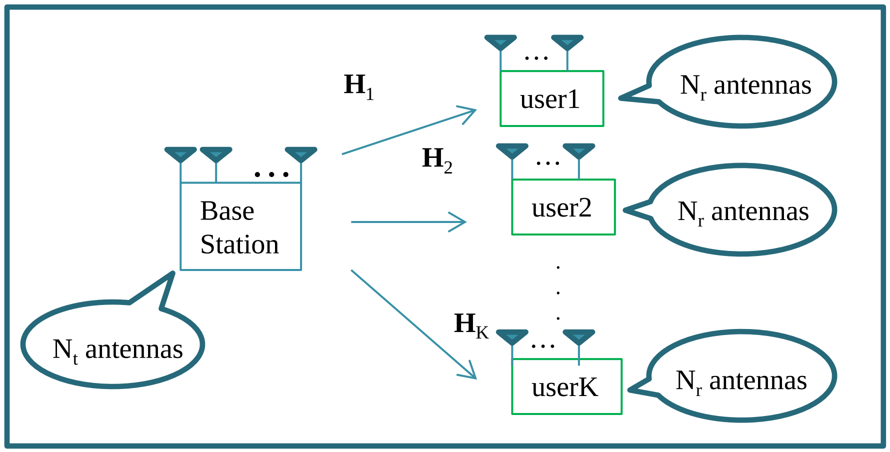
Problems: the number of supportable users is usually less than the number of active users due to the limitation of the transmit antennas

Solution: user scheduling is needed to select a group of users from all active users

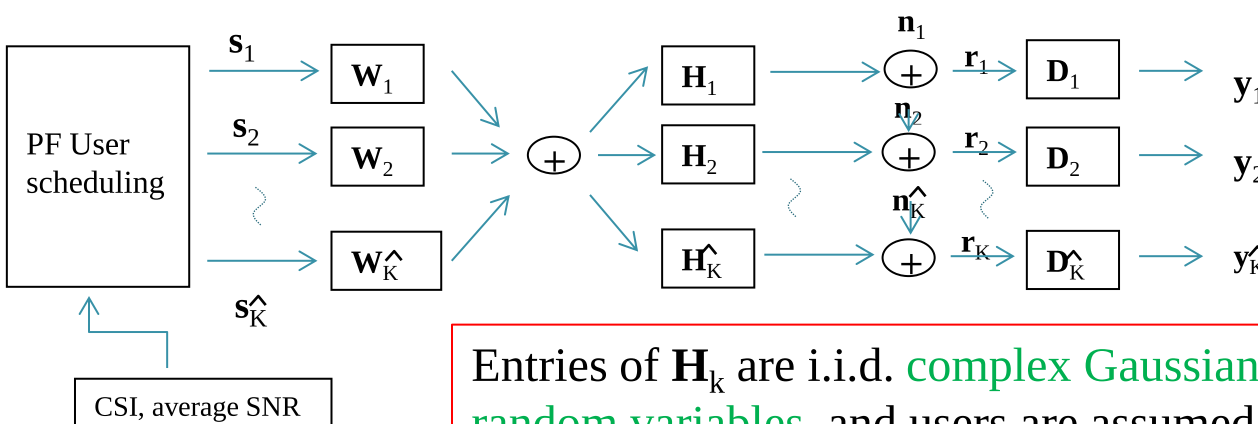
System model

$$\mathbf{r}_k = \mathbf{H}_k \sum_{j=1}^{K} \mathbf{W}_j \mathbf{s}_j + \mathbf{n}_k$$

- Multiuser MIMO system



- Each user has the same number of antenna
- If $N_t < KN_r$, the user scheduling is needed



Entries of $\mathbf{H}_k$ are i.i.d. complex Gaussian random variables, and users are assumed to have different average SNR

