Suboptimal Maximum Likelihood Detection Using Gradient-based Algorithm for MIMO Channels

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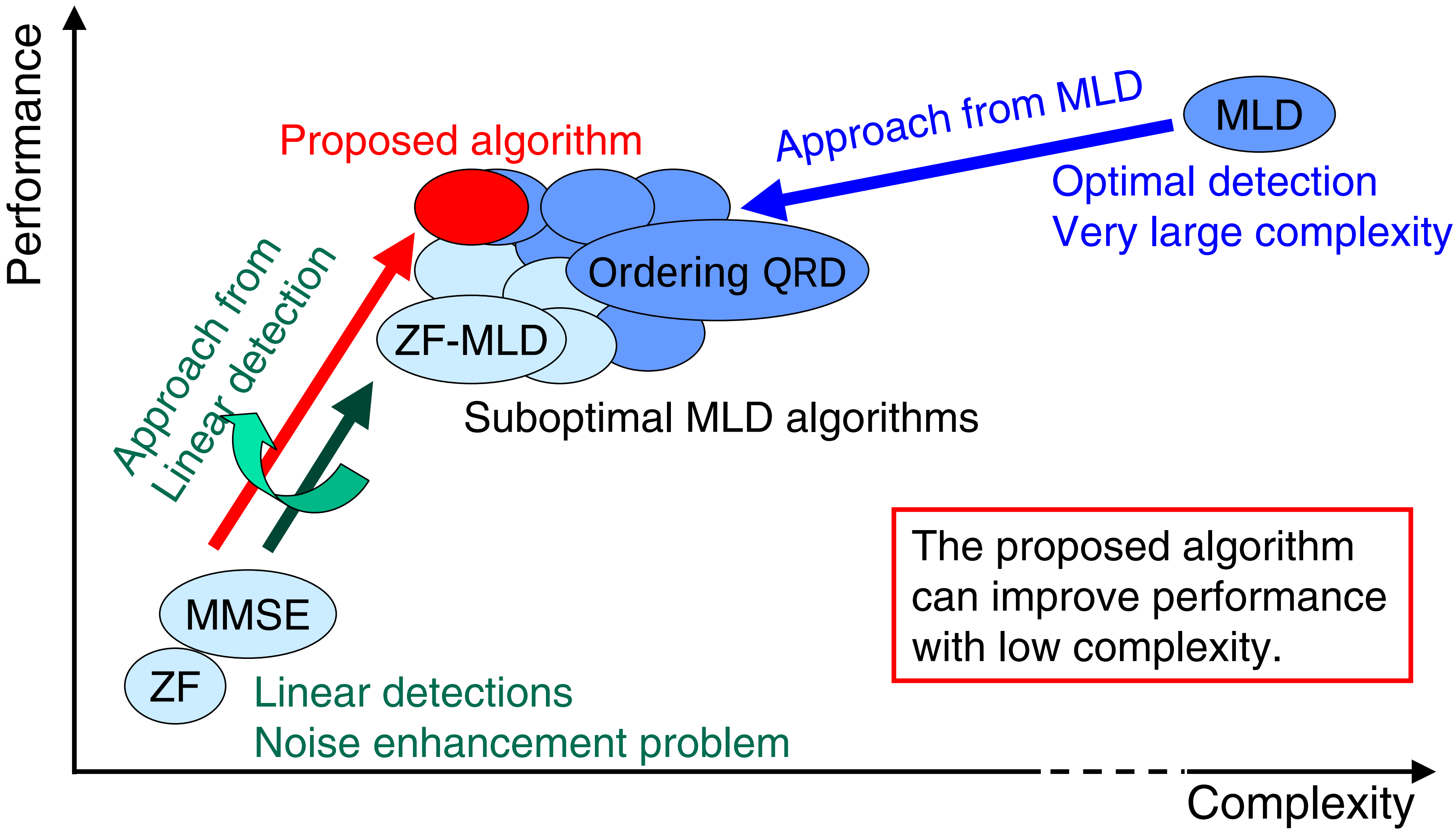
Background

MIMO signal detection



The proposed algorithm can improve performance with low complexity.

