

# MIMO-OFDM Precoder for Minimizing BER Upper Bound of MLD under Imperfect CSI

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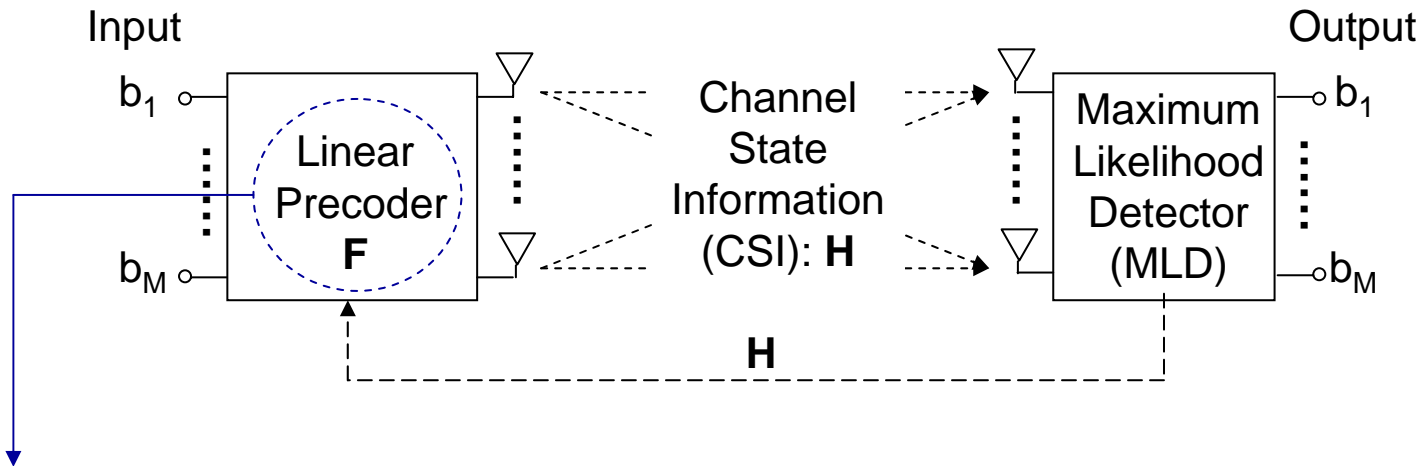
# Presentation Outline

- Introduction
  - Research background
- Literature Review
  - Precoder for minimizing BER upper bound under perfect CSI
  - Problem identification under imperfect CSI and research objective
- Proposed Precoding Method under Imperfect CSI
  - Criterion formulation and optimization
  - Computer simulation results
- Conclusions
  - Summary and suggestions for future work

# Research Background

- Demands for future mobile communications (High Speed + High Reliability)

- Framework: MIMO-OFDM system with feedback channel



- Linear precoding criteria

- Maximum Capacity

- Adaptive Modulation Coding (AMC)
- Waterfilling-based power allocation

- Non-Capacity Based

- Fixed modulation and coding
- Phase and power optimization:

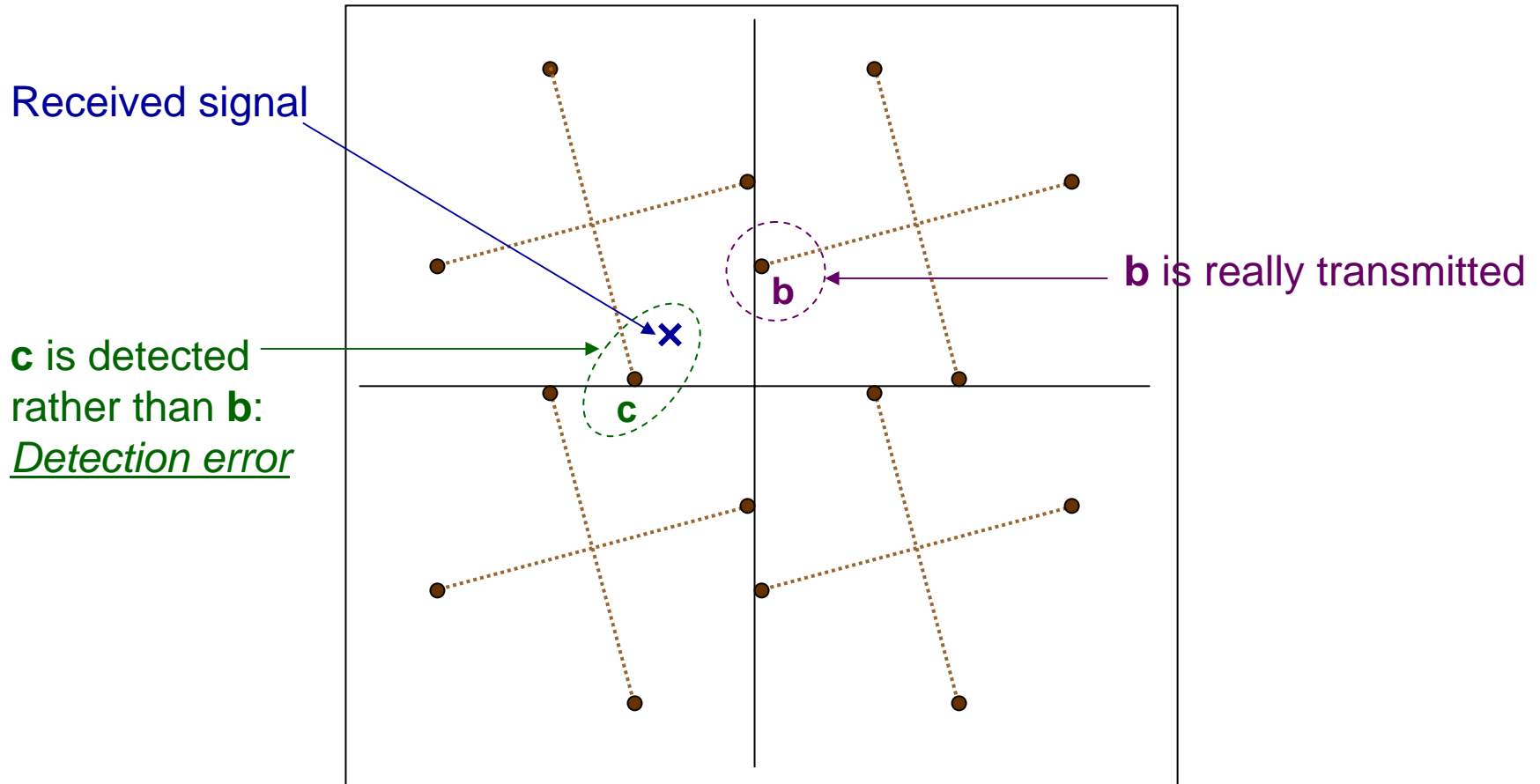
- Satisfy QoS
- Minimize transmit power

- Minimize bit error rate (MBER)

Increase reliability (BER performance):  
Focus of this research

# Detection Error and MBER Precoding

Transmitted signal replica of MIMO  
with QPSK and  $M=2$  data streams



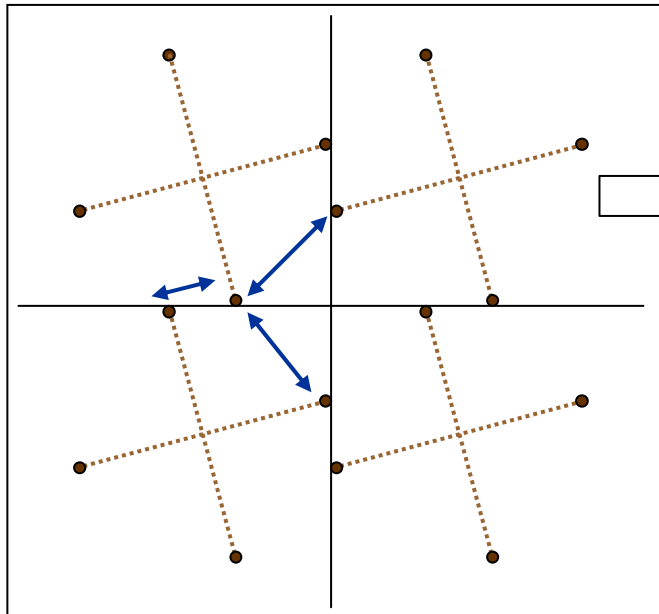
- Detection error: Received signal closer to another signal than transmitted one
- MBER precoding: Minimize detection error → Increase reliability

# MBER Precoder under Perfect CSI

- Transmitter and receiver: Same CSI
- Procedures at transmitter

## No precoder

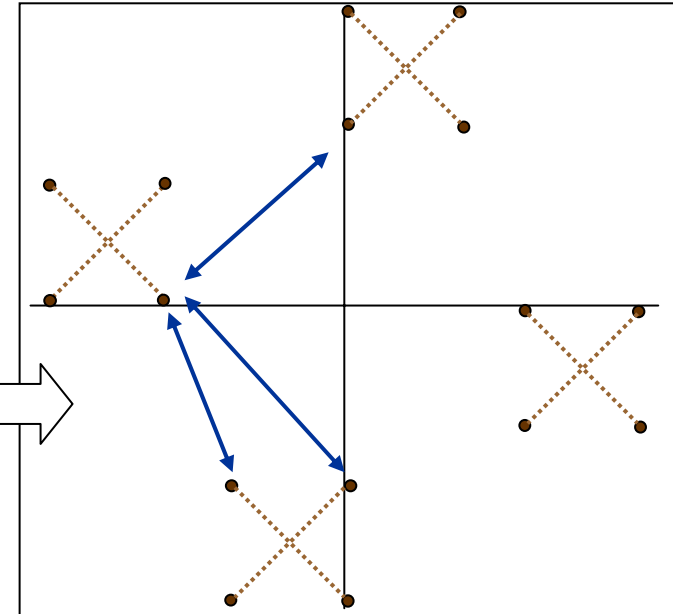
Transmitted signal replica of MIMO with QPSK and two data streams