System model

The received signal of the *k*th user with postcoding matrix $\mathbf{D}_k$ is applied, then

$$y_k = \mathbf{D}_k r_k = \mathbf{D}_k \mathbf{H}_k \sum_{j=1}^{\hat{K}} \mathbf{W}_j s_j + \mathbf{D}_k n_k$$

$$= \mathbf{D}_k \mathbf{H}_k \mathbf{W}_k s_k + \mathbf{D}_k n_k + \mathbf{D}_k \mathbf{H}_k \sum_{j \neq k}^{\hat{K}} \mathbf{W}_j s_j$$

Inter-user interference

We need $\mathbf{H}_k \mathbf{W}_j = \mathbf{0}$, $k \neq j$

$$y_k = \mathbf{D}_k r_k = \mathbf{D}_k \mathbf{H}_k \boldsymbol{W}_k s_k + \mathbf{D}_k n_k$$

Block Diagonalization (BD)

- Introduced to cancel inter-user-interference (IUI) by designing precoding matrices
- For the *k*th user, define

$$\tilde{\mathbf{H}}_k = \begin{bmatrix} \mathbf{H}_1^T & \cdots & \mathbf{H}_{k-1}^T & \mathbf{H}_{k+1}^T & \cdots & \mathbf{H}_K^T \end{bmatrix}^T$$

then determine the singular value decomposition (SVD) of the matrix

$$\tilde{\mathbf{H}}_k = \tilde{\mathbf{U}}_k \begin{bmatrix} \tilde{\mathbf{\Sigma}}_k & \mathbf{0} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{V}}_{k,1} & \tilde{\mathbf{V}}_{k,0} \end{bmatrix}^H$$

Precoding matrix is columns of $\tilde{\mathbf{V}}_{k,0}$

Noninterfering single user MIMO effective channel becomes

$$\mathbf{H}_{k,e} = \mathbf{H}_k \tilde{\mathbf{V}}_{k,0}$$

TOKYO TECH Pursuing Excellence

Block Diagonalization (BD)

- The situation becomes single MIMO capacity maximization problem, (find postcoding and another precoding matrix)
- The postcoding matrix is determined by applying SVD to the effective channel $\mathbf{H}_{k,e}$

$$\mathbf{H}_{k,e} = \mathbf{U}_{k,e} \left[\begin{array}{cc} \mathbf{\Sigma}_{k,e} & 0 \end{array} \right] \left[\begin{array}{cc} \mathbf{V}_{k,1e} & \mathbf{V}_{k,0e} \end{array} \right]^H$$

- Postcoding matrix is columns of $\mathbf{U}_{k,e}$
- Precoding matrix for effective channel is columns of $\mathbf{V}_{k,1e}$

TOKYO TECH
Pursuing Excellence

Block Diagonalization (BD)

- Summary of BD

  From

$$\mathbf{y}_k = \mathbf{D}_k\mathbf{r}_k = \mathbf{D}_k\mathbf{H}_k\sum_{j=1}^{\hat{K}}\mathbf{W}_j\mathbf{s}_j + \mathbf{D}_k\mathbf{n}_k$$
$$= \mathbf{D}_k\mathbf{H}_k\mathbf{W}_k\mathbf{s}_k + \mathbf{D}_k\mathbf{n}_k$$

  where $\mathbf{W}_k = \widetilde{\mathbf{V}}_{k,0}\mathbf{V}_{k,1e}$ and $\mathbf{D}_k = \mathbf{U}_{k,e}^H$

- Finally, the sum rate capacity of the BD MIMO system becomes

$$C_{BD} = \max_{\Lambda_k, \sum_{k=1}^{\hat{K}} tr(\Lambda_k)=P_t} \sum_{k=1}^{\hat{K}} \log_2 \left| \mathbf{I} + \frac{1}{\sigma_{n,k}^2}\mathbf{\Sigma}_{k,e}^2\mathbf{\Lambda}_k \right|$$

TOKYO TECH
Pursuing Excellence

Proportional Fair User scheduling

- If $N_t < KN_r$, the base station has to select $\widehat{K}$ out of K users where $\widehat{K} = \left\lfloor \frac{N_t}{N_r} \right\rfloor$