Conventional Detection Algorithms

Approach from MLD

Ordering QRD algorithm

- QRD: $\mathbf{H} = \mathbf{QR}$

Unitary matrix (Q) — Upper triangular matrix (R)

- feedback detection

$$\mathbf{Q}^{\mathrm{H}}\mathbf{y}(i) = \mathbf{Rs}(i) + \mathbf{Q}^{\mathrm{H}}\mathbf{n}(i)$$

- ordering of the channel matrix

(based on MMSE)

$$\mathbf{G} = (\mathbf{H}^{\mathrm{H}}\mathbf{H} + \sigma_n^2\mathbf{I})^{-1}\mathbf{H}^{\mathrm{H}}$$

Lower $(\mathbf{GG}^{\mathrm{H}})_{ii}$ ➡ Higher SNR

QRD and ordering H still needs high complexity.

Approach from linear detection

ZF-MLD algorithm

- Initial detection of ZF

$$\mathbf{s}_{ZF}(i) = \mathrm{Dec}\left[(\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{H}}\mathbf{y}(i)\right]$$

- searching similar vectors to $\mathbf{s}_{ZF}$

$\{\mathbf{s}_{near}\}$: set of signal vectors that differ from $\mathbf{s}_{ZF}$ only in one symbol.

- MLD execution

$$\hat{\mathbf{s}}(i) = \arg\min_j \| \mathbf{y}(i) - \mathbf{Hs}_j \|^2$$

$$\mathbf{s}_j \in \{\mathbf{s}_{near}\}$$

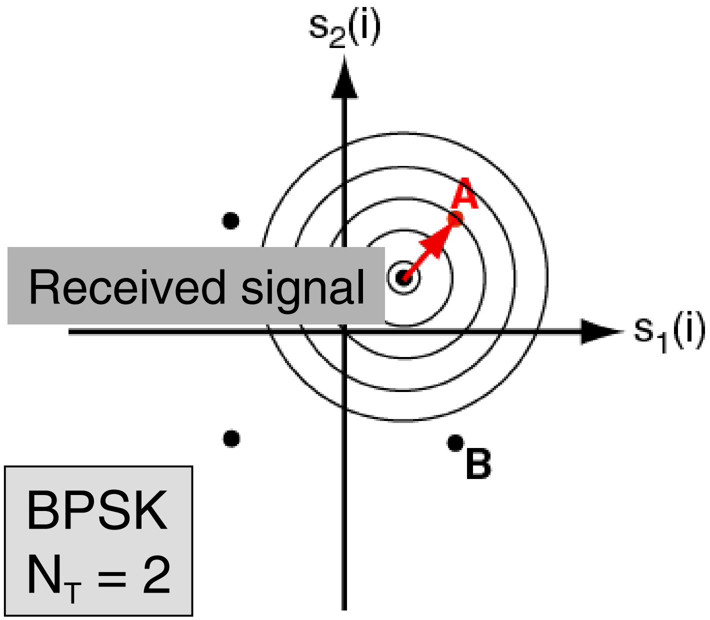
Noise enhancement occurs & BER performance is still poor.