1. Calculate BER upper bound  $J_e(\mathbf{F})$
2. Optimize  $\mathbf{F}$  to minimize  $J_e(\mathbf{F})$

## With MBER precoder

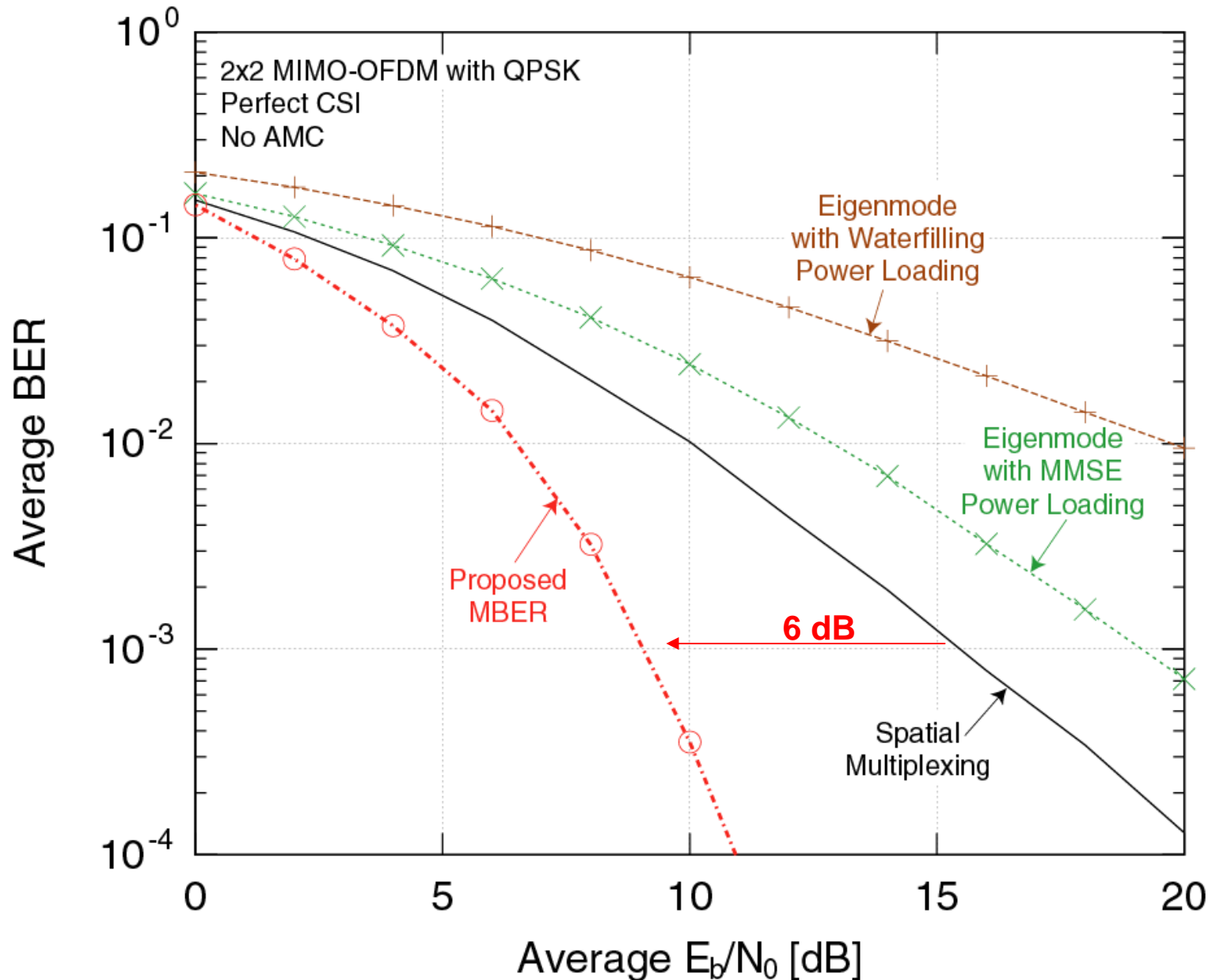
Transmitted signal replica of MIMO with QPSK and two data streams



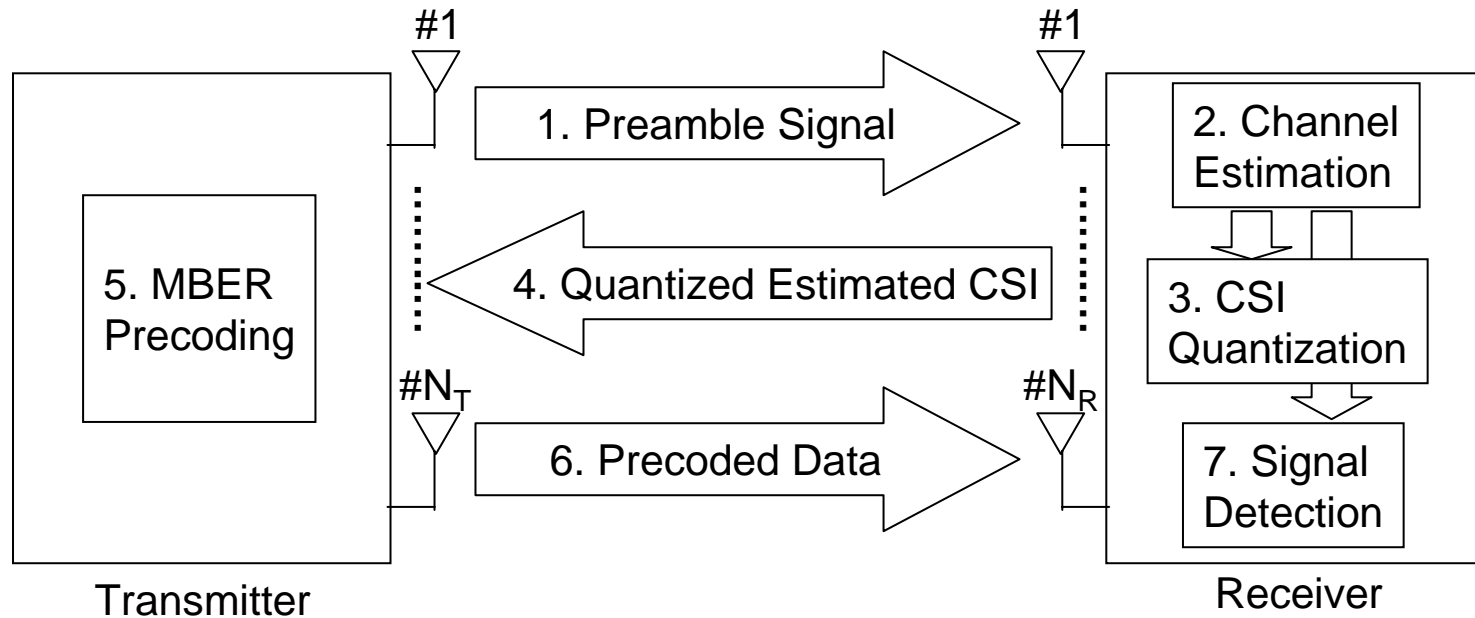
- Significant BER improvement over spatial multiplexing (nonprecoded) system<sup>1</sup>

<sup>1</sup> B. Pitakdumrongkija, K. Fukawa, H. Suzuki, and T. Hagiwara, "Linear precoding with minimum BER criterion for MIMO-OFDM systems employing ML detection," IEEE ICC 2007, pp. 2522-2527, June 2007.

