Mobile Group Seminar **7 December 2006**

Reduction of AOA estimation error due to perturbation in array response by spatial smoothing preprocessing

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Background (1)

- Subspace-based AOA estimation, e.g. MUSIC/ ESPRIT, requires the precise array response.
- In practice, it is difficult to obtain the precise array response even if the antenna calibration is applied.
- Therefore, methods to compensate the calibration error are required.

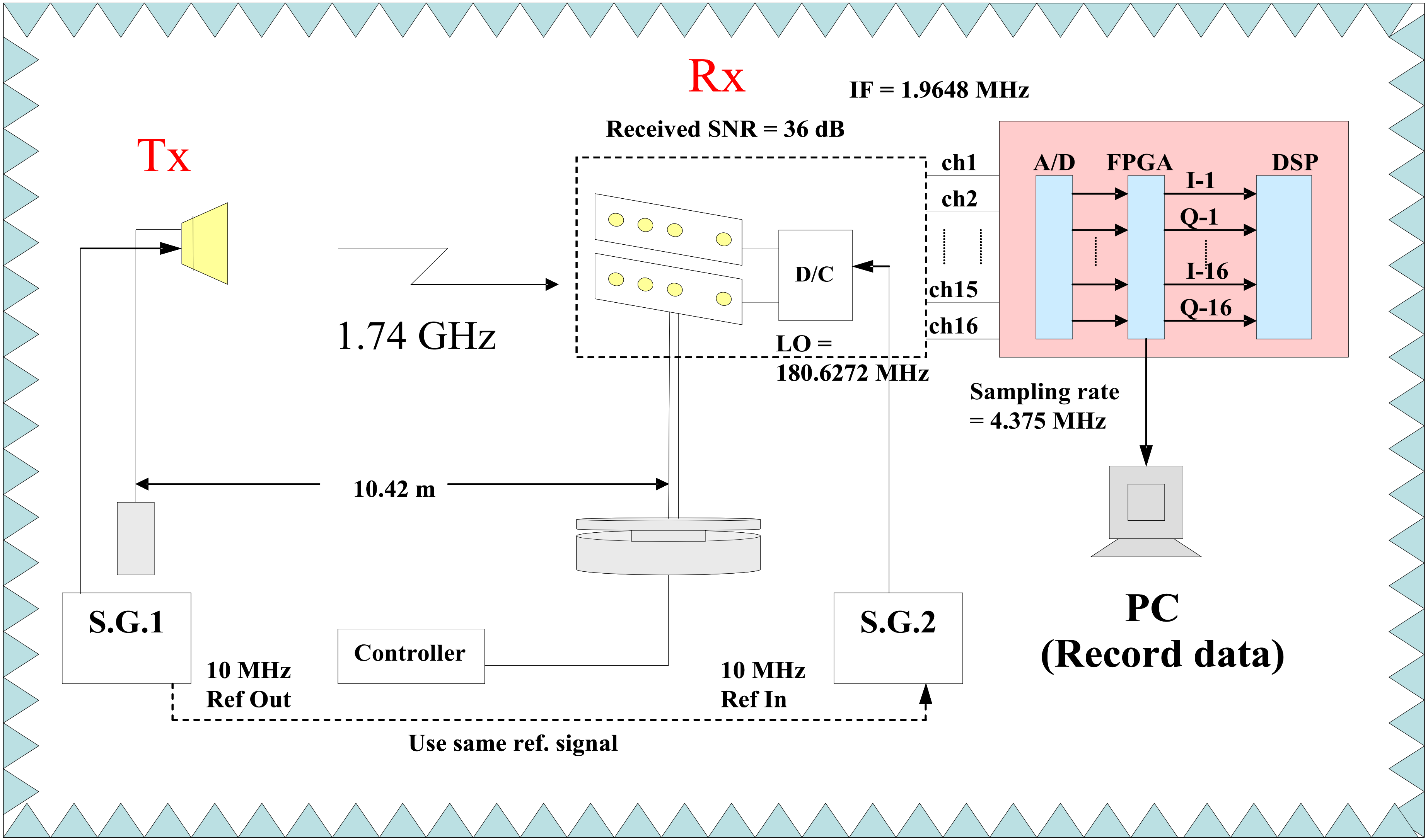
Background (2)

How to compensate the error in the array response?

- In 1993, Hari and Gummadavelli proposed the performance analysis of subspace methods for estimating AOA due to SSP in the presence of array model errors.
  - SSP can improve the performance of ESPRIT and Min-Norm in the presence of errors in the array response, but it is not so for MUSIC.
  - The theoretical expressions were well derived, but complicated and verified by only simulation.
- Objective

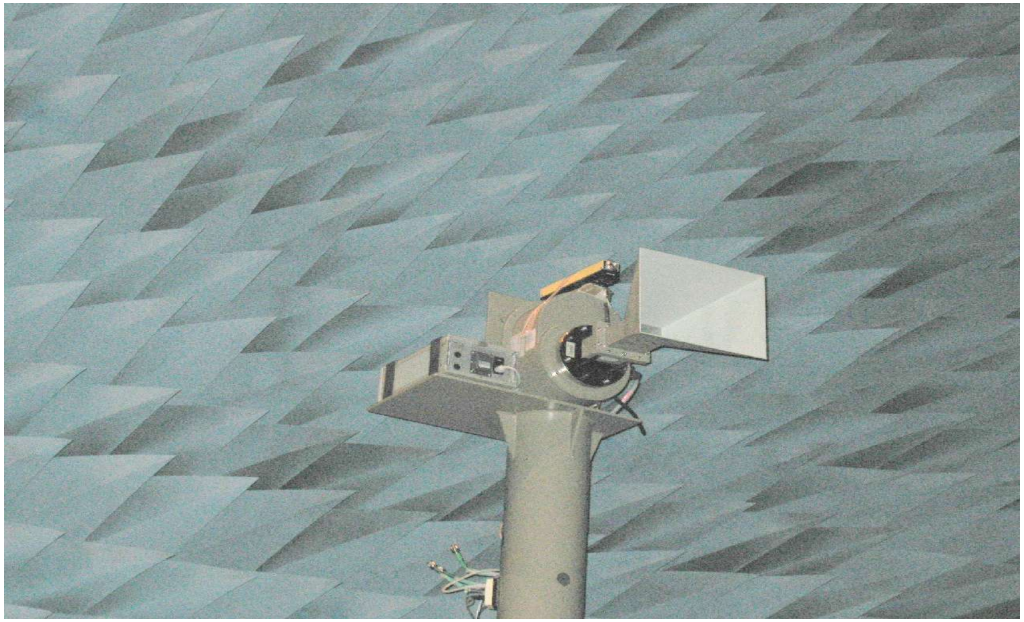
To investigate the possibility of SSP to reduce the random error in the array response in real systems.

