

# Optimization of Base Station Antenna Directivity for Base Station Cooperation Cellular Systems

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# Background

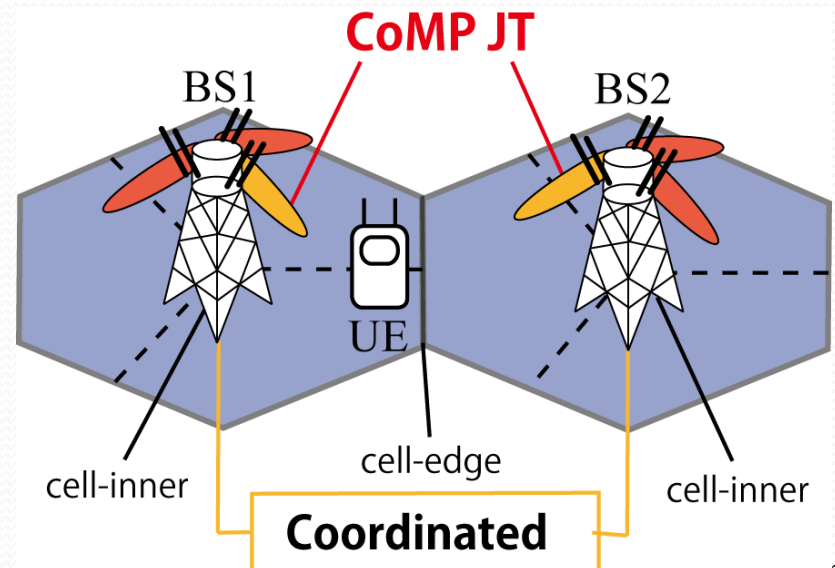
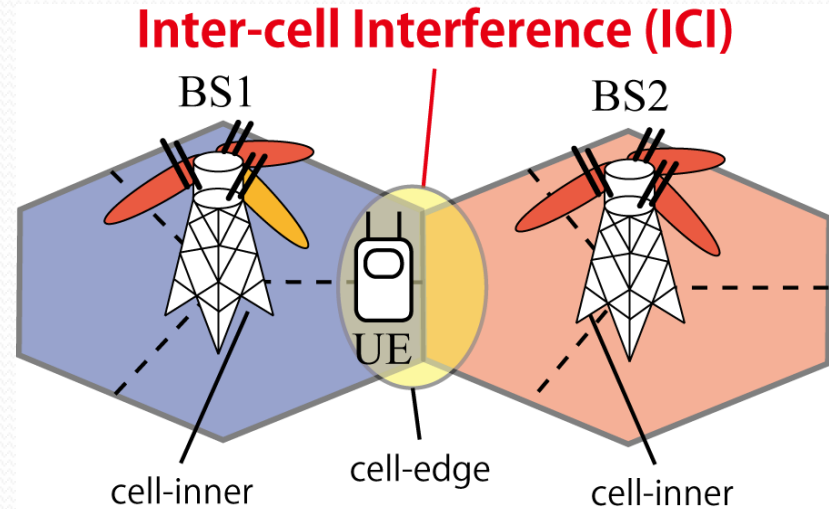
## ◆ Single transmission

- ✓ Capacity at the cell-edge is degraded
  - pathloss
  - **Inter-cell interference (ICI)**
  - antenna correlation



## ◆ Base station cooperation (BSC)

- ✓ Capacity at the cell-edge is improved
  - **CoMP JT**
  - transmit signal to control **ICI**



# Background

## ◆ Base station antenna directivity

- Single transmission

**Maximize the received power of own cell**

**To reduce the ICI, minimize the power to adjacent cell**

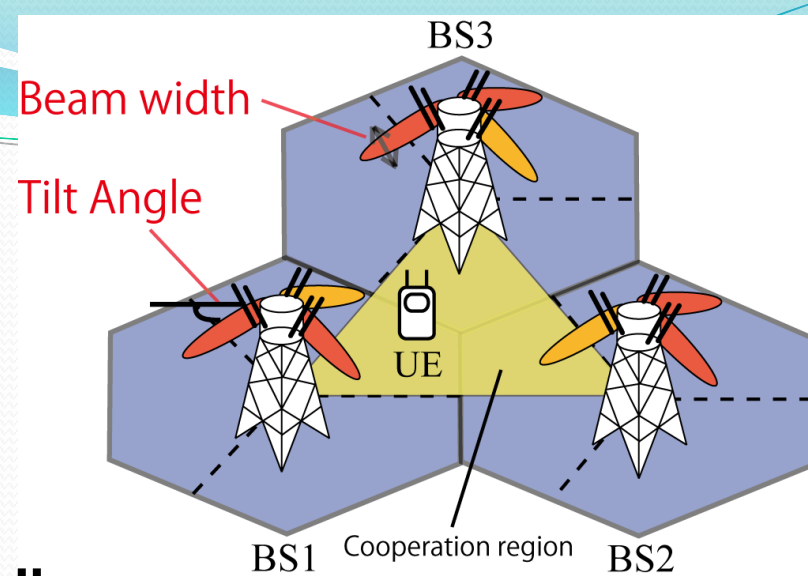
- Base station cooperation

**Maximize the received of cooperative cell without the problem of ICI**

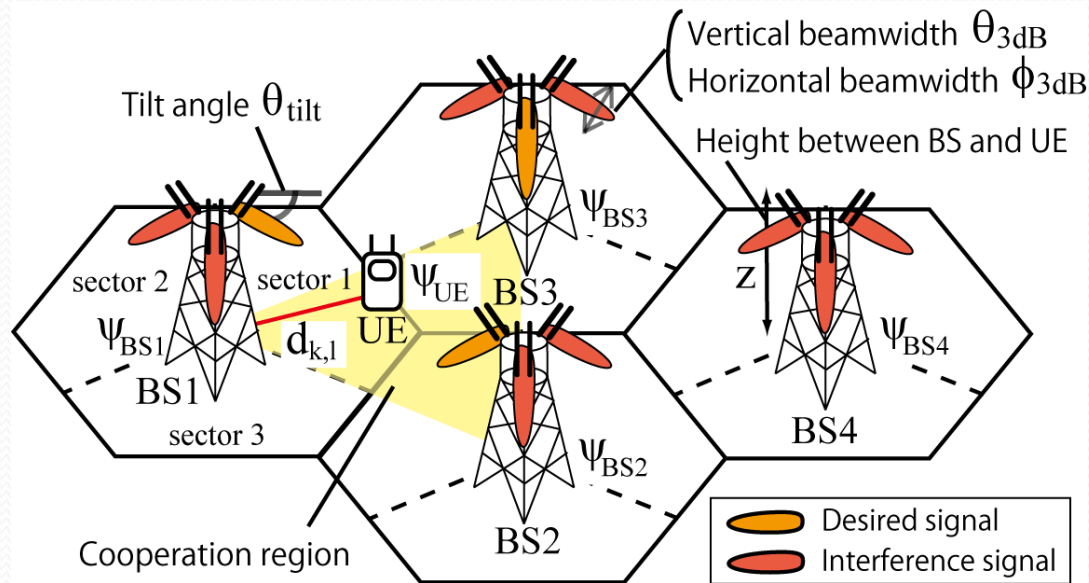


**Base station antenna directivity is required to be designed differently from conventional way**

**Optimize base station antenna directivity**



# Base station cooperation system



| System model                           |   |
|--|---|
| The number of cooperative BSs $K_{BS}$ | 3 |
| The number of BS antennas $N_{BS}$     | 2 |
| The number of UE per sector $K_{UE}$   | 1 |
| The number of UE antennas $N_{UE}$     | 2 |