# Uncoded BER under Perfect CSI



# System Model under Imperfect CSI



- CSI model

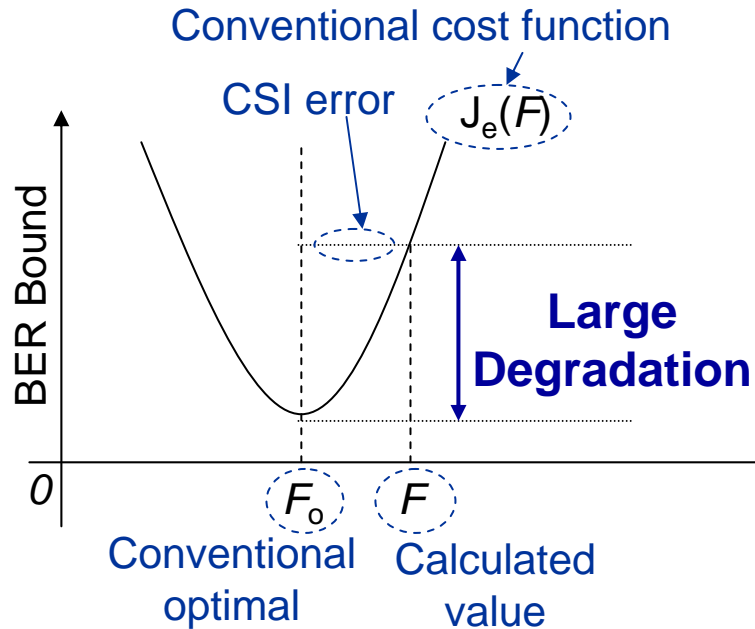
- Receiver: Channel Estimation Error

- Transmitter: Channel Estimation Error + Quantization Error

- Transmitter and receiver: Different CSI

# Problem and Proposed Solution

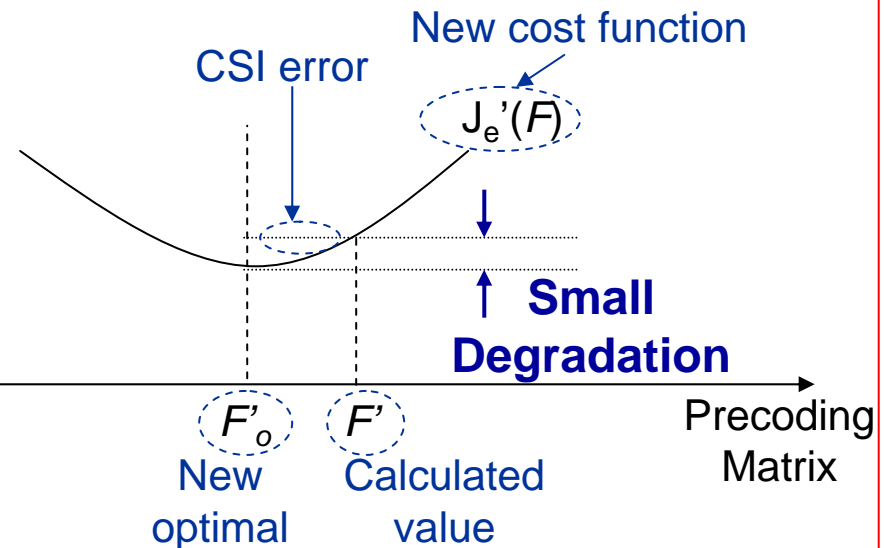
- Problem with conventional MBER precoder (designed by assuming perfect CSI)