- In conventional work, the user scheduling is based on $\|\mathbf{H}_k\|^2$ (channel quality)

- With proportional fair strategy and unequal SNR users case, the scheduling is then modified to consider $\|\mathbf{H}_k\|^2$, SNR of each user and the fairness among users

TOKYO TECH Pursuing Excellence

Proportional Fair User scheduling

(1) Initialization ($i = 1$), let $\mathcal{U} = \{1, 2, ..., K\}$ and $\mathcal{U}_s = \phi$

a. Select the first user u_1 such that $u_1 = \arg\max_{k\in\mathcal{U}} \mu_k(t) \log_2(1 + \rho_k \|\mathbf{H}_k\|^2)$

b. Set $\mathcal{U}_s = \mathcal{U}_s \cup \{u_1\}$ and $\mathcal{U} = \mathcal{U} \backslash \{u_1\}$

(2) For $i = 1$ to $\widehat{K}$

a. Find the projection matrix $\mathcal{P}_i^{\perp} = \mathbf{I}_{N_t} - \mathbf{V}_{i-1}^H \mathbf{V}_{i-1}$ where $\mathbf{V}_{i-1}$ is the row basis of $\mathbf{H}(\mathcal{U}_s)$ and $\mathbf{H}(\mathcal{U}_s) = \begin{bmatrix} \mathbf{H}_{u_1}^T & \mathbf{H}_{u_2}^T & \cdots & \mathbf{H}_{u_{i-1}}^T \end{bmatrix}^T$

b. Find the set of users $\mathcal{U}_i = \{k \in \mathcal{U}_{i-1} | (\|\mathbf{H}_k \mathbf{V}_{i-1}^H\| / \|\mathbf{H}_k\| \, \|\mathbf{V}_{i-1}\|) < \varepsilon\}$

c. If $\mathcal{U}_i = \phi$, terminate the algorithm and $\mathcal{U}_s$ is the selected user set