## • Received signal

$$\begin{aligned}
 \mathbf{y} &= \mathbf{H}_{1,1} \tilde{\mathbf{s}}_{1,1} + \mathbf{H}_{2,2} \tilde{\mathbf{s}}_{2,2} + \mathbf{H}_{3,3} \tilde{\mathbf{s}}_{3,3} + \sum_{\substack{\text{non-cooperative} \\ (k,l) \neq (1,1), \\ (2,2), (3,3)}} \mathbf{H}_{k,l} \tilde{\mathbf{s}}_{k,l} + \mathbf{n} \\
 &= \begin{bmatrix} \mathbf{H}_{1,1} & \mathbf{H}_{2,2} & \mathbf{H}_{3,3} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{s}}_{1,1} \\ \tilde{\mathbf{s}}_{2,2} \\ \tilde{\mathbf{s}}_{3,3} \end{bmatrix} + \sum_{\text{non-cooperative}} \mathbf{H}_{k,l} \tilde{\mathbf{s}}_{k,l} + \mathbf{n} \\
 &= \mathbf{H} \tilde{\mathbf{s}} + \sum_{\text{non-cooperative}} \mathbf{H}_{k,l} \tilde{\mathbf{s}}_{k,l} + \mathbf{n}
 \end{aligned}$$

Desired signal    
 Interference    
 Noise

## • Channel Matrix

$$\mathbf{H}_{k,l} = \sqrt{\beta_{k,l}} \mathbf{H}_{k,l}^{iid} A_{k,l}(\theta, \phi)$$

Evaluation angle    Azimuth angle  
 ↙                      ↘

pathloss  $\beta_{k,l} = \alpha \cdot \sqrt{d_{k,l}^2 + z^2}^{-m}$

Channel matrix with independent and identically distributed elements  $\mathbf{H}_{k,l}^{iid}$

Base station antenna directivity  $A_{k,l}(\theta, \phi)$

Distance between BS and UE  $d_{k,l} = |\psi_{UE} - \psi_{BSk}|$

# Instantaneous capacity

## • Singular Value Decomposition

$$\mathbf{y} = \mathbf{H}\tilde{\mathbf{s}} + \sum_{\text{non-cooperative}} \mathbf{H}_{k,l} \tilde{\mathbf{s}}_{k,l} + \mathbf{n} = (\mathbf{U}\mathbf{\Lambda}\mathbf{V}^H)\tilde{\mathbf{s}} + \sum_{\text{non-cooperative}} \mathbf{H}_{k,l} \tilde{\mathbf{s}}_{k,l} + \mathbf{n}$$

$$\begin{aligned} \tilde{\mathbf{y}} &= \mathbf{U}^H \mathbf{y} \\ &= \mathbf{U}^H (\mathbf{U}\mathbf{\Lambda}\mathbf{V}^H) (\mathbf{V}\mathbf{s}) + \sum_{\text{non-cooperative}} \mathbf{U}^H \mathbf{H}_{k,l} \tilde{\mathbf{s}}_{k,l} + \mathbf{U}^H \mathbf{n} \\ &= \mathbf{\Lambda}\mathbf{s} + \sum_{\text{non-cooperative}} \mathbf{U}^H \mathbf{H}_{k,l} \tilde{\mathbf{s}}_{k,l} + \tilde{\mathbf{n}} \end{aligned}$$

Precoding  $\tilde{\mathbf{s}} = \mathbf{V}\mathbf{s}$   
 Postcoding  $\tilde{\mathbf{y}} = \mathbf{U}^H \mathbf{y}$   
 Eigen matrix  $\mathbf{\Lambda}$

## • Received signal (*i*-th eigenmode)

$$\tilde{y}_i = \sqrt{\lambda_i} s_i + \sum_{\text{non-cooperative}} \mathbf{U}_i^H \mathbf{H}_{k,l} \tilde{\mathbf{s}}_{k,l} + \tilde{n}_i$$

## • Received SINR (*i*-th eigenmode)

$$\gamma_i = \lambda_i \frac{P_s}{\sum_{\text{non-cooperative}} \|\mathbf{U}_i^H \mathbf{H}_{k,l}\|^2 P_{\text{BS}} + P_n}$$

Transmit power per BS  $P_{\text{BS}} = E[\tilde{\mathbf{s}}_{k,l}^H \tilde{\mathbf{s}}_{k,l}]$   
 Total transmit power  $P = K_{\text{BS}} P_{\text{BS}}$   
 Transmit power per stream  $P_s = P / q$   
 Noise power  $P_n$   
 The number of streams  $q = \min(K_{\text{BS}} N_{\text{BS}}, K_{\text{UE}} N_{\text{UE}})$

**Instantaneous capacity**  
 $C_i = \log_2(1 + \gamma_i)$  (*i*-th eigenmode)

# Average channel capacity

Instantaneous channel capacity

$$C_i = \log_2(1 + \gamma_i) \quad (i\text{-th eigenmode})$$

PDF of each eigenvalue:  $f_i(\lambda_i)$

• **Average channel capacity**

$$\bar{C}(\psi_{\text{UE}}, A(\theta, \phi)) = \sum_{i=1}^q \int_0^{\infty} C_i \cdot f_i(\lambda_i) d\lambda_i$$

## • Eigenvalue Theory of Wishart Matrix

◆ BSC-MIMO  $(3,2) \times (1,2)$  system  $\leftarrow$   $(K_{\text{BS}}, N_{\text{BS}}) \times (K_{\text{UE}}, N_{\text{UE}})$

The number of BSs  $\nearrow$   
The number of BS antennas  $\nearrow$

$\nwarrow$  The number of UEs  
 $\nwarrow$  The number of UE antennas

• PDF of 1<sup>st</sup> eigen mode

$$f_1(\lambda_1) = \sum_{\substack{\{k,l,m\} \\ (1,2,3), (2,3,1), (3,1,2)}} \frac{\sigma_k e^{-\frac{\lambda_1}{\sigma_k}}}{(\sigma_k - \sigma_l)^3 (\sigma_k - \sigma_m)^3} [(\sigma_k - \sigma_l)(\sigma_k - \sigma_m) \lambda_1^2 - 2\{\sigma_k^3 + (\sigma_l + \sigma_m)\sigma_k^2 - (\sigma_l^2 + 3\sigma_l\sigma_m + \sigma_m^2)\sigma_k + \sigma_l\sigma_m(\sigma_l + \sigma_m)\} \lambda_1 + 2\sigma_k \{\sigma_k^3 + (\sigma_l + \sigma_m)\sigma_k^2 + (\sigma_l^2 + \sigma_l\sigma_m + \sigma_m^2)\sigma_k - 3\sigma_l\sigma_m(\sigma_l + \sigma_m)\}] - f_2(\lambda_1)$$