Experiment (1): Anechoic chamber



Remark: Because of near-field measurement, the signal has spherical wavefront. So, the phase curvature of spherical wavefront is compensated to far-field planar wavefront before data processing.

Experiment (2): Specification of experiment

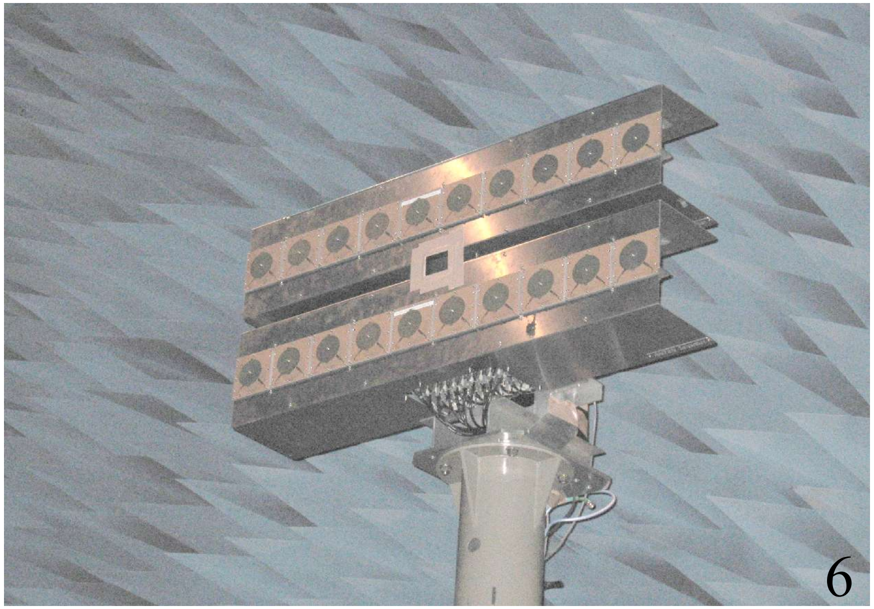


Transmitter

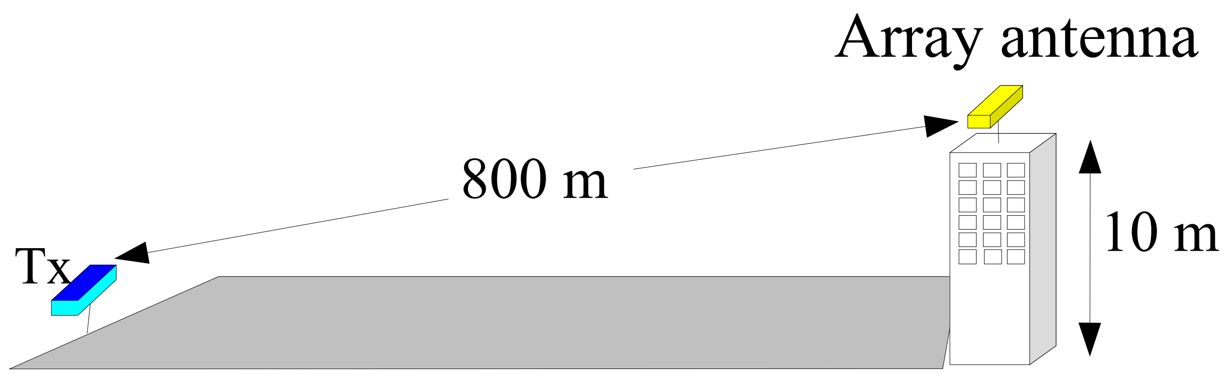
Frequency	1.74 GHz
Antenna	Horn antenna
Antenna gain	14.67 dBi
Tx power	-30 dBm
Modulation	GMSK

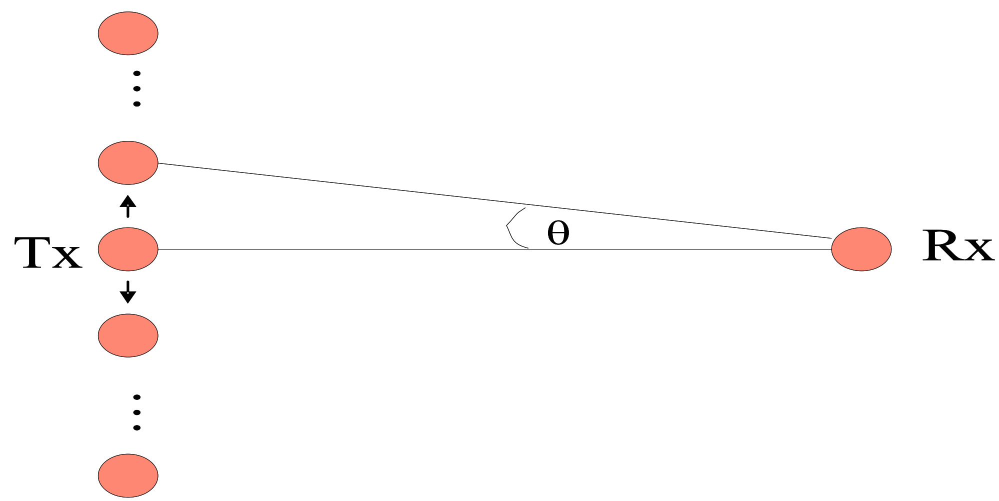
Antenna array

Shape of array	Uniform linear array
Number of elements	10
Element spacing	0.8 λ
Antenna element	Patch antenna
Antenna gain	7 dBi