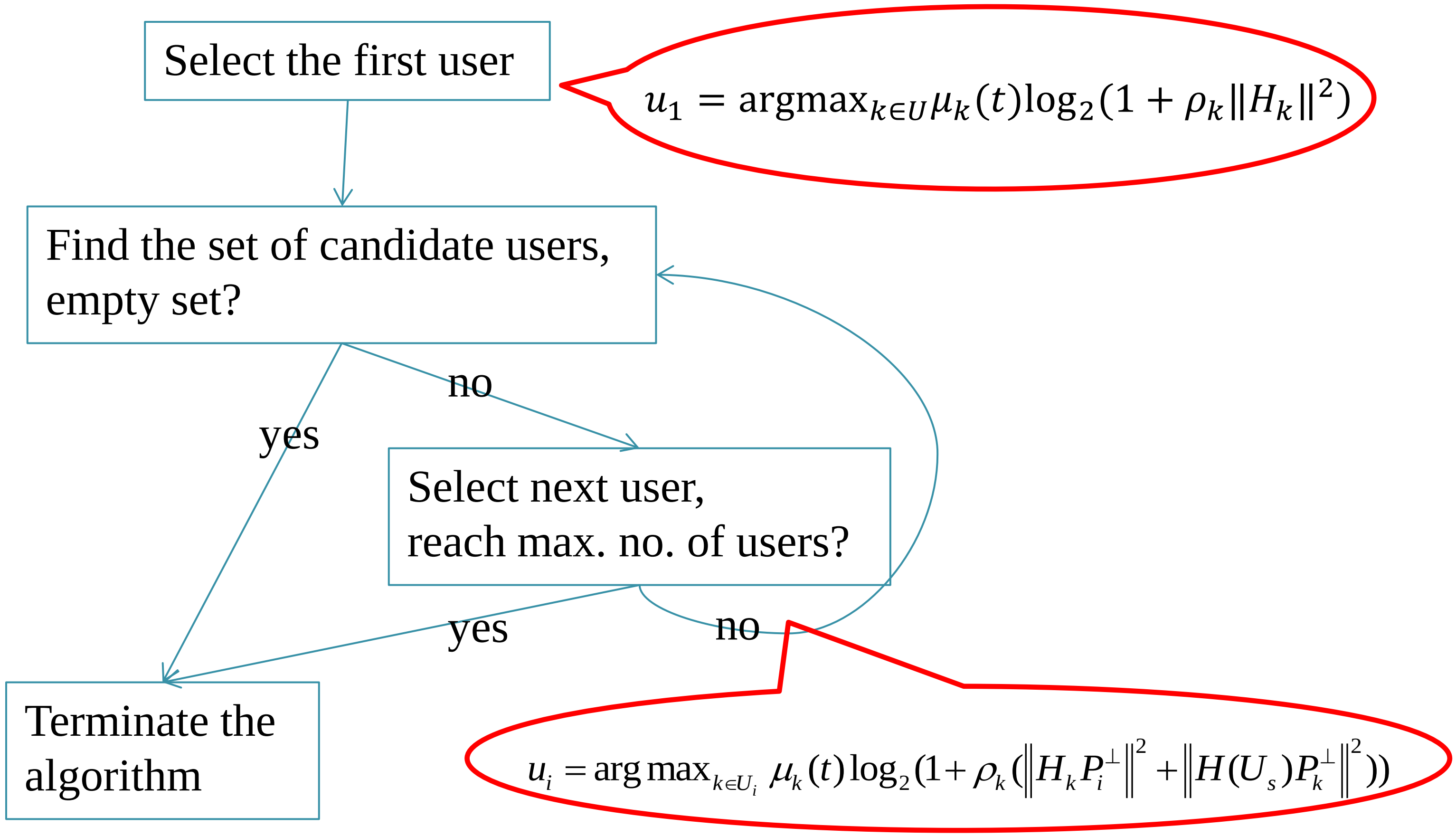
d. Select a user such that $u_i = \arg\max_{k\in\mathcal{U}_i} \mu_k(t) \log_2(1 + \rho_k(\|\mathbf{H}_k \mathcal{P}_i^{\perp}\|^2 + \|\mathbf{H}(U_s)\mathcal{P}_k^{\perp}\|^2))$

e. Set $\mathcal{U}_s = \mathcal{U}_s \cup \{u_1\}$ and $\mathcal{U} = \mathcal{U} \backslash \{u_1\}$

(3) End: The selected user set is $\mathcal{U}_s$

TOKYO TECH Pursuing Excellence

PF User scheduling



PF User scheduling

- The first user is selected such that

$$u_1 = \text{argmax}_{k \in U} \mu_k(t) \log_2(1 + \rho_k \|H_k\|^2)$$

where ρ_k is the SNR and $\mu_k(t)$ is the proportional fairness of the *k*th user

  - Proportional fairness factor

$$\mu_k(t) = \frac{1}{\overline{R_k}(t)} \quad \text{and}$$

$$\overline{R}_k(t+1) = \begin{cases} \delta \overline{R}_k(t) + (1-\delta) R_k(t), & k \in \mathcal{U}_s \\ \delta \overline{R}_k(t), & k \notin \mathcal{U}_s \end{cases}$$

$\delta = \frac{1}{T_c}$ is the forgetting factor which average the rate of users over T_c timeslots.