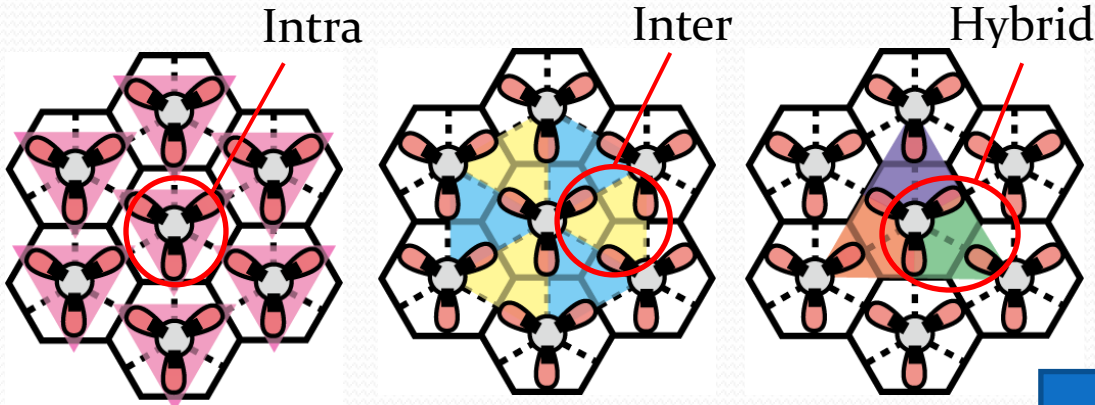
• PDF of 2<sup>nd</sup> eigen mode

$$f_2(\lambda_2) = \frac{1}{(\sigma_1 - \sigma_2)^4 (\sigma_2 - \sigma_3)^4 (\sigma_3 - \sigma_1)^4} \cdot \left[ \sum_{\substack{\{k,l,m\} \\ (1,2,3), (2,3,1), (3,1,2)}} 2\sigma_k^7 (\sigma_l - \sigma_m)^4 \cdot e^{-\frac{2\lambda_2}{\sigma_k}} - \sigma_k \sigma_l (\sigma_k - \sigma_m) (\sigma_l - \sigma_m) \cdot e^{-\left(\frac{1}{\sigma_k} + \frac{1}{\sigma_l}\right)\lambda_2} [(\sigma_k - \sigma_l)^2 (\sigma_k + \sigma_l) (\sigma_k - \sigma_m) (\sigma_l - \sigma_m) \lambda_2^2 + 2(\sigma_k - \sigma_l)^2 \{(2\sigma_k^2 + 3\sigma_k\sigma_l + 2\sigma_l^2)\sigma_m^2 - (\sigma_k + \sigma_l)(\sigma_k^2 + 3\sigma_k\sigma_l + \sigma_l^2)\sigma_m + \sigma_k\sigma_l(\sigma_k^2 + \sigma_k\sigma_l + \sigma_l^2)\} \lambda_2 + 2\sigma_k\sigma_l \{(\sigma_k + \sigma_l)(6\sigma_k^2 - 11\sigma_k\sigma_l + 6\sigma_l^2)\sigma_m^2 - (3\sigma_k^4 - \sigma_k^3\sigma_l - \sigma_k\sigma_l^3 + 3\sigma_l^4)\sigma_m + \sigma_k\sigma_l(\sigma_k^3 + \sigma_l^3)\}] \right]$$

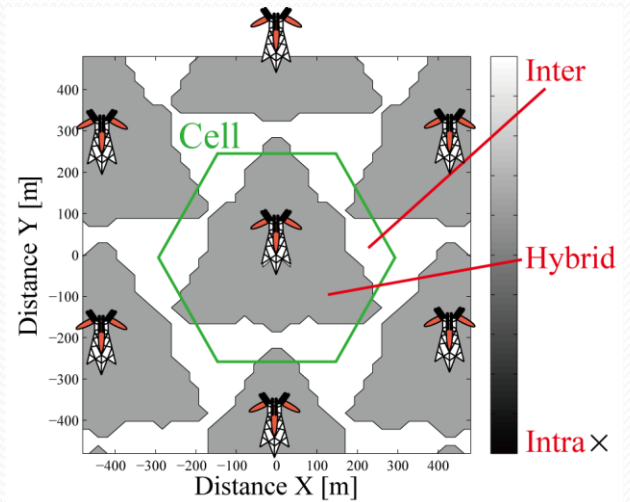
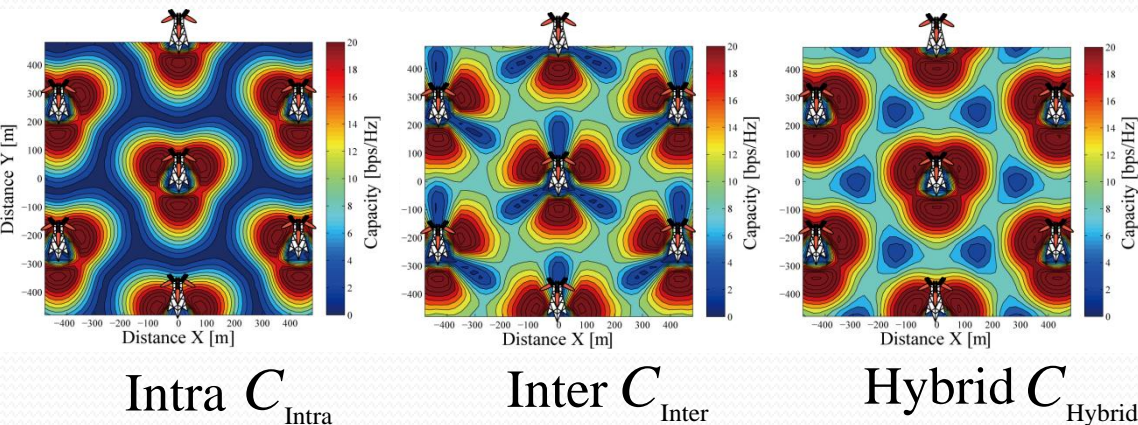
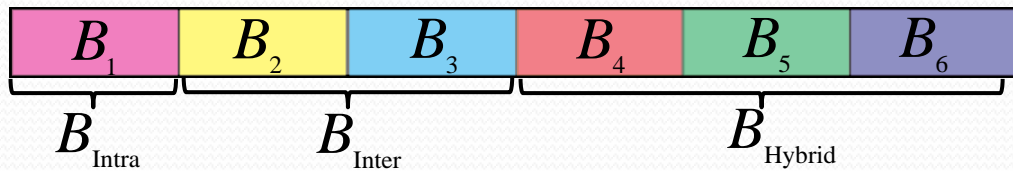
$\sigma_i = \left(\sqrt{\beta_i} A_i(\theta, \phi)\right)^2$  : eigenvalue of correlation matrix  $\mathbf{H}^H \mathbf{H}$

# Objective function

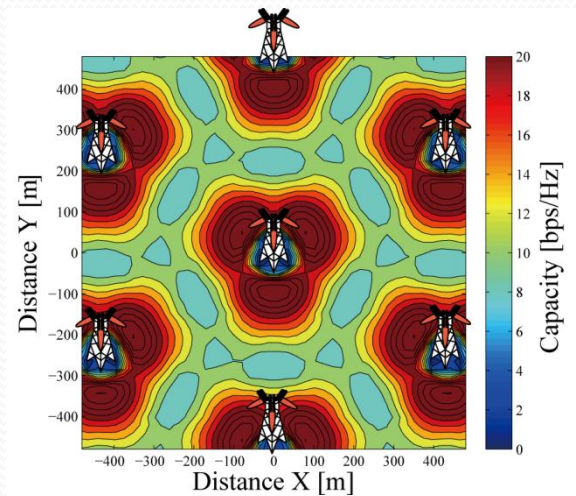
## Cooperative pattern (3GPP)



Resource



## Cooperation region



**BSC**

$$\max(C_{\text{Intra}}, C_{\text{Inter}}, C_{\text{Hybrid}})$$



# Objective function

- Resource ( $i$ -th cooperation)