CSI error → Large degradation

- Proposed solution

New BER cost function:  
Robust to imperfect CSI



CSI error → Small degradation

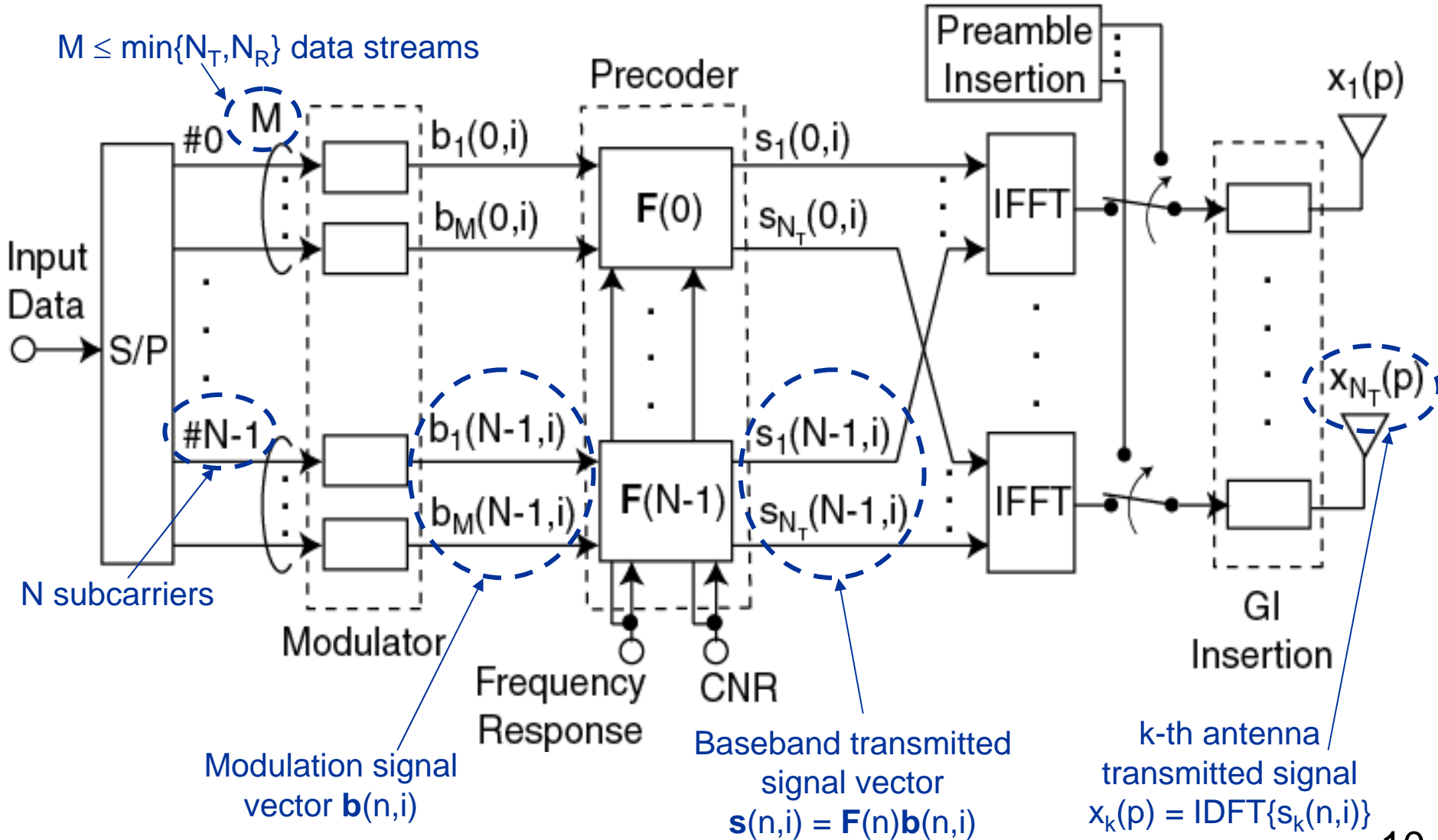


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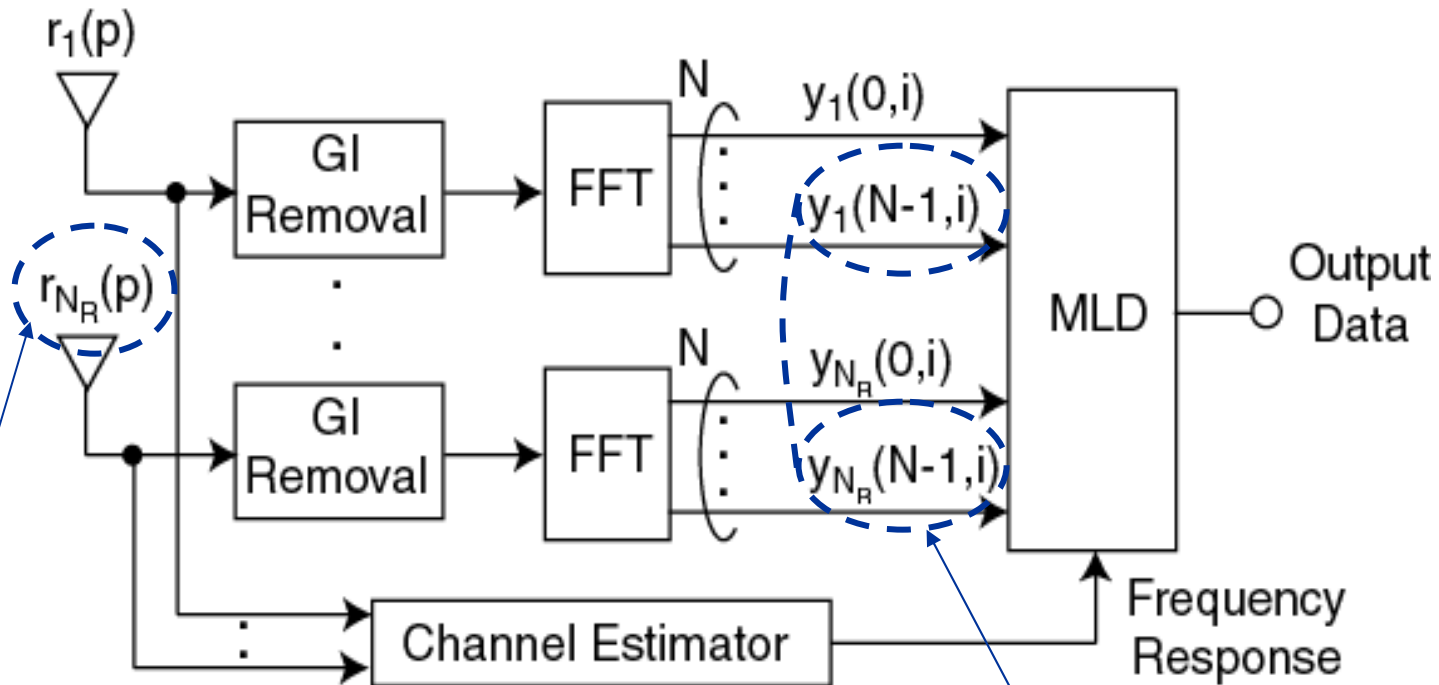
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# MIMO-OFDM Transmitter