Experiment (3): Open site



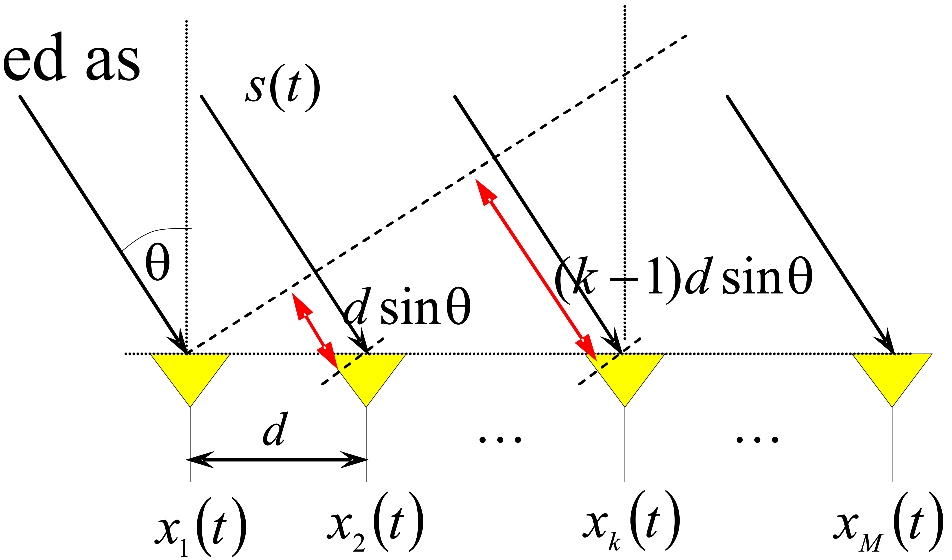


Moving Tx where AOA at Rx are known in range -6 to 6 degrees.

Signal Model

The array output vector can be modeled as

$$\boldsymbol{x}(t)=\boldsymbol{a}\,s(t)+\boldsymbol{n}(t)$$



where $s(t)$ is an arriving signal

$\boldsymbol{n}(t)$ is a noise vector

$\boldsymbol{a}=[1\ ,\ e^{j\phi_1},\ e^{j\phi_2},\ \ldots,\ e^{j\phi_M}]^T$

and ϕ_m is the nominal phase of the m^{th} element; $\phi_m=(m-1)\omega$

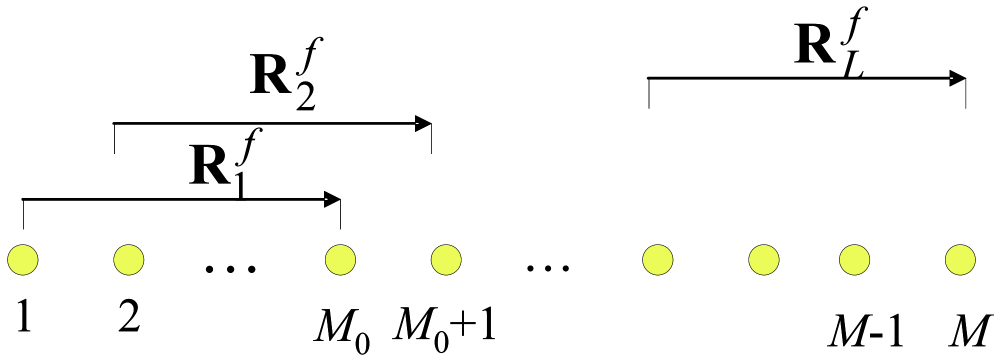
$\omega=2\pi d\sin(\theta)/\lambda$ is the phase difference between adjacent elements.

The output covariance matrix of the single source can be written as

$$\begin{aligned}\boldsymbol{R}=E[\boldsymbol{x}(t)\boldsymbol{x}^H(t)]&=\sigma_s^2\boldsymbol{a}\boldsymbol{a}^H+\sigma_n^2\boldsymbol{I}\\&=\lambda_s\boldsymbol{u}_s\boldsymbol{u}_s^H+\sigma_n^2\boldsymbol{I}\end{aligned}$$

AOA estimation using subspace-based approaches: ESPRIT and MUSIC.

Spatial smoothing preprocessing:
Forward-only SSP



The output vector of the l^{th} subarray,

$$\boldsymbol{x}_l^f(t) = \boldsymbol{a}_l s(t) + \boldsymbol{n}_l(t),\ l = 1,2,\ldots,L$$