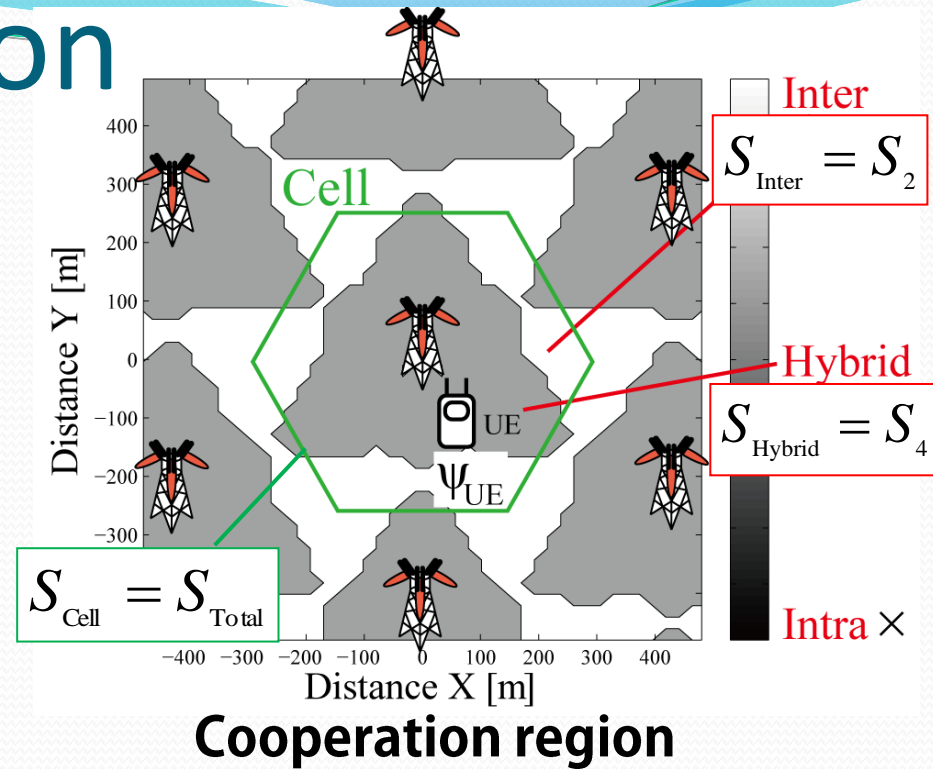
$$B_i = (S_i / S_{\text{Total}}) B_{\text{Total}}$$

- Average cell capacity ( $i$ -th cooperation)

$$C_i^\diamond = \int_{S_i} \bar{C}(\psi_{\text{UE}}, A(\theta, \phi)) dS / S_i$$

## Average Cell Capacity (BSC)

$$C_{\text{BSC}}^\diamond = \sum_i \frac{B_i}{B_{\text{Total}}} C_i^\diamond$$



- **optimal antenna directivity**

$$A^*(\theta, \phi) = \arg \max_{A(\theta, \phi)} C^\diamond$$

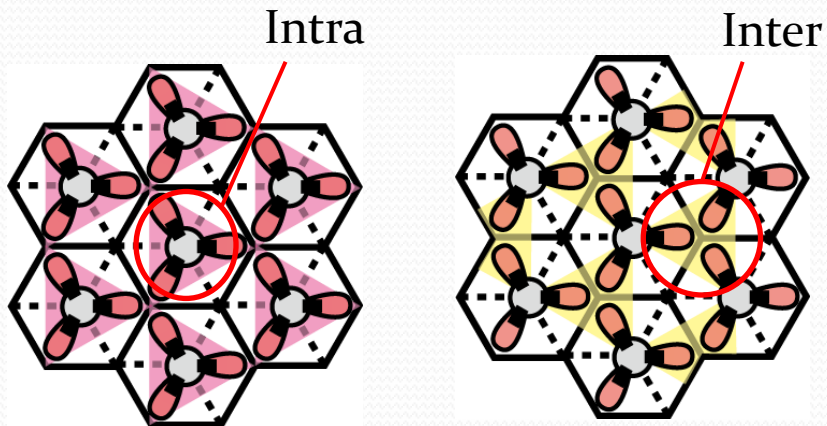
Assume that BS antenna directivity  $A(\theta, \phi)$  as defined in 3GPP



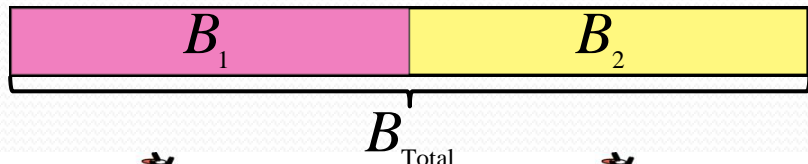
Maximize average cell capacity by varying the parameters

# Objective function

Cooperative pattern (face to face)



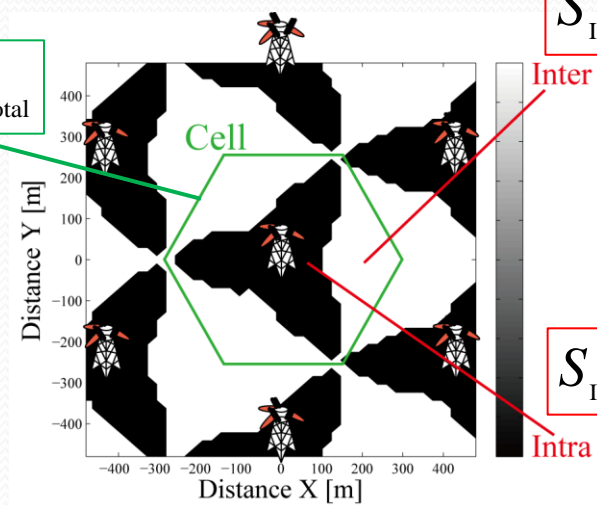
resource



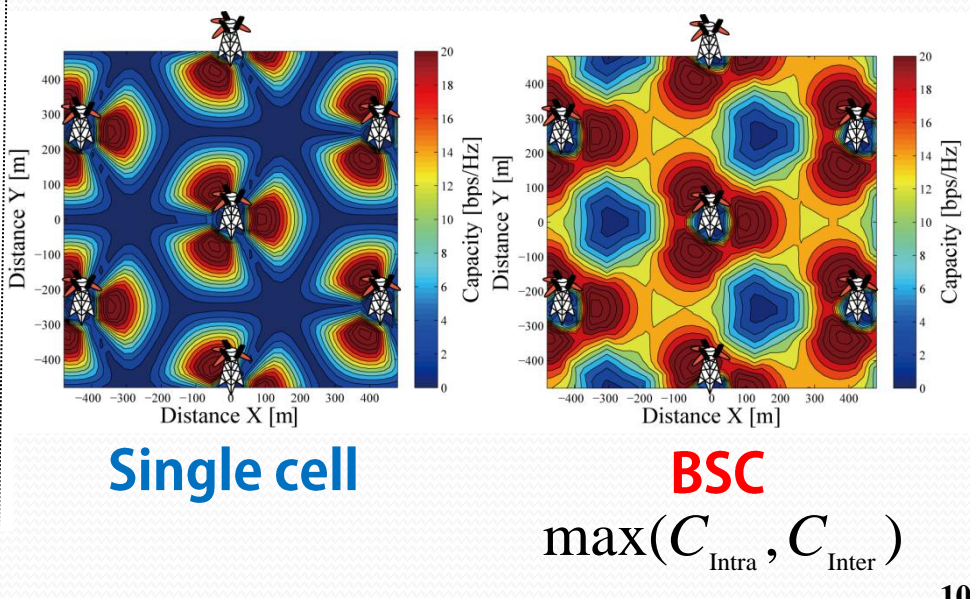
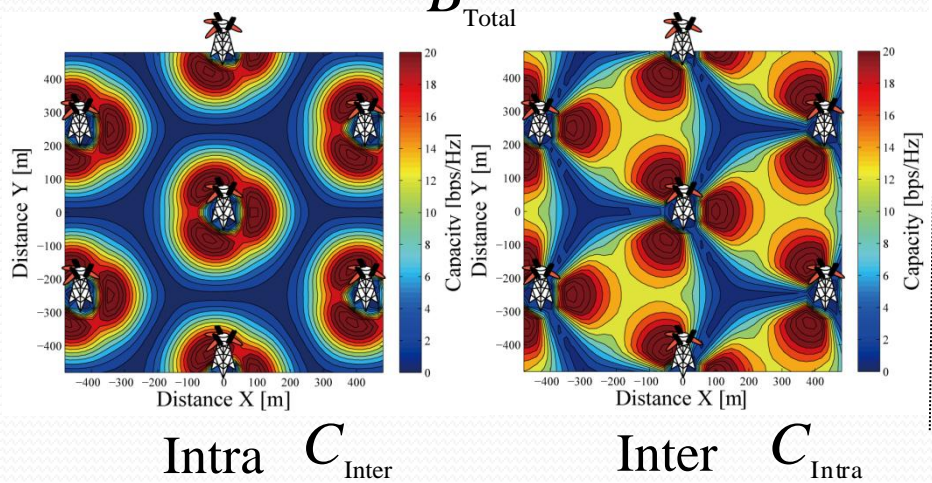
$$S_{\text{Cell}} = S_{\text{Total}}$$