- System with  $N_T$  transmit and  $N_R$  receive antennas



# MIMO-OFDM Receiver



$l$ -th antenna received signal :

$$r_l(p) = \sum_{k=1}^{N_T} \sum_{d=0}^D \boxed{h_{lk,d}} x_k(p-d) + \boxed{n_l(p)}$$

$h_{lk,d}$  :  $d$ -th path impulse response between  $l$ -th and  $k$ -th antennas

$n_l(p)$  : AWGN zero mean and variance  $\sigma_n^2$

Baseband received signal vector :

$$\mathbf{y}(n,i) = \boxed{\mathbf{H}(n)} \mathbf{F}(n) \mathbf{b}(n,i) + \boxed{\mathbf{z}(n,i)}$$

$\mathbf{H}(n)$  :  $N_R$ -by- $N_T$  channel frequency response matrix

$\mathbf{z}(n,i)$  :  $N_R$ -by-1 noise vector

# CSI Error Model

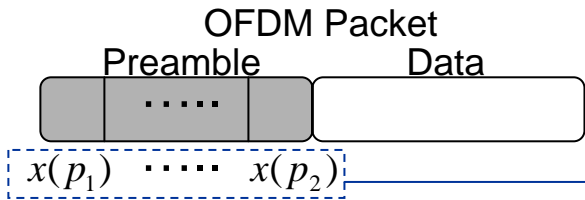
• CSI at receiver:

$$\tilde{h}_{lk,d} = h_{lk,d} + \psi_{lk,d} \leftarrow \text{Estimation error}$$

• CSI at transmitter:

$$\hat{h}_{lk,d} = h_{lk,d} + \psi_{lk,d} + \nu_{lk,d} \leftarrow \text{Quantization error}$$

• MMSE channel estimation



$$\tilde{\mathbf{h}}_l = \arg \min_{\mathbf{h}_l} \sum_{p=p_1}^{p_2} \lambda^{p_2-p} |r_l(p) - \mathbf{h}_l^H x(p)|^2$$

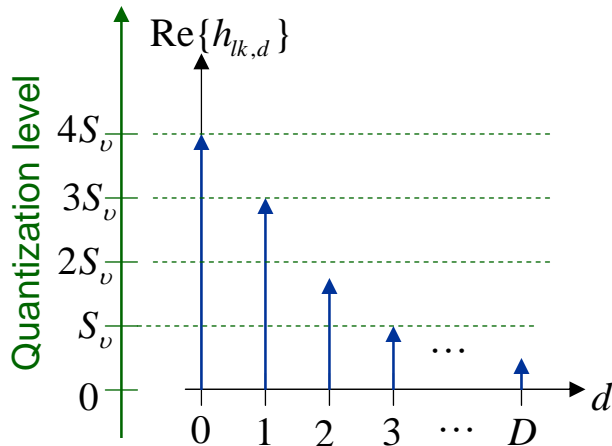
$$= \mathbf{h}_l + \boldsymbol{\psi}_l$$

- Estimation error statistics:

$$\langle \boldsymbol{\psi}_l \rangle = \mathbf{0}$$

$$\langle \boldsymbol{\psi}_l \boldsymbol{\psi}_l^H \rangle \approx \sigma_n^2 \mathbf{I}$$

• Uniform quantization



- Quantization error statistics:

$$\langle \nu_{lk,d} \rangle = 0$$

$$\langle \nu_{l_1 k_1, d_1}^* \nu_{l_2 k_2, d_2} \rangle = \sigma_v^2 \delta_{l_1 l_2} \delta_{k_1 k_2} \delta_{d_1 d_2}$$

$\sigma_v^2 / 2 = S_v^2 / 12$

- If  $\text{Re}\{h_{lk,d}\} \in [-1, 1]$ , then

$$B_l = \log_2(2 / S_v)$$

No. of quantizing bit per path

# Pairwise Error Probability (PEP)

- Detection error condition when  $\sigma_v^2$  is small

$$\underbrace{\left\| \mathbf{y}(n, i) - \hat{\mathbf{H}}(n) \mathbf{F}(n) \mathbf{b}(n, i) \right\|^2}_B > \underbrace{\left\| \mathbf{y}(n, i) - \hat{\mathbf{H}}(n) \mathbf{F}(n) \mathbf{c}(n, i) \right\|^2}_C$$

$\xrightarrow{\quad} \varepsilon = B - C$

$(\hat{\mathbf{H}}(n))_{lk} = H_{lk}(n) + \Psi_{lk}(n) + Y_{lk}(n) \leftrightarrow h_{lk,d} + \psi_{lk,d} + u_{lk,d}$

- PEP averaged with respect to CSI errors

$$P(\tilde{\mathbf{b}} \rightarrow \tilde{\mathbf{c}} | \hat{H}) = \int_0^\infty \underbrace{p(\varepsilon)}_{\text{Joint p.d.f. between AWGN and CSI errors}} d\varepsilon = \frac{1}{2} \operatorname{erfc} \sqrt{\gamma(\tilde{\mathbf{b}} \rightarrow \tilde{\mathbf{c}})}$$

$$\gamma(\tilde{\mathbf{b}} \rightarrow \tilde{\mathbf{c}}) = \left\| \hat{\mathbf{H}}(n) \mathbf{F}(n) [\mathbf{b}(n, i) - \mathbf{c}(n, i)] \right\|^2 / 4 \sigma_e^2(\mathbf{F}(n))$$

$$\text{Error variance: } \sigma_e^2(\mathbf{F}(n)) = \underbrace{N^{-1} \sigma_n^2}_{\text{Conventional}} + \underbrace{[\mathbf{F}(n) \mathbf{b}(n, i)]^H \Delta(n) [\mathbf{F}(n) \mathbf{b}(n, i)]}_{\text{Proposed additional term}}$$

$$\text{CSI errors: } \Delta(n) = \underbrace{\langle \Psi^H(n) \Psi(n) \rangle}_{\text{Estimation error}} + \underbrace{\langle \mathbf{Y}^H(n) \mathbf{Y}(n) \rangle}_{\text{Quantization error}}$$