Proposed Algorithm

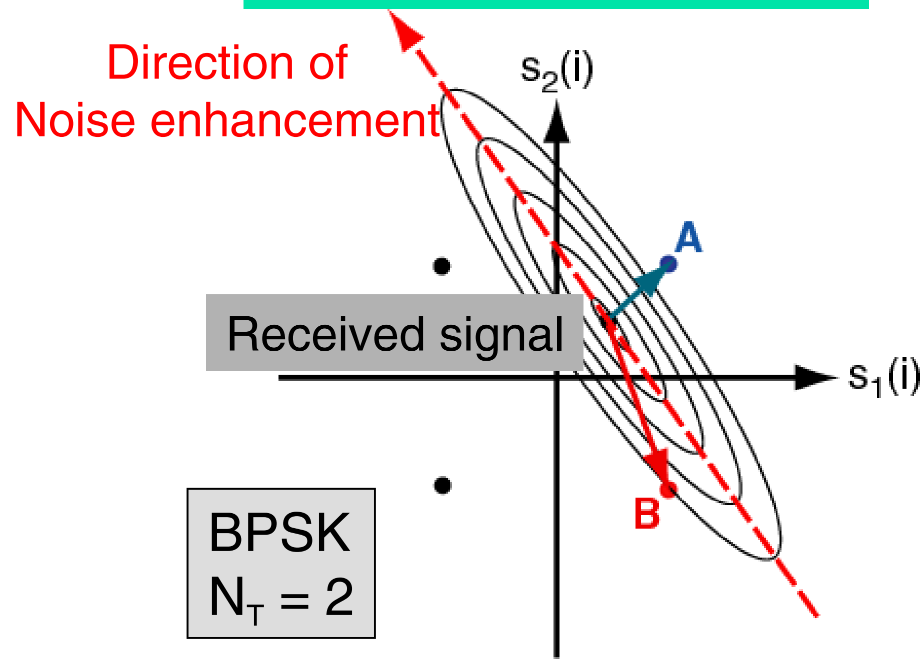
To reduce complexity

- approaches from linear detection (ZF/MMSE)

Without noise enhancement



With noise enhancement



To overcome noise enhancement

- search the signal candidates in the direction of noise enhancement
- search the signal candidate that minimizes the metric by gradient-based method

Analysis of Noise Enhancement

SVD of channel matrix: $\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{H}}$

Diagonal matrix:

$$\boldsymbol{\Sigma} = \begin{bmatrix} \mathbf{D} & \mathbf{O}_{W,N_T-W} \\ \mathbf{O}_{N_R-W,W} & \mathbf{O}_{N_R-W,N_T-W} \end{bmatrix}$$

$$\mathbf{D} = \mathrm{diag}\left[\lambda_1^{1/2}\ \lambda_2^{1/2}\ \cdots\ \lambda_W^{1/2}\right],$$

$$\mathrm{rank}(\mathbf{H}) = W (\leq \min(N_T, N_R))$$

Unitary matrices:

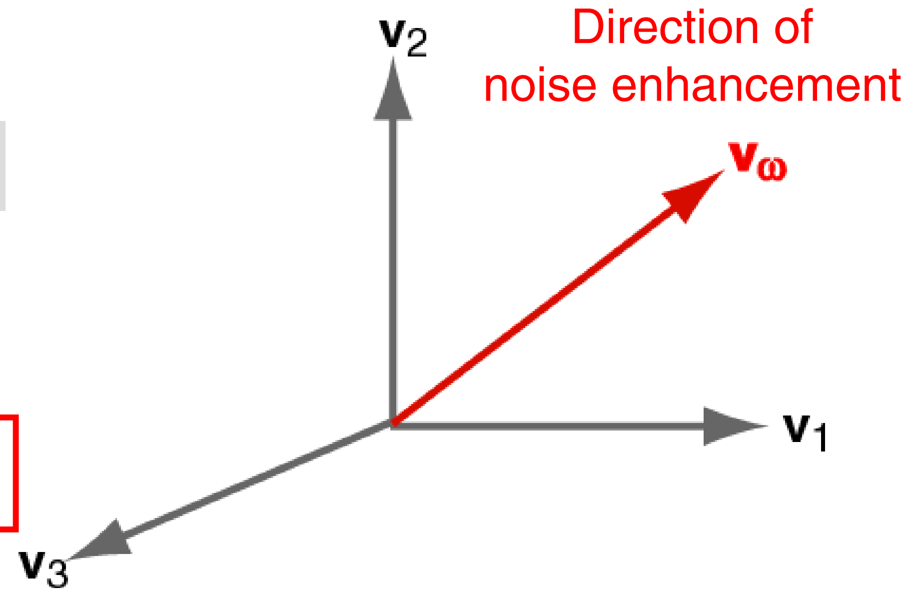
$$\mathbf{U} = \left[\mathbf{u}_1\ \mathbf{u}_2 \cdots \mathbf{u}_{N_R}\right]$$