$$S_{\text{Inter}} = S_1$$

$$S_{\text{Intra}} = S_2$$



Cooperation region



# BS Antenna Directivity

- **3GPP Directivity**

$$A(\theta, \phi) = K_{\text{omni}} \cdot -\min[-[A_H(\phi) + A_V(\theta)], A_m]$$

|                        |   |                       |
|------------------------|---|-----------------------|
| Horizontal Directivity | $A_H(\phi) = -\min\left[12\left(\frac{\phi}{\phi_{3\text{dB}}}\right)^2, A_m\right]$                                | $A_m = 25\text{dB}$   |
| Vertical Directivity   | $A_V(\theta) = -\min\left[12\left(\frac{\theta - \theta_{\text{tilt}}}{\theta_{3\text{dB}}}\right)^2, SLA_V\right]$ | $SLA_V = 20\text{dB}$ |

- **parameter**

$\theta_{\text{tilt}}$  Vertical Tilt angle

$\theta_{3\text{dB}}$  Vertical Tilt angle

$\phi_{3\text{dB}}$  Horizontal beamwidth



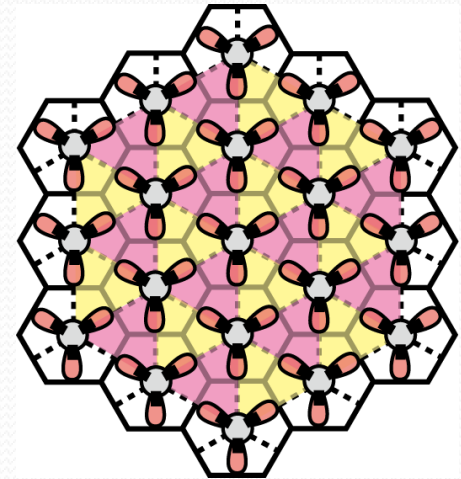
Derive parameters maximize average cell capacity

# Numerical Analysis

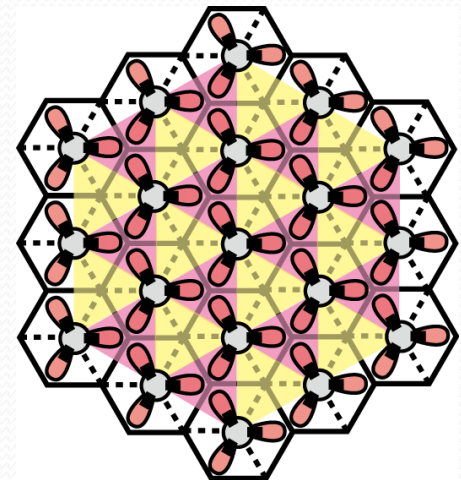
| Numerical Condition                     |              |
|---|--------------|
| The number of Cooperative BSs $K_{BS}$  | 3            |
| The number of BS antennas $N_{BS}$      | 2            |
| The number of UE per sector $K_{UE}$    | 1            |
| The number of UE antennas $N_{UE}$      | 2            |
| BS transmission power $P_s$             | 46dBm        |
| Noise power $P_n$<br>(10MHz Bandwidths) | -99dBm       |
| Inter BS distance $D$                   | 500m         |
| Pathloss coefficient $\alpha$           | $10^{-3.45}$ |
| Pathloss decay $m$                      | 3.5          |
| The height of antenna $z$               | 25m          |

## • 19 cell model

3GPP  
model



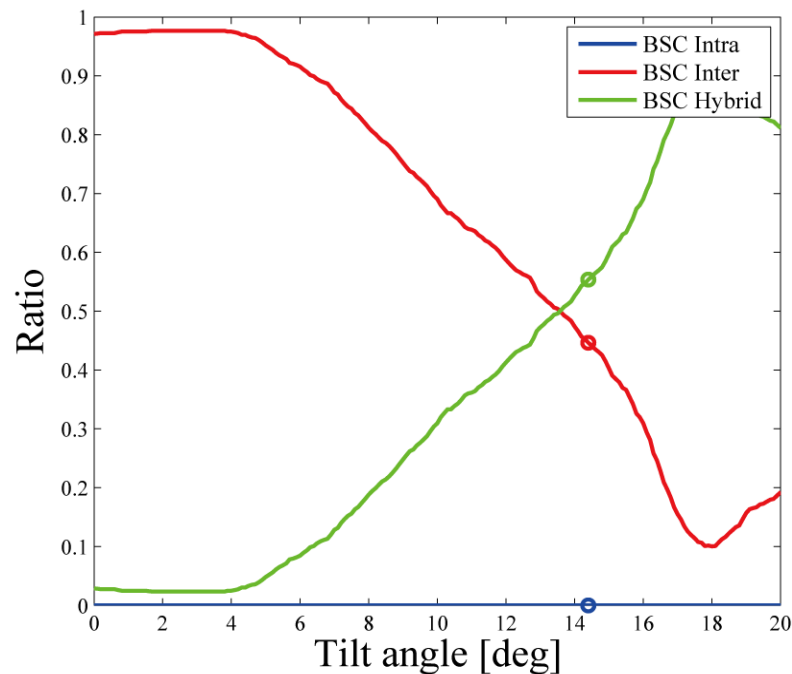
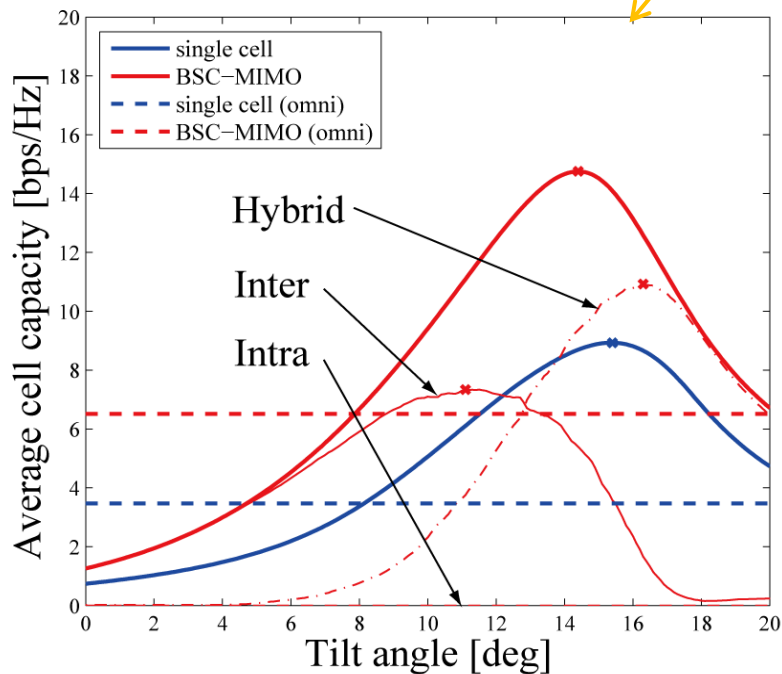
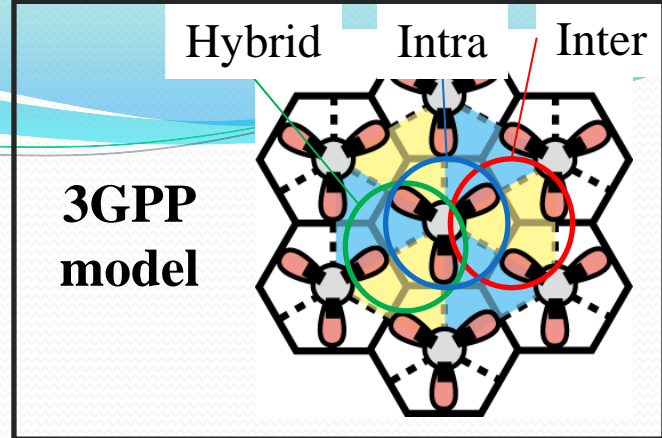
face to face  
model