where $\boldsymbol{a}_l = [a_l,\ a_{l+1},\ \ldots,\ a_{l+M_0-1}]^T$,

and $\boldsymbol{n}_l(t) = [n_l(t),\ n_{l+1}(t),\ \ldots,\ n_{l+M_0-1}(t)]^T$,

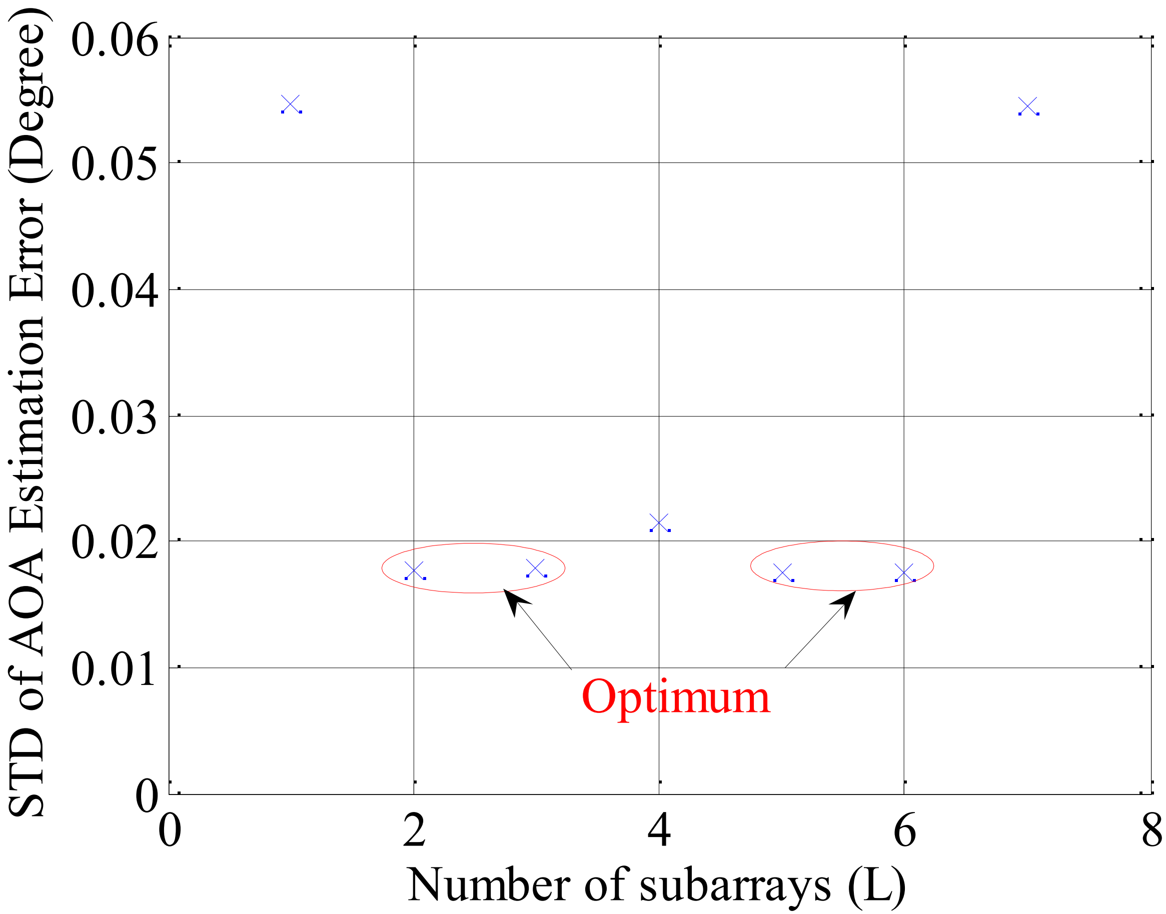
The covariance matrix of the l^{th} subarray is given by

$$\boldsymbol{R}_l^f = E[\boldsymbol{x}_l^f(t)(\boldsymbol{x}_l^f(t))^H] = \sigma_s^2 \boldsymbol{a}_l \boldsymbol{a}_l^H + \sigma_n^2 \boldsymbol{I},$$

therefore the forward spatially smoothed covariance matrix is

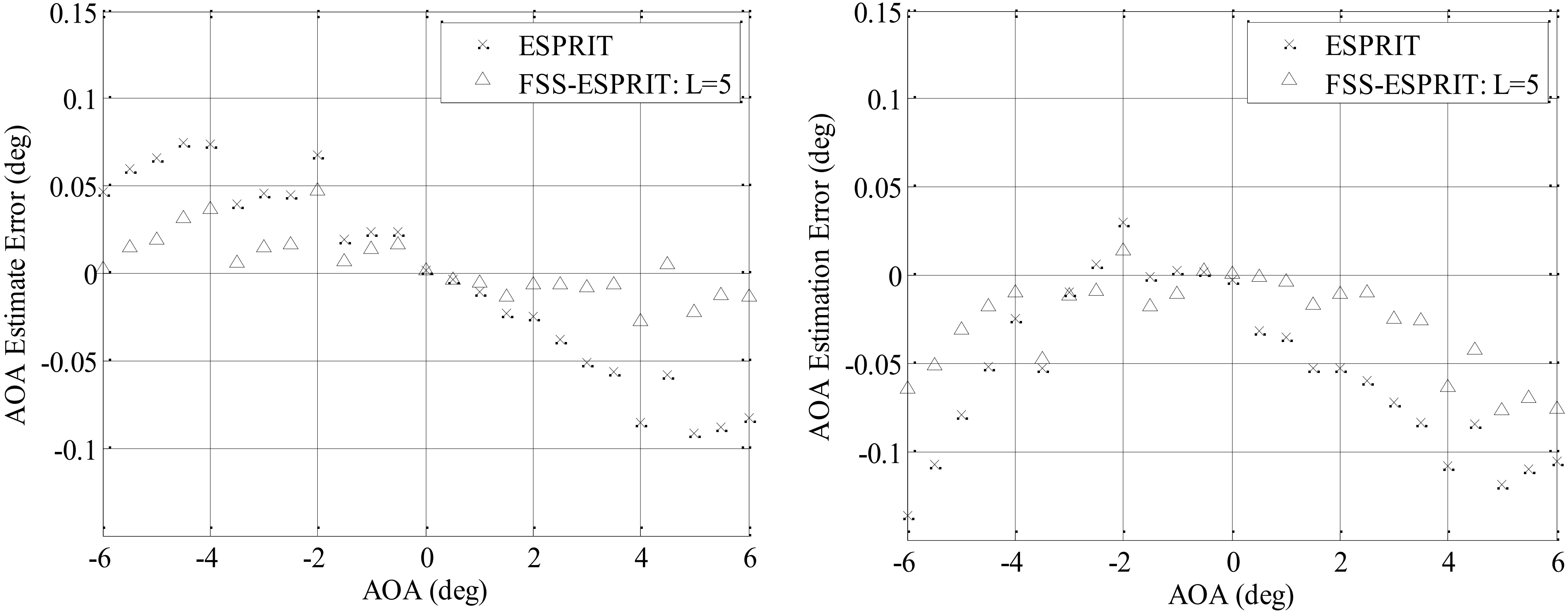
$$\boldsymbol{R}_F = \frac{1}{L}\sum_{l=1}^{L} \boldsymbol{R}_l^f = \sigma_s^2 \left\{ \frac{1}{L}\sum_{l=1}^{L} \boldsymbol{a}_l \boldsymbol{a}_l^H \right\} + \sigma_n^2 \boldsymbol{I}.$$