$$\mathbf{V} = \left[\mathbf{v}_1\ \mathbf{v}_2 \cdots \mathbf{v}_{N_T}\right]$$

The noise term in MMSE detection:

$$\left(\mathbf{H}^{\mathrm{H}}\mathbf{H} + \sigma_n^2\mathbf{I}\right)^{-1}\mathbf{H}^{\mathrm{H}}\mathbf{n}(i)$$

$$\simeq \sum_{w=1}^{W} \mathbf{v}_w \lambda_w^{-1/2}\left[\mathbf{u}_w^{\mathrm{H}}\mathbf{n}(i)\right] \quad (\sigma_n^2 \simeq 0)$$



The noise is enhanced in the direction of $\mathbf{v}_w$ with very small λ_w.

Gradient-based Algorithm

Calculate several values of μ_r

$0 \leq r \leq (M-1)N_T$

$$\mu_r = \left[a(r) - \hat{\mathbf{x}}_k\right] / \mathbf{g}(i)_k$$

$$a(r) = \left[\tilde{\mathbf{x}}(r)\right]_k$$

Next guess

$\tilde{\mathbf{x}}(r) = \hat{\mathbf{x}} + \mu_r \mathbf{g}(i)$

Initial guess by MMSE : $\hat{\mathbf{x}}$

Gradient vector :

$$\mathbf{g}(i) = -\mathbf{P}^{q} \frac{\partial L(\mathbf{x})}{\partial \mathbf{x}^*}\bigg|_{\mathbf{x}=\hat{\mathbf{s}}(i,0)}$$

$$\mathbf{P} = \left(\mathbf{H}^{\mathrm{H}}\mathbf{H} + \sigma_n^2 \mathbf{I}_{N_T}\right)^{-1}$$