# Optimal tilt angle $\theta_{\text{tilt}}$

## • Average cell capacity

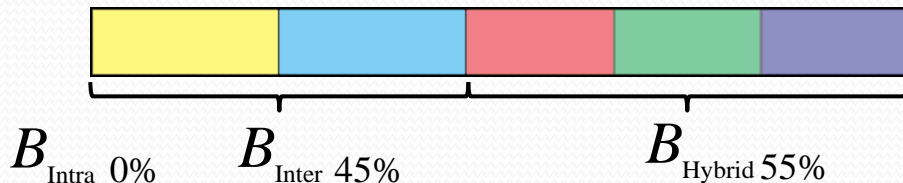
$$\theta_{\text{tilt}}^{\star} = \arg \max_{\theta_{\text{tilt}}} C^{\diamond}(\theta_{3\text{dB}} = 10\text{deg}, \phi_{3\text{dB}} = 70\text{deg})$$



## Optimal tilt angle

|                        | Single cell | BSC-MIMO |
|------------------------|-------------|----------|
| $\theta_{\text{tilt}}$ | 15.4deg     | 14.4deg  |

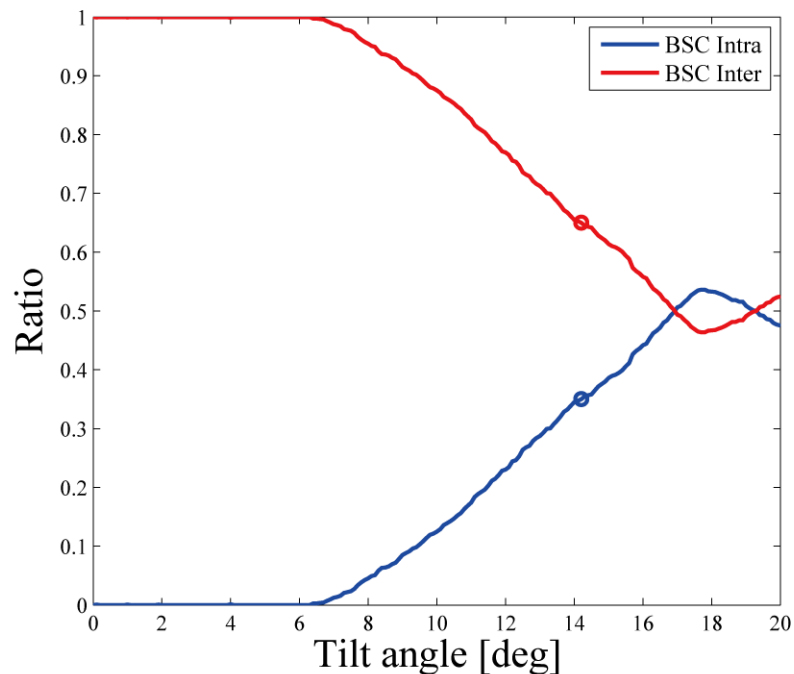
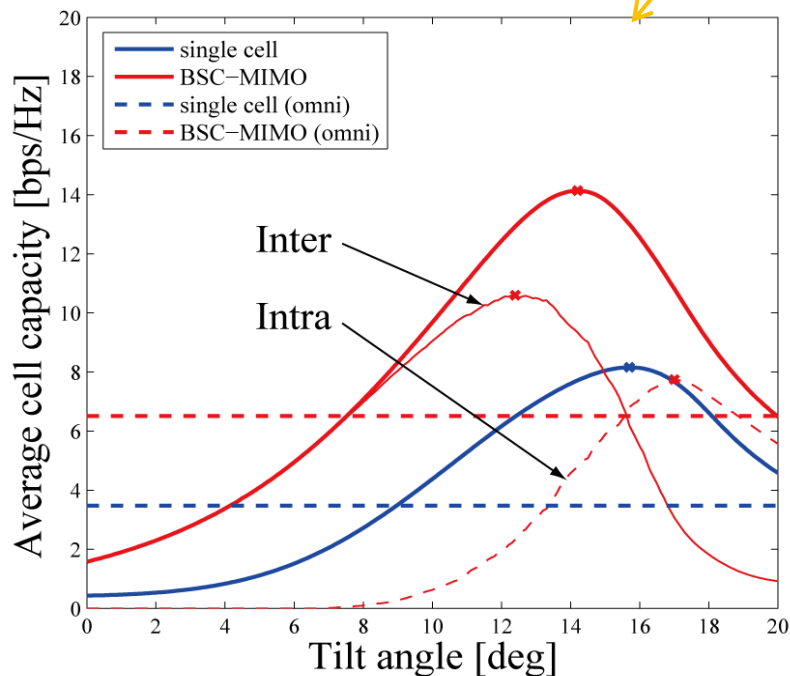
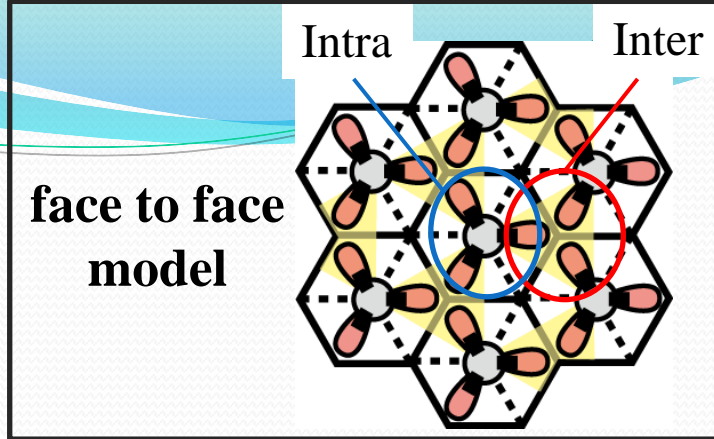
## Resource



# Optimal tilt angle $\theta_{\text{tilt}}$

- Average cell capacity**

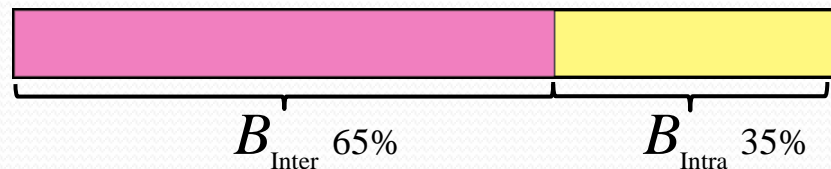
$$\theta_{\text{tilt}}^{\star} = \arg \max_{\theta_{\text{tilt}}} C^{\diamond}(\theta_{3\text{dB}} = 10\text{deg}, \phi_{3\text{dB}} = 70\text{deg})$$



## Optimal tilt angle

|                        | Single cell | BSC-MIMO |
|------------------------|-------------|----------|
| $\theta_{\text{tilt}}$ | 15.7deg     | 14.2deg  |