TOKYO TECH Pursuing Excellence

PF User scheduling

- Candidate users

  Set of candidate user can be found by determining correlation coefficient

$$\frac{\|\mathbf{H}_k \mathbf{V}_{i-1}^H\|}{\|\mathbf{H}_k\| \|\mathbf{V}_{i-1}\|} < \varepsilon$$

where $\mathbf{V}_{i-1}$ is the row basis of $\mathbf{H}(U_s)$ and

$$\mathbf{H}(U_s) = \begin{bmatrix} \mathbf{H}_{u_1}^T & \mathbf{H}_{u_2}^T & \cdots & \mathbf{H}_{u_{i-1}}^T \end{bmatrix}^T$$

(aggregate channel of selected users)

TOKYO TECH Pursuing Excellence

PF User scheduling

- Other users are selected such that

$$u_i = \text{argmax}_{k \in U_i} \mu_k(t) \log_2\left(1 + \rho_k\left(\left\|\mathbf{H}_k \mathcal{P}_i^{\perp}\right\|^2 + \left\|\mathbf{H}(U_s)\mathcal{P}_k^{\perp}\right\|^2\right)\right)$$

Backward Projection

Forward Projection

where projection matrix $\mathcal{P}_i^{\perp} = \mathbf{I}_{N_t} - \mathbf{V}_{i-1}^H \mathbf{V}_{i-1}$

and projection matrix $\mathcal{P}_k^{\perp} = \mathbf{I}_{N_t} - \widehat{\mathbf{V}}_k^H \widehat{\mathbf{V}}_k$,

($\widehat{\mathbf{V}}_k$ is the row basis of $\mathbf{H}_k$)

TOKYO TECH Pursuing Excellence

Power allocation

- An important key to increase the sum capacity of the system
- Firstly, denote the power matrix as

$$\mathbf{\Lambda}_k = \begin{bmatrix} P_1^k & & 0 \\ & \ddots & \\ 0 & & P_{N_\mathrm{r}}^k \end{bmatrix}$$

- The power is allocated within the condition

$$\sum_{k=1}^{\widehat{K}} tr(\mathbf{\Lambda}_k) = P_\mathrm{t}$$

Power allocation

- Equal power allocation (EP)
  - Simple method to allocate power to users
  - The transmission power of each stream of the *k*th user can be determined by

$$P_i^k = \frac{P_{\mathrm{t}}}{\widehat{K} N_{\mathrm{r}}}$$

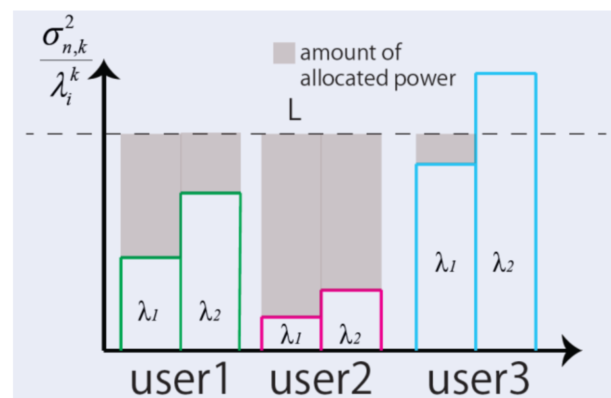
TOKYO TECH Pursuing Excellence

Power allocation

- Water filling (WF)
  - Well-known optimal scheme in many systems
  - Allocate power based on SNR and channel gain of all subchannels of all users.
  - The transmission power of each stream of the kth user can be determined by

$$P_i^k = \begin{cases} L - \frac{\sigma_{n,k}^2}{\lambda_i^k}, & L \geq \frac{\sigma_{n,k}^2}{\lambda_i^k} \\ 0, & L < \frac{\sigma_{n,k}^2}{\lambda_i^k} \end{cases},$$