$$L(\mathbf{x}) = \| \mathbf{y}(i) - \mathbf{H}\mathbf{x} \|^2$$

Hard decision

$\hat{\mathbf{s}}(i,r) = \mathrm{Dec}[\tilde{\mathbf{x}}(\mathbf{r})]$

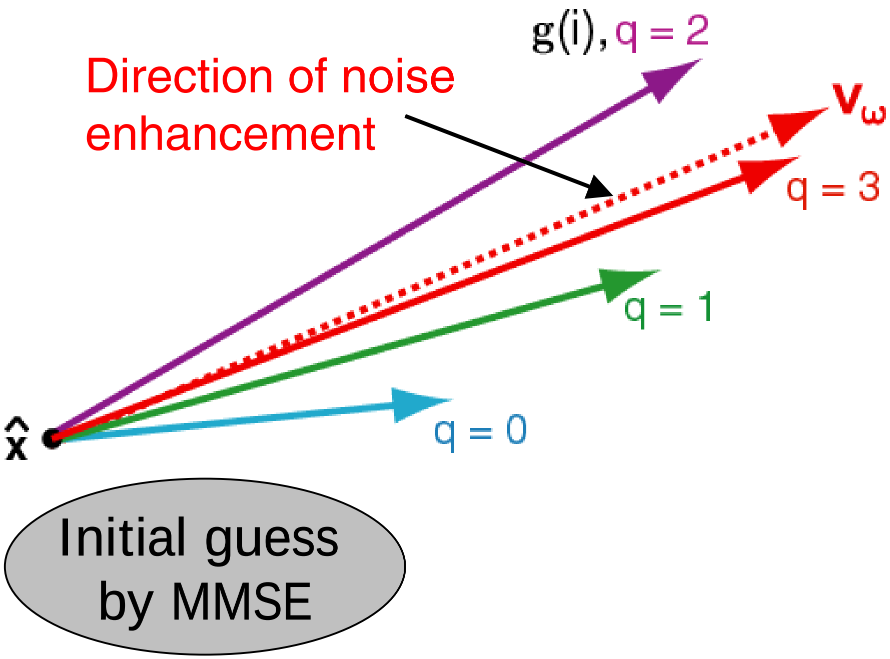
Final stage

$$\hat{\mathbf{s}}(i) = \arg\min_{\hat{\mathbf{s}}(i,r)} \|\mathbf{y}(i) - \mathbf{H}\hat{\mathbf{s}}(i,r)\|^2$$

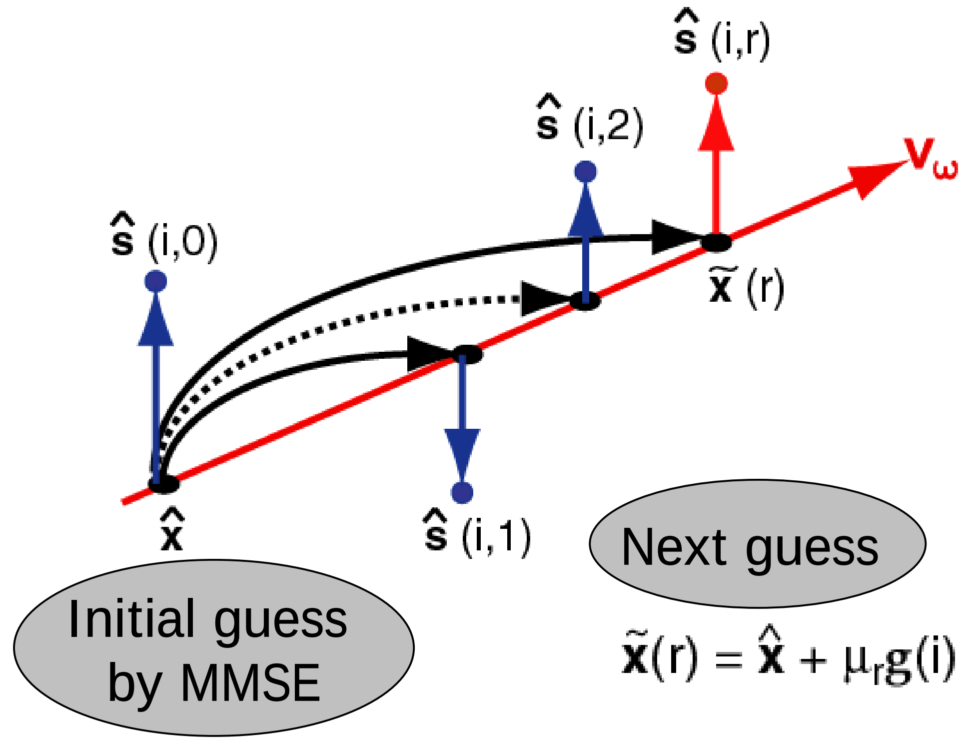
The finally detected signal that minimizes the metric is selected.

The Gradient Vector

Direction of the gradient vector



Searching for the next guess



The direction of the gradient vector vary with parameter q.

The next guess is obtained by initial guess and gradient vector.

Recursive Form of Initial Guess

- The initial guess $\hat{\mathbf{x}}$ is given by MMSE.

$$\hat{\mathbf{x}} = \mathbf{P}\mathbf{H}^{\mathrm{H}}\mathbf{y}(i) \qquad \mathbf{P} = \left(\mathbf{H}^{\mathrm{H}}\mathbf{H} + \sigma_n^2 \mathbf{I}_{N_T}\right)^{-1}$$

- Its RLS-like recursive form that can reduce complexity.