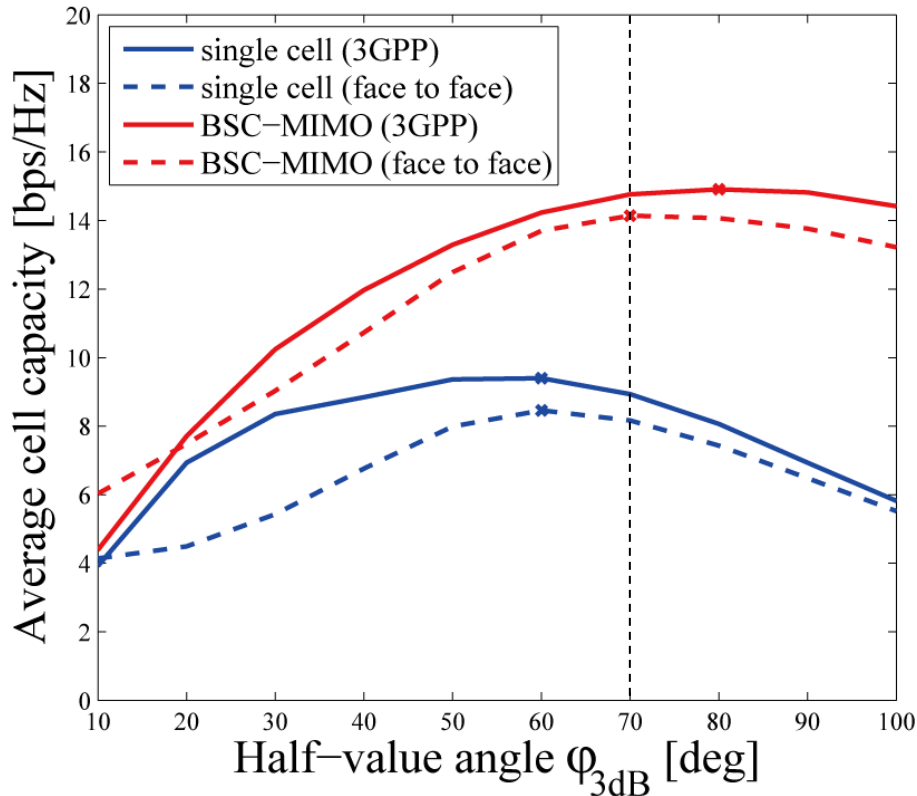
## Resource



# Horizontal beam width $\phi_{3dB}$

- Average cell capacity

$$\phi_{3dB}^* = \arg \max_{\phi_{3dB}} C^\diamond(\theta_{\text{tilt}}^*, \theta_{3dB} = 10 \text{ deg})$$



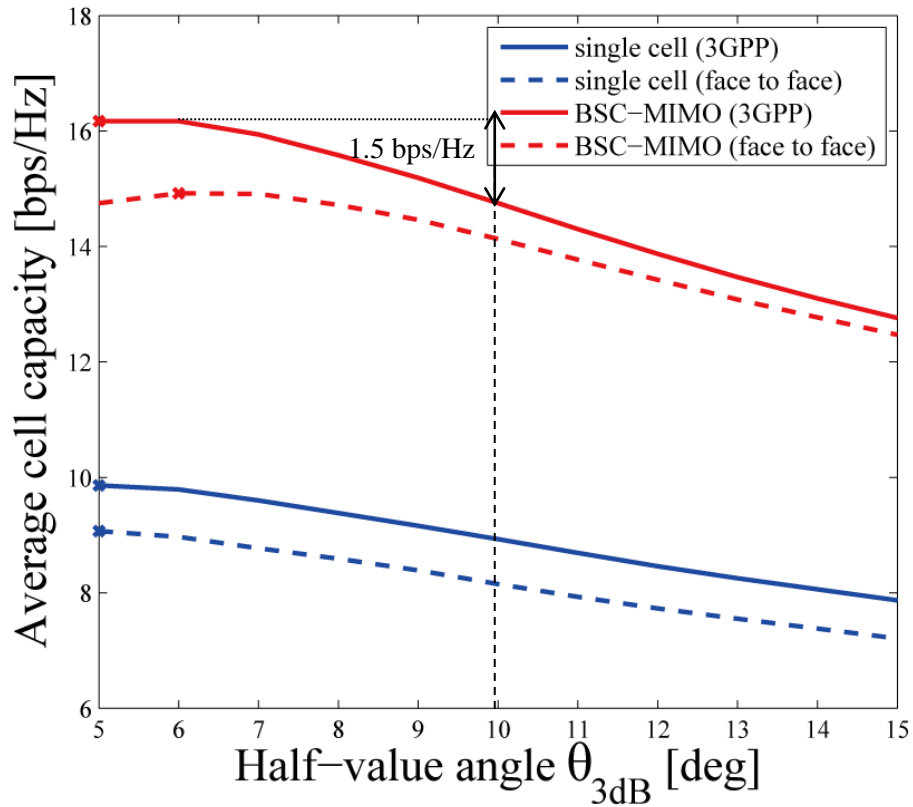
## Optimal Horizontal Beamwidth

|              | Single cell           | 3GPP                          |
|--------------|-----------------------|-------------------------------|
| $\phi_{3dB}$ | 60deg                 | 70deg                         |
|              | <b>BSC<br/>(3GPP)</b> | <b>BSC<br/>(face to face)</b> |
| $\phi_{3dB}$ | 80deg                 | 70deg                         |

# Vertical beam width $\theta_{3dB}$

- Average cell capacity**

$$\theta_{3dB}^* = \arg \max_{\theta_{3dB}} C^\diamond(\theta_{\text{tilt}}^*, \phi_{3dB} = 70 \text{ deg},)$$

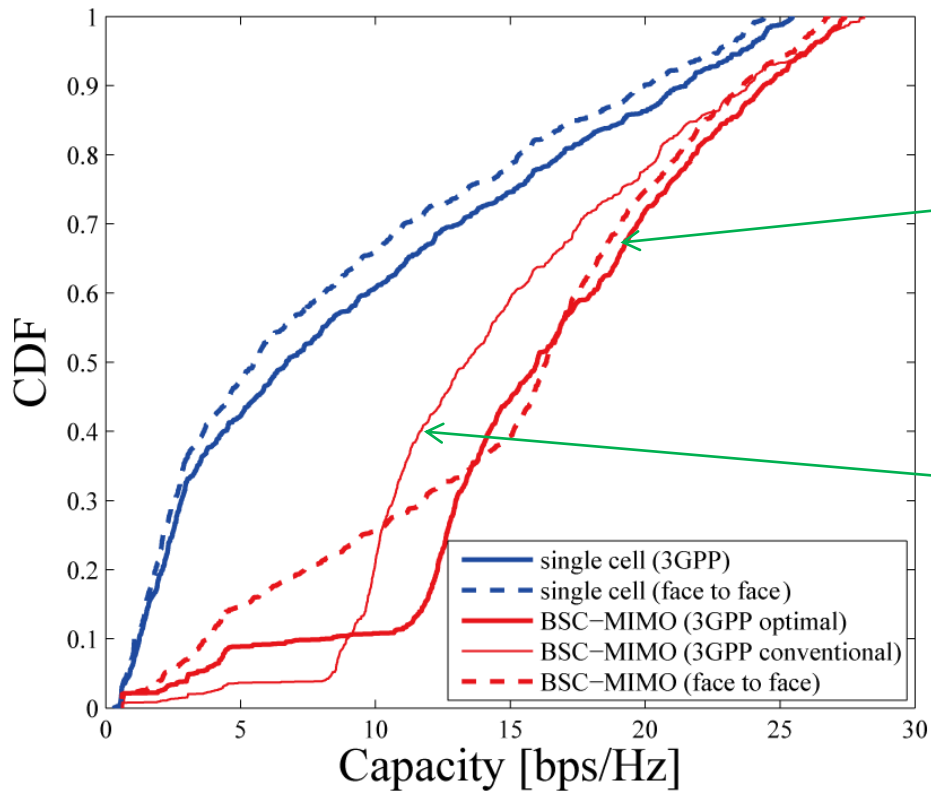


## Optimal Vertical Beamwidth

|                | Single cell           | 3GPP                          |
|----------------|-----------------------|-------------------------------|
| $\theta_{3dB}$ | 5deg                  | 10deg                         |
|                | <b>BSC<br/>(3GPP)</b> | <b>BSC<br/>(face to face)</b> |
| $\theta_{3dB}$ | 5deg                  | 6deg                          |