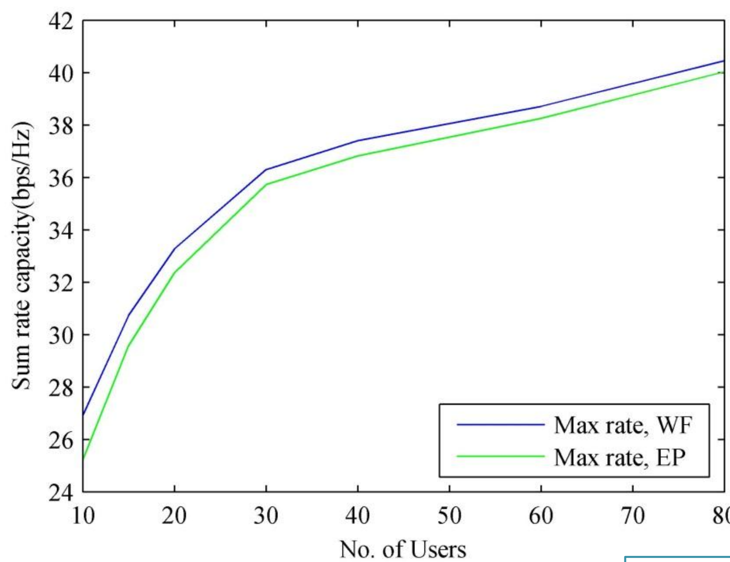
$$\sum_{k=1}^{\hat{K}} \sum_{i=1}^{N_r} P_i^k = P_t$$

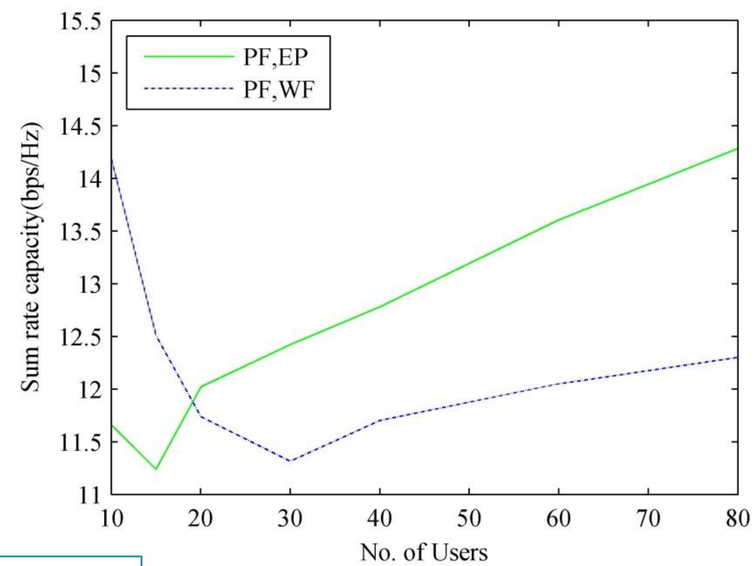


  - where λ_i^k is the ith diagonal element of $\Sigma_{k,e}^2$

Power allocation

- Problem
  - With PF scheduler, some low-channel gain selected users don't have power due to WF strategy
  - This problem also occurs in equal SNR users case but not as large as in unequal SNR users case





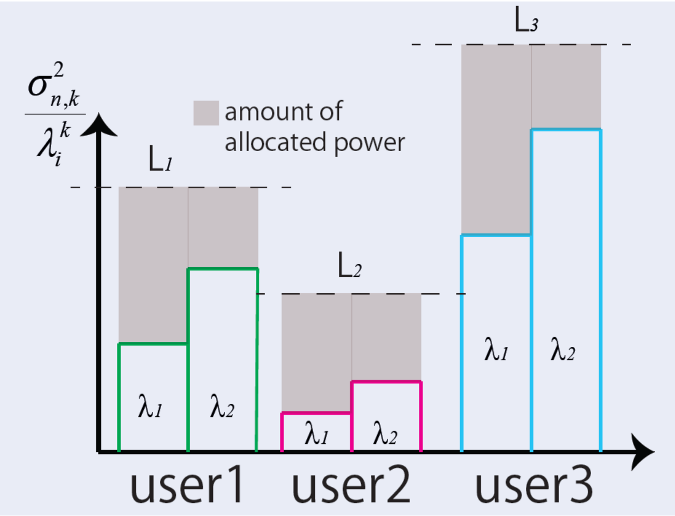
N_t=12, N_r=2

Power allocation

- Proposed power allocation
  - WF has a problem in MU-MIMO system with proportional fairness scheduling
  - Combining concepts of EP and WF, the power is firstly divided equally then WF is applied over subchannels of each user individually.