Result of AOA Estimation Error
Measured result in anechoic chamber (1)



The optimal number of subarrays can be observed at L =5 and 6.

Result of AOA Estimation Error
Measured result in anechoic chamber (2)



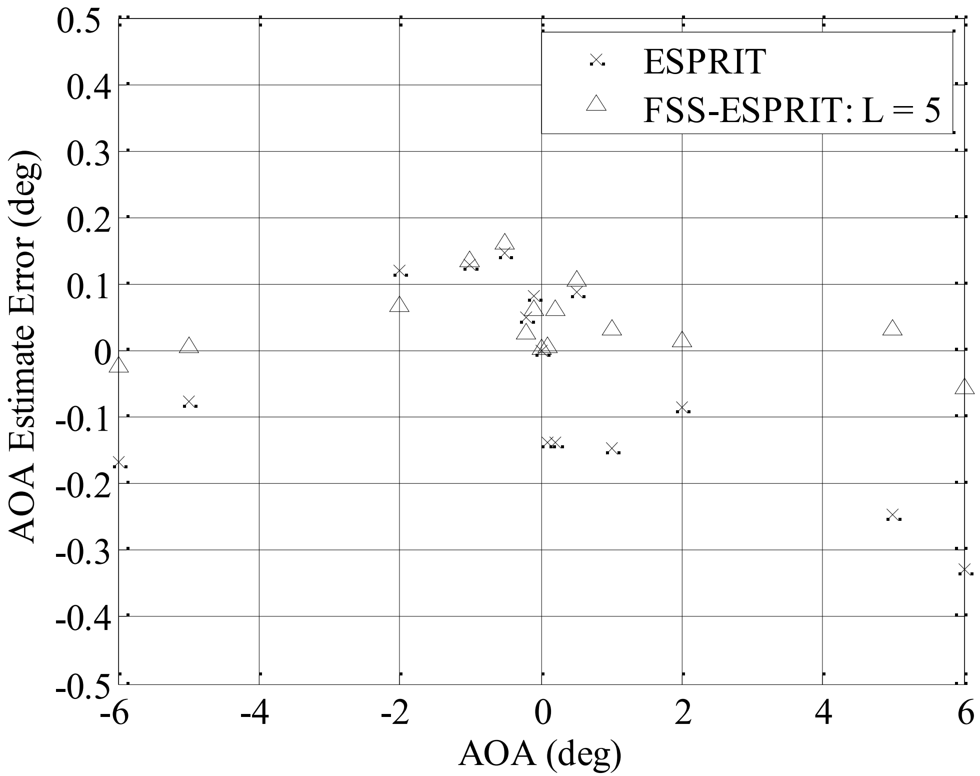
STD of est. AOA error (deg)

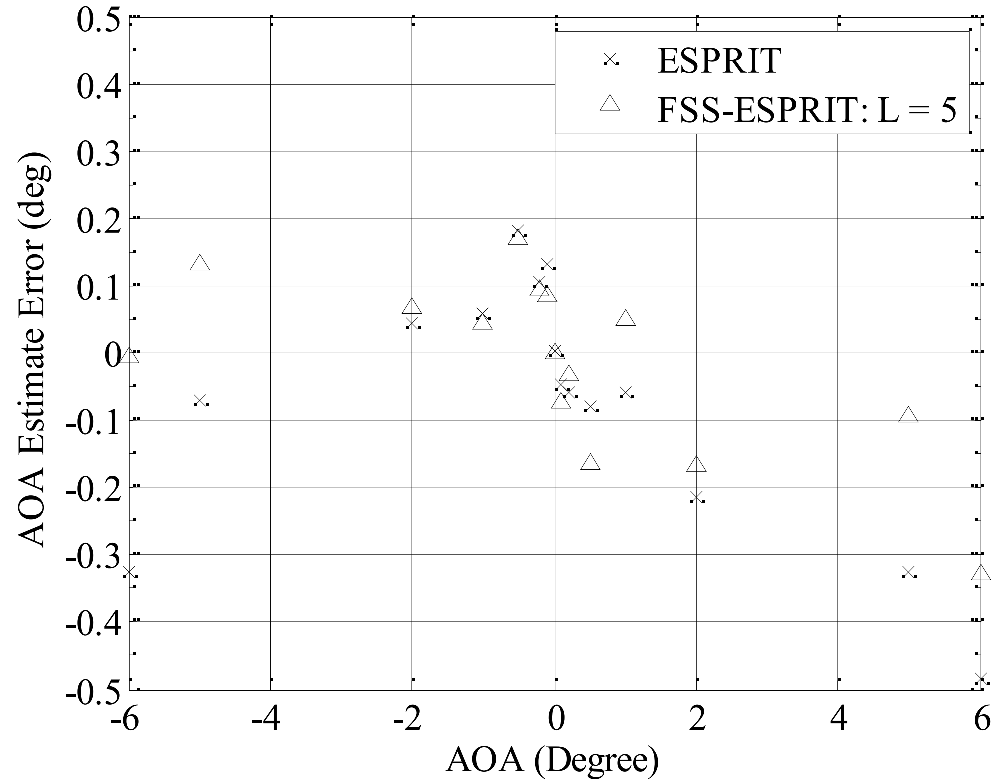
	Array 1	Array 2
ESPRIT	0.0547	0.0454
FSS-ESPRIT	0.0176	0.0263

The performance of ESPRIT is improved by applying SSP.