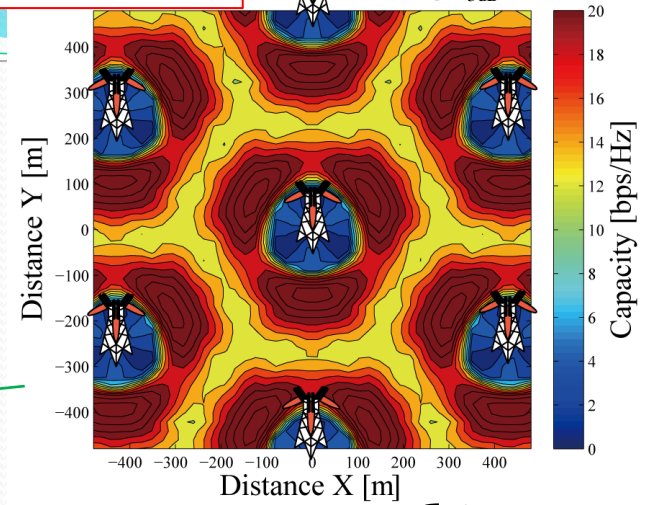


# CDF of capacity



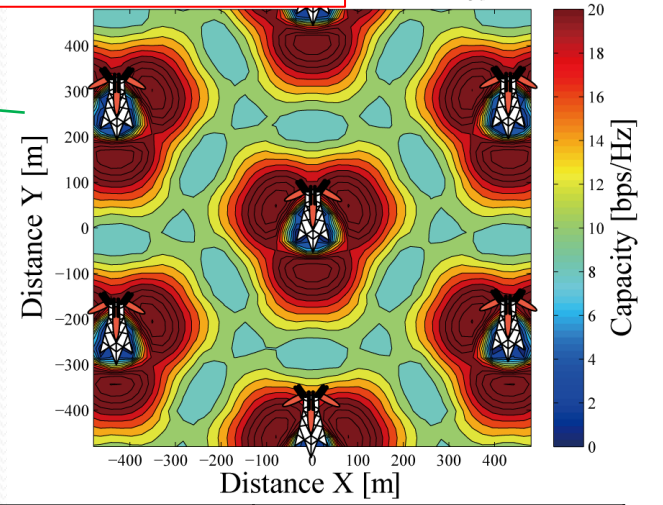
3GPP optimal

$$\begin{cases} \theta_{3dB} = 5 \text{ deg} \\ \phi_{3dB} = 70 \text{ deg} \end{cases}$$



3GPP conventional

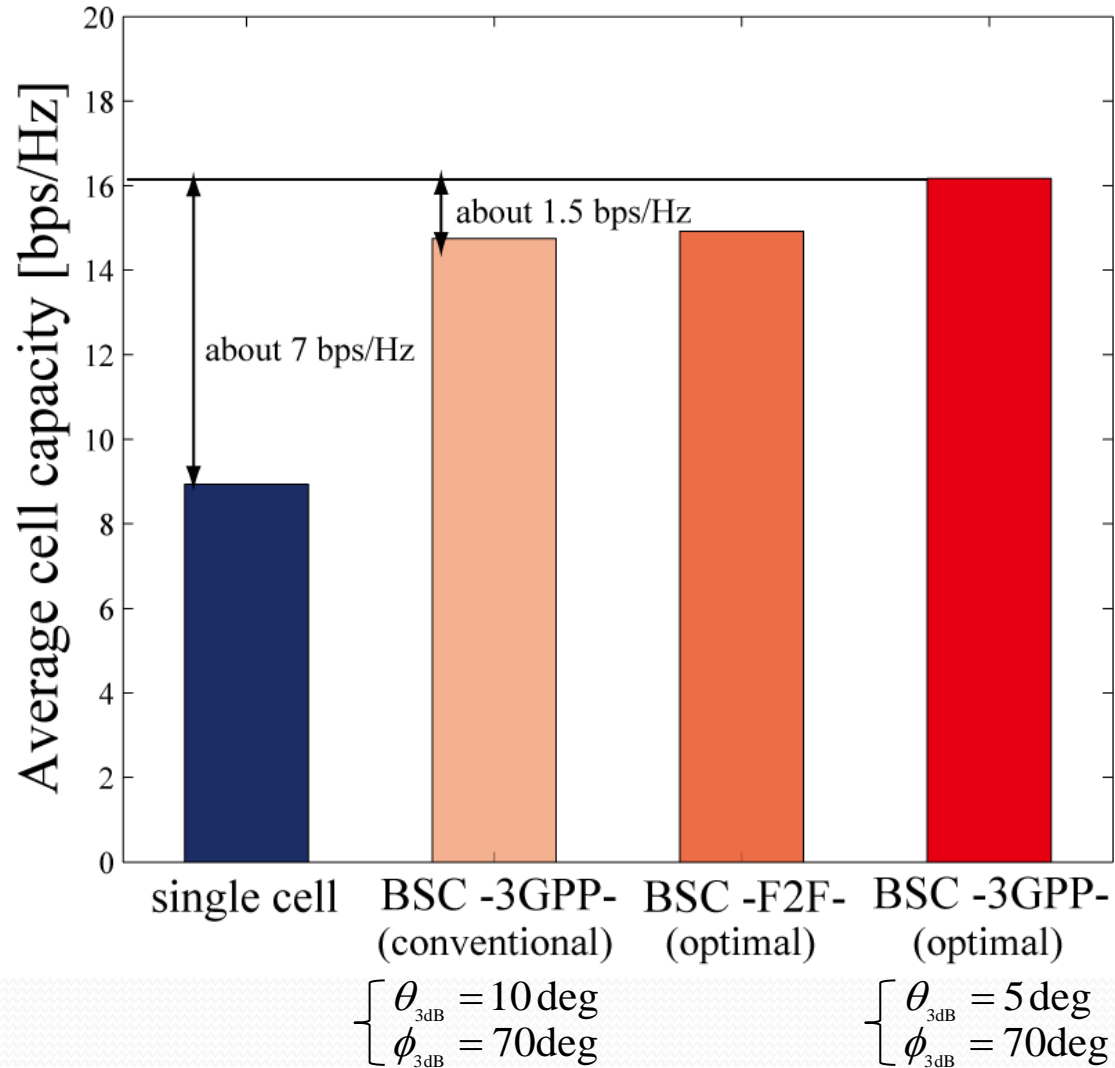
$$\begin{cases} \theta_{3dB} = 10 \text{ deg} \\ \phi_{3dB} = 70 \text{ deg} \end{cases}$$



## 5% outage capacity

|                   | Single cell<br>(3GPP) | BSC-MIMO<br>(face to face) | BSC-MIMO<br>(3GPP optimal) | BSC-MIMO<br>(3GPP conventional) |
|-------------------|-----------------------|----------------------------|----------------------------|---------------------------------|
| Capacity [bps/Hz] | 0.895                 | 2.196                      | 3.291                      | 8.398                           |

# Average cell capacity



# Conclusion

- The optimal tilt angle of BSC-MIMO is smaller than that of single cell transmission
- Average cell capacity of BSC is increased about 1.5 bps/Hz by optimizing base station antenna directivity
- Average cell capacity of BSC system is increased about 7 bps/Hz than that of single cell