$$P_i^k = \begin{cases} L_k - \frac{\sigma_{n,k}^2}{\lambda_i^k}, & L_k \geq \frac{\sigma_{n,k}^2}{\lambda_i^k} \\ 0, & L_k < \frac{\sigma_{n,k}^2}{\lambda_i^k} \end{cases},$$

$$\sum_{i=1}^{N_r} P_i^k = \frac{P_t}{K}$$



Simulation results

- Parameters setting

Parameters	Value
SNR	Ranged from 0-20 dB
No. of users	10, 15, 20, 30, 40, 60, 80
No. of transmit antenna, N_t	12
No. of receive antenna, N_r	2
Channel model	i.i.d Rayleigh channel
Total power transmission	1
Width of sliding window, T_c	100 time slots
Correlation coefficient threshold, ε	1

Simulation results

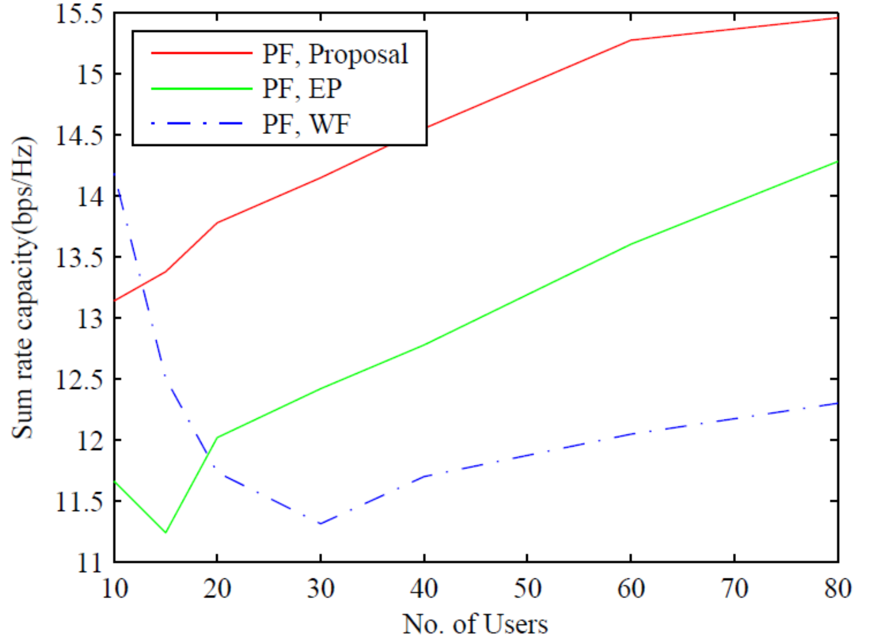


Fig. 2 Average sum rate capacities of PF scheduling systems.

- As the number of users increases, the combining power allocation gives the best sum rate capacity.
- The equal power allocation performs even better than WF.