# BER Upper Bound Minimization

- BER upper bound cost function

$$J_e(\mathbf{F}(n)) = \sum_{\tilde{\mathbf{b}}} \sum_{\tilde{\mathbf{b}} \neq \tilde{\mathbf{c}}} N_e(\tilde{\mathbf{b}} \rightarrow \tilde{\mathbf{c}}) P(\tilde{\mathbf{b}} \rightarrow \tilde{\mathbf{c}} | \hat{H})$$

All pairs of  $\mathbf{b}(n,i)$  and  $\mathbf{c}(n,i)$

No. of error bit when  $\mathbf{c}(n,i)$  rather than  $\mathbf{b}(n,i)$  is detected

- Solution for optimal precoding matrix

$$\mathbf{F}_o(n) = \arg \min_{\tilde{\mathbf{F}}(n)} J_e(\tilde{\mathbf{F}}(n)) \longrightarrow \text{Mathematically Untraceable}$$

- Optimization by the steepest descent algorithm

$$\mathbf{F}^{(q)}(n) = \mathbf{F}^{(q-1)}(n) - \mu \left. \frac{\partial J_e(\mathbf{F}(n))}{\partial \mathbf{F}^*(n)} \right|_{\mathbf{F}^{(q-1)}(n)}$$

Iteration index      Real positive constant      Gradient of  $J_e(\mathbf{F}(n))$

subjected to

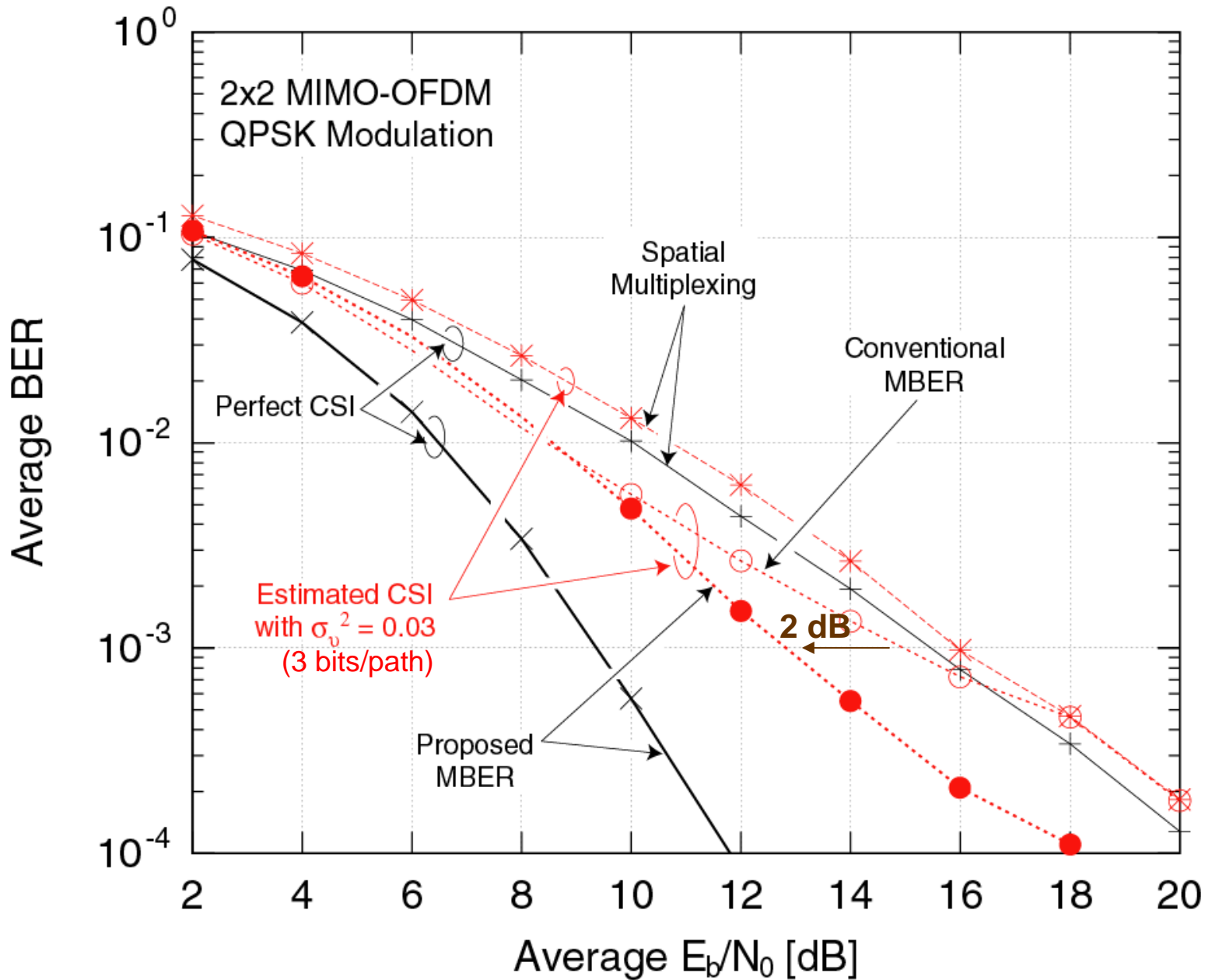
$$\sum_{n=0}^{N-1} \left\langle \|\mathbf{F}(n)\mathbf{b}(n,i)\|^2 \right\rangle = \sum_{n=0}^{N-1} \text{tr} \{ \mathbf{F}(n)\mathbf{F}^H(n) \} = P_0$$