Result of AOA Estimation Error

Measured result in open site





STD of est. AOA error (deg)

	Array 1	Array 2
ESPRIT	0.1427	0.1814
FSS-ESPRIT	0.0563	0.1246

The performance of ESPRIT is improved by applying SSP.

Problem Formulation:
Error in Array Model

$$a_m = e^{j\phi_m}$$ Gain and phase errors →

$$\tilde{a}_m = (1+\tilde{g}_m)\, e^{j(\phi_m+\tilde{\phi}_m)}$$

$$\tilde{a}_m = \gamma_m a_m ;\ \gamma_m = (1+\tilde{g}_m)\, e^{j\tilde{\phi}_m}$$

Assume: $\tilde{\phi} \sim N(0, \sigma_\phi^2)$

$\tilde{g} \sim N(0, \sigma_g^2)$

The array output vector with error in the array response

$$\tilde{\boldsymbol{x}}(t) = \Gamma \boldsymbol{a}\, s(t) + \boldsymbol{n}(t)$$

$$;\ \Gamma = diag[\gamma_1, \gamma_2, \ldots, \gamma_M]$$

$$\tilde{\boldsymbol{x}}(t) = \tilde{\boldsymbol{a}}\, s(t) + \boldsymbol{n}(t)$$

The output covaraince matrix becomes

$$\tilde{\boldsymbol{R}} = E[\tilde{\boldsymbol{x}}(t)\tilde{\boldsymbol{x}}^H(t)] = \sigma_s^2 \tilde{\boldsymbol{a}}\tilde{\boldsymbol{a}}^H + \sigma_n^2 \boldsymbol{I}$$

By applying FSS, $\tilde{\boldsymbol{R}}_F = \frac{1}{L}\sum_{l=1}^{L} \tilde{\boldsymbol{R}}_l = \sigma_s^2 \left\{ \frac{1}{L}\sum_{l=1}^{L} \tilde{\boldsymbol{a}}_l \tilde{\boldsymbol{a}}_l^H \right\} + \sigma_n^2 \boldsymbol{I}.$

Capability of SSP in reducing error (1):
Phase error case

Focusing only on the part of the array response of the above eq.;

$$\boldsymbol{B}=\frac{1}{L}\sum_{l=1}^{L}\boldsymbol{B}_l=\frac{1}{L}\sum_{l=1}^{L}\tilde{\boldsymbol{a}}_l\tilde{\boldsymbol{a}}_l^H$$

$$=\frac{1}{L}\sum_{l=1}^{L}\begin{bmatrix} 1 & e^{-j\omega}e^{-j(\tilde{\phi}_{l+1}-\tilde{\phi}_l)} & \cdots & e^{-j(M_0-1)\omega}e^{-j(\tilde{\phi}_{l+M_0-1}-\tilde{\phi}_l)} \\ e^{j\omega}e^{j(\tilde{\phi}_{l+1}-\tilde{\phi}_l)} & 1 & \cdots & e^{-j(M_0-2)\omega}e^{-j(\tilde{\phi}_{l+M_0-1}-\tilde{\phi}_{l+1})} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j(M_0-1)\omega}e^{j(\tilde{\phi}_{l+M_0-1}-\tilde{\phi}_l)} & e^{j(M_0-2)\omega}e^{j(\phi_{l+M_0-1}-\phi_{l+1})} & \cdots & 1 \end{bmatrix}$$

very small

To first order of Taylor expansion:

$$\frac{1}{L}\sum_{l=1}^{L}e^{j(\tilde{\phi}_{l+1}-\tilde{\phi}_l)}=1+\frac{j}{L}\sum_{l=1}^{L}(\tilde{\phi}_{l+1}-\tilde{\phi}_l)$$

$$=1+\frac{j}{L}(\tilde{\phi}_{L+1}-\tilde{\phi}_1)$$

$$\sim N(0,\ \frac{2\sigma^2}{L^2})$$

Phase error at p-row, q-column is distributed with

$$\zeta_{pq}\sim N(0,\ 2|p-q|\frac{\sigma_\phi^2}{L^2})$$

for $p\neq q$

STD of the phase error reduces according to L.

Capability of SSP in reducing error (2)

Gain error case:

Gain error at p-row and q-column is distributed with

$$\eta_{pq} \sim N(0, \frac{2\sigma_g^2}{L})$$

STD of the gain error reduces according to L.