Simulation results

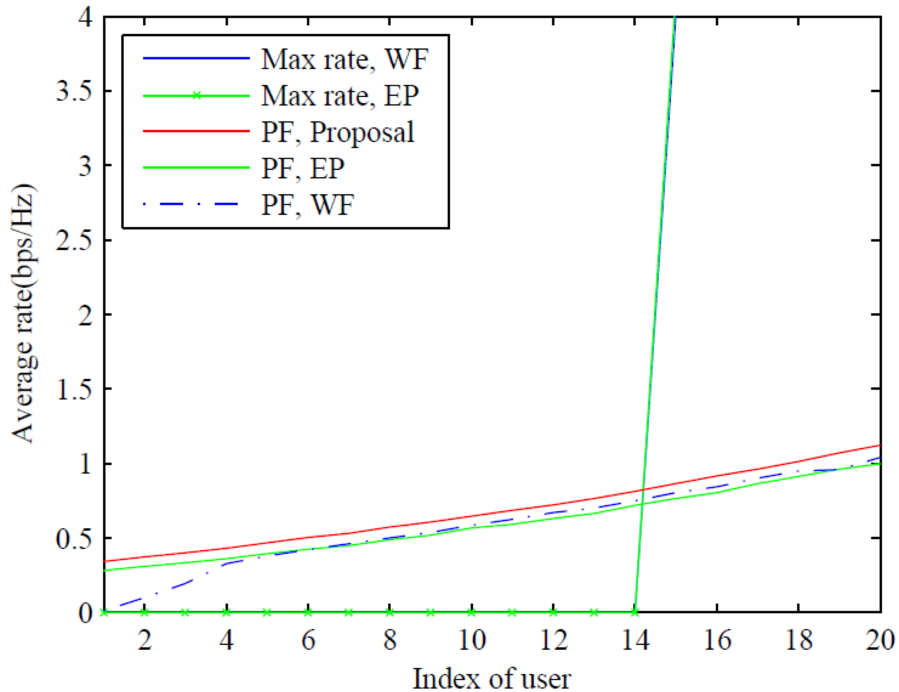


Fig. 3 Average rate of each user.

- For the system with fixed 20 users, the first 14 users are discarded with Max rate scheduler.
- PF scheduler provide fairness among users, the combining power allocation method gives the best result.

Simulation results

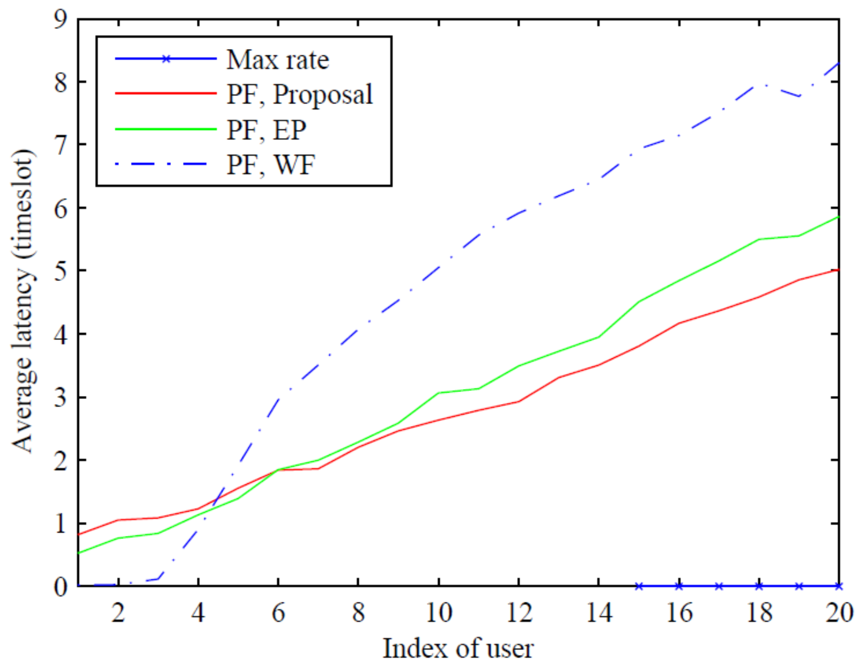


Fig. 4 Average latency time of each user.

- WF scheduler selects the low SNR users 'too' frequent
- Combining method provides better latency time and fairness among users

Conclusion and Future work

- Conclusion
  - Although the Water filling is effective in MU-MIMO system without scheduler or in the equal SNR users system. Water filling power allocation does not work well in the PF scheduling for MU-MIMO system. The proposed scheme combines the method of EQ and WF, thus increase the sum capacity of the system and average rate of each user.
- Future work
  - Consider the system with unequal number of receive antennas of users
  - Consider the system with Quality of Service (QoS)
  - Study the feedback scheme for CSI

TOKYO TECH Pursuing Excellence

…Thank you for your attention…