Total average transmit power

# Simulation Conditions

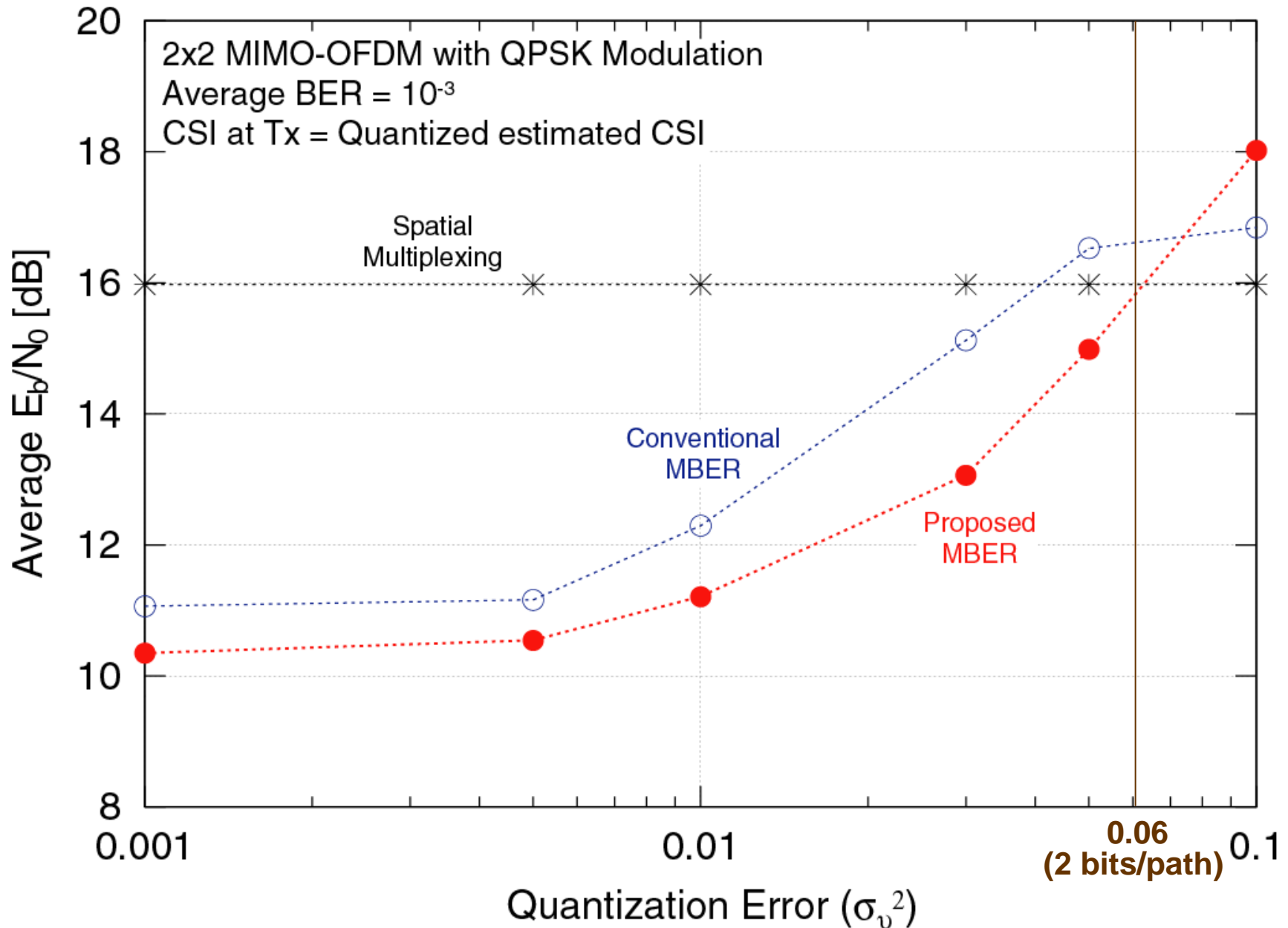
Modulation	QPSK
Tx, Rx antenna ( $N_T \times N_R$ )	2 x 2
Data stream M	2
OFDM packet format	IEEE802.11a (Preamble: 2 symbols, Data: 10 symbols)
Total subcarrier	64
Effective subcarrier	52 (Pilot:4, Data:48)
GI length	16 points
Channel model	17 path exponential decay Rayleigh fading
Maximum Doppler freq.	0 Hz
CSI at Rx	MMSE channel estimation ( $\lambda=1.0$ )
CSI at Tx	CSI from Rx + Quantization error ( $\sigma_v^2=0.001-0.1$ )
Signal detection	MLD

# Average BER Performance





# Robustness to Quantization Error



# Conclusions and Future Works

Conclusions: Proposed MBER precoding technique under imperfect CSI

- Consider channel estimation and quantization errors as CSI imperfection
- Average PEP with respect to CSI errors
- Employ steepest descent algorithm for optimization
- Improve BER robustness to imperfect CSI

Future Works:

- Improvement of optimization and reduction of CSI amount
- Precoding technique for time-varying channel