Tradeoff:

* if L increases, STD of gain error decrease...
  But M_0 also decreases.. This means resolution of AOA estimation decrease.
  (Since resolution is proportional to no. of elements)

AOA estimation error caused by gain error:

$$\varepsilon_g \propto \frac{1}{LM_0}$$

$$\varepsilon_g \propto \frac{1}{L(M-L+1)}$$

Maximizing by taking its first derivative

The optimal number of subarrays:

$$L_{opt} = \frac{M+1}{2}$$

Phase error case:

Phase error at p-row and q-column is distributed with

$$\zeta_{pq} \sim N(0, 2|p-q| \frac{\sigma_\phi^2}{L^2}) \quad \text{for} \quad p \neq q$$

The optimal number of subarrays:

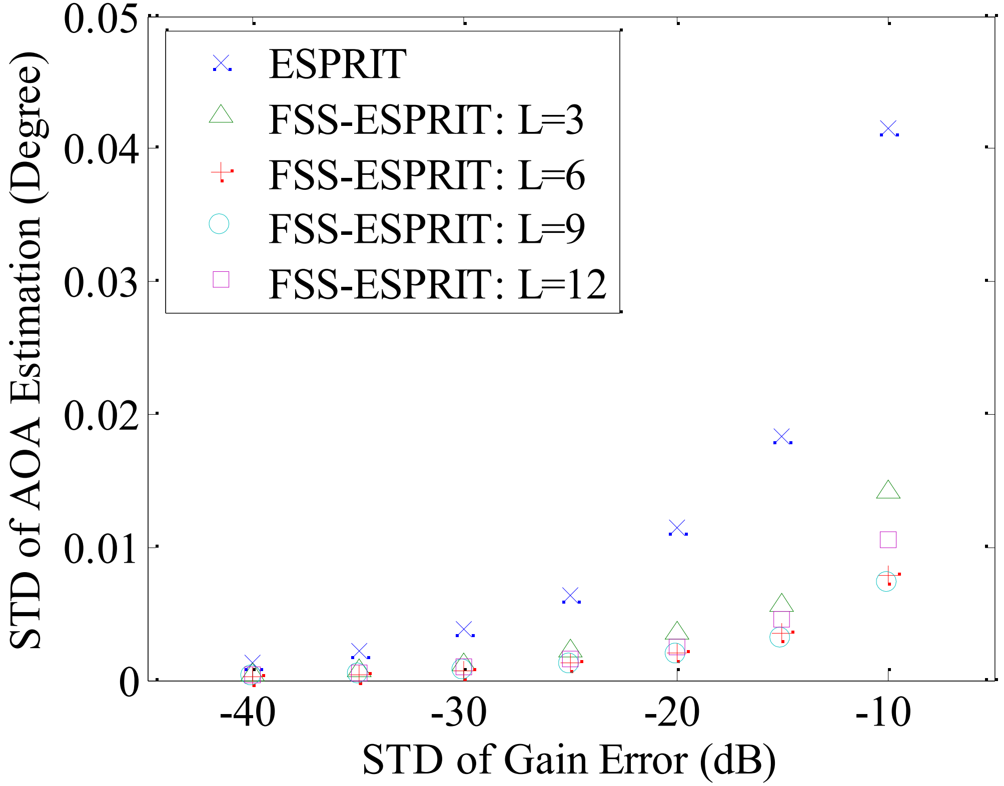
$$L_{opt} = \frac{M}{3} \quad \text{for} \quad L < M_0$$

$$L_{opt} = \frac{2M}{3} \quad \text{for} \quad L > M_0$$

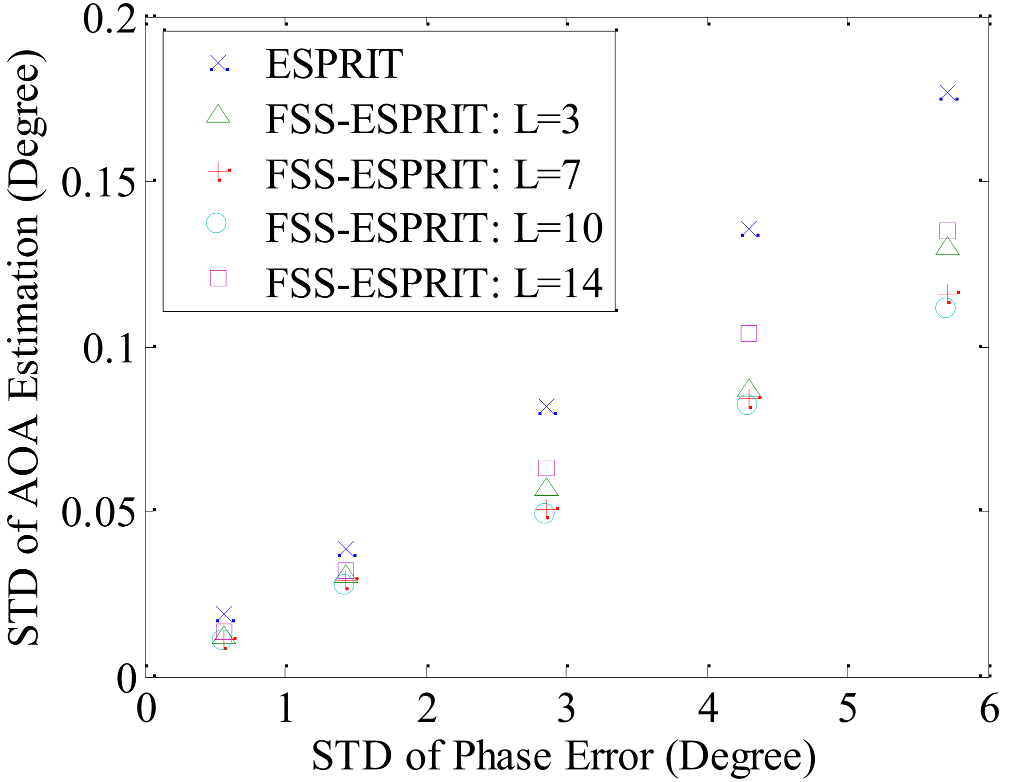
Simulation Condition

Number of element (M)	16
Element Spacing (d)	Half a wavelength
AOA	6 [deg]
Signal SNR	10 dB
Number of snapshots	500
Number of trials	100