Initial conditions: $\mathbf{P}(0) = \sigma_n^{-2}\mathbf{I}_{N_T}, \mathbf{x}(0) = \mathbf{0}_{N_T}$

$$\mathbf{k}(l) = \frac{\mathbf{P}(l-1)\mathbf{h}_l}{1 + \mathbf{h}_l^{\mathrm{H}}\mathbf{P}(l-1)\mathbf{h}_l} \qquad \mathbf{z}(l) = \mathbf{z}(l-1) + \mathbf{k}(l)e^*(l)$$

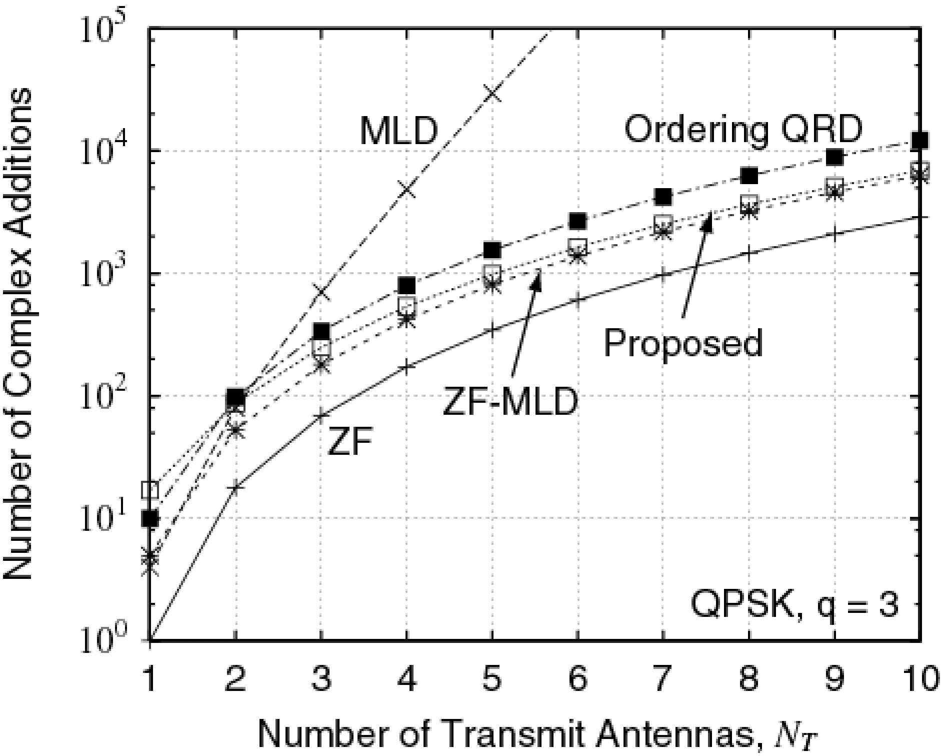
$$e(l) = y_l^*(i) - \mathbf{z}^{\mathrm{H}}(l-1)\mathbf{h}_l \qquad \mathbf{P}(l) = \mathbf{P}(l-1) - \mathbf{k}(l)\mathbf{h}_l^{\mathrm{H}}\mathbf{P}(l-1)$$

$$\mathbf{z}(l) = \left(\sum_{l=1}^{N_R} \mathbf{h}_l\mathbf{h}_l^{\mathrm{H}} + \sigma_n^2\mathbf{I}_{N_T}\right)^{-1} \sum_{l=1}^{N_R} \mathbf{h}_l\mathbf{y}_l(i)$$

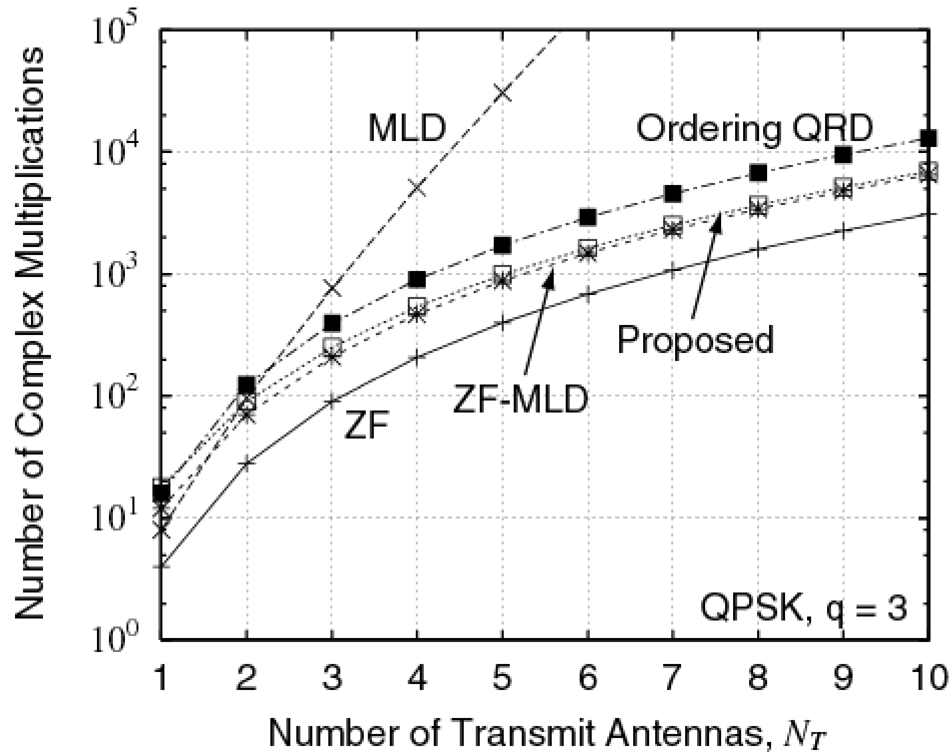
- The desired initial guess: $\hat{\mathbf{x}} = \mathbf{z}(N_R)$

Computational Complexity

Complex additions



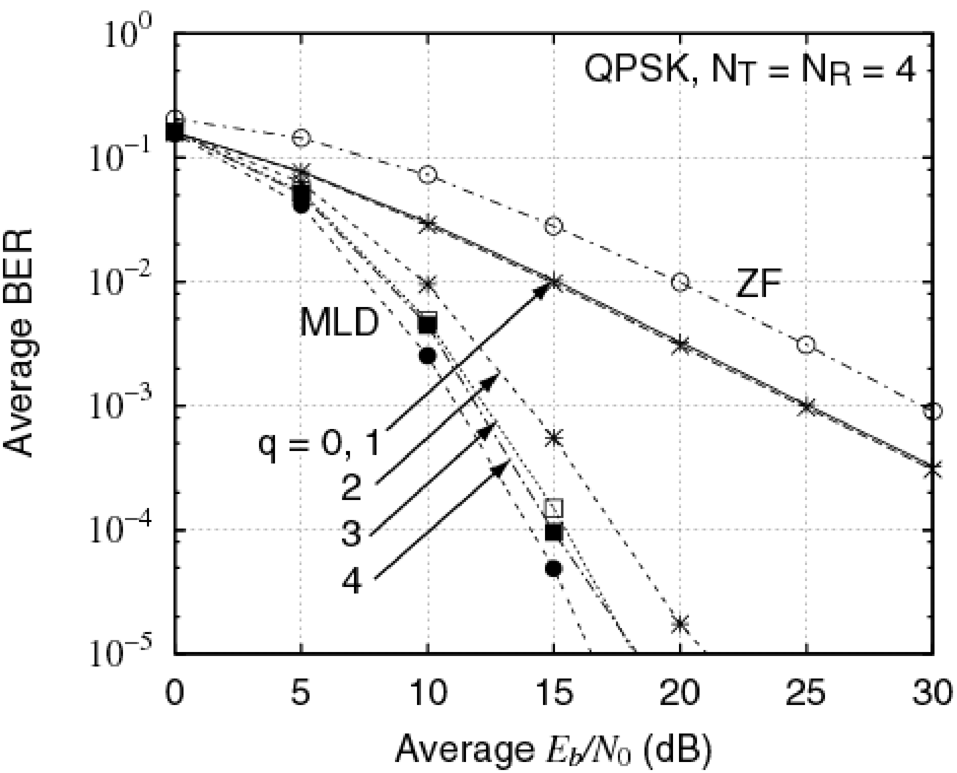
Complex multiplications



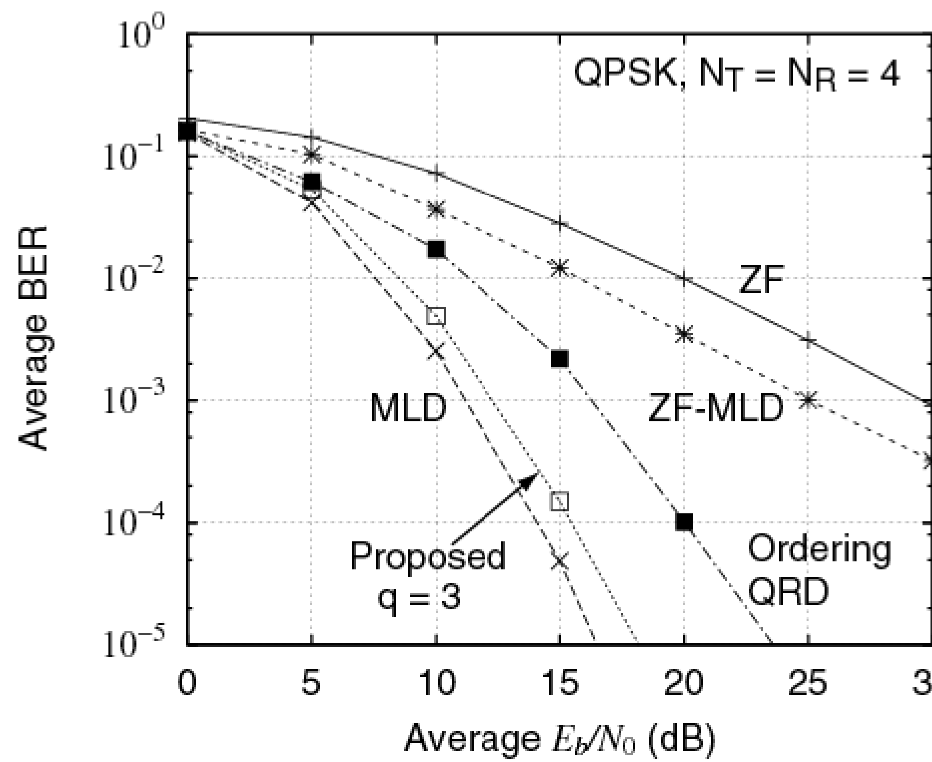
The computational complexity of the proposed algorithm is less than that of the ordering QRD and almost same as that of ZF-MLD.

BER Performances

with *q* as a parameter



with QPSK scheme



As *q* increases, the BER improves. BER improvement is saturated with $q = 3$.

The proposed algorithm is superior in BER performance to the conventional ones.

Conclusion

- A suboptimal MIMO MLD algorithm, which uses gradient-based method, is proposed.
- The proposed algorithm sets the initial guess to the solution by MMSE algorithm to reduce complexity.
- Signal candidates are searched in the noise-enhanced direction by the gradient-based method to overcome noise enhancement problem.
- The computational complexity of the proposed algorithm is less than that of ordering QRD and almost same as that of ZF-MLD.
- The proposed algorithm is superior in BER performance to the conventional ones.

Thank You for Your Attention