Simulation Result



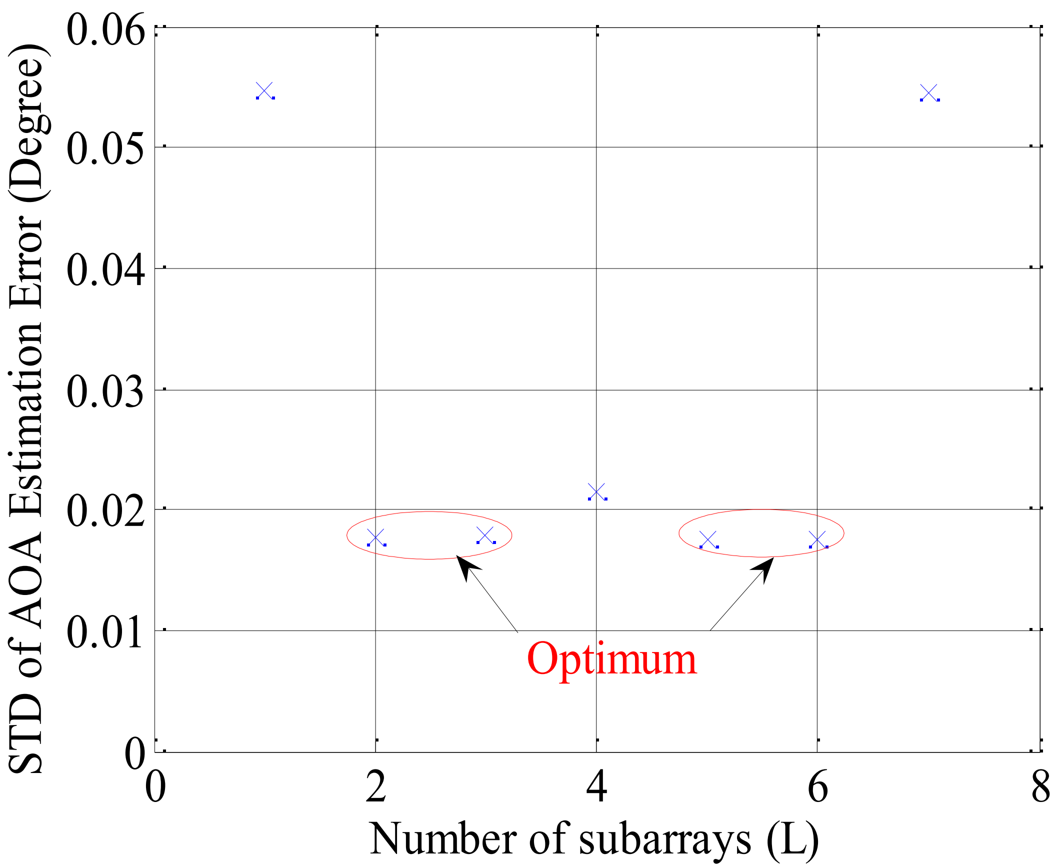
$$L_{opt} = \frac{M+1}{2} = 9$$



$$L_{opt} = \frac{2M}{3} = 10$$

STD of estimated AOA by applying FSS with the number of subarrays (L) shown is smaller that that without applying FSS for both cases of gain and phase errors.

Optimal number of subarrays
Measured data in anechoic chamber



- The optimal number of subarrays can be observed and agrees with the theoretical formular the case of phase error.

- Our measured data were possibly only affected by phase errors in the array response.

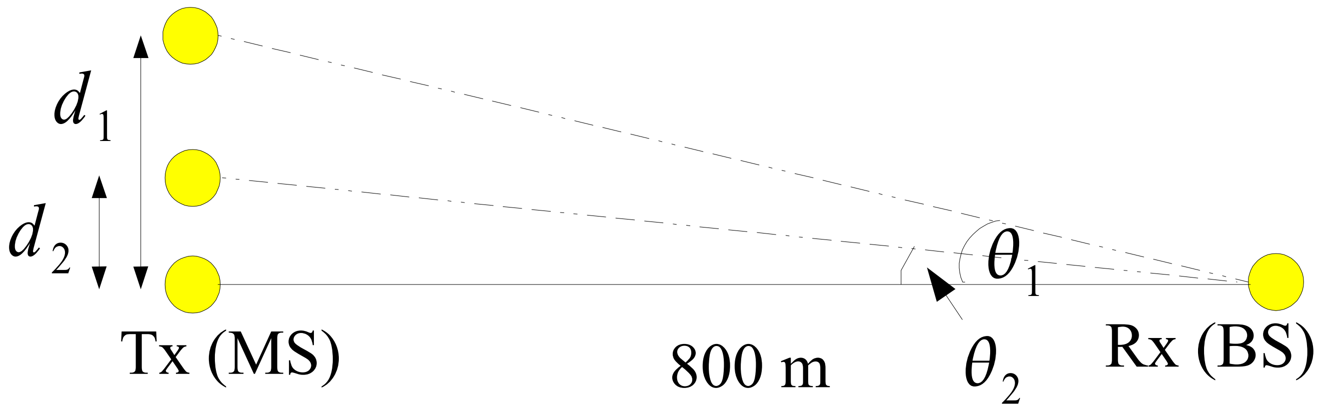
Conclusion and Future work

- SSP can reduce the random error in the array response occurred in the real senarios, which is verified by the improved result of the meaured-data estimation.
  - Lessening of the error of estimated AOAs.
  - Applicable in real scenarios
    (at least to the extent verified by our measurement system and under the field experiment conditions).
- The problem formulas according to the gain and phase error in the array response are generalized and their performances were verified by simulation.
- The optimal number of sub-arrays was also investigated to obtain the most effectiveness of SSP.
- Future works: To investigate its applicability for the multiple sources case in real systems.

Thank you very much for your attention.

What is the advantage if AOA errors reduce?

Application for Mobile localization system



d_1 and d_2 are distance errors associated with θ_1 and θ_2 respectively.

Error in AOA estimation deceases (ex. $\theta_1 - \theta_2 = 0.09$ deg.)

Error in the deteched distance deceases ($d_1 - d_2 = 1.2$ m)

More precise in dectecting the required mobile